Housing Demand and Expenditures: How Rising Rent Levels Affect Behavior and Costs-of-Living over Space and Time

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Figure 1: Expenditure Share on Housing 1970-2013

Note: For non-BEA series, a moving average with weight of 0.5 for the year after and the year before is shown in the curve.
“Academics typically deem rent unaffordable if it eats up more than 30% of a household’s income.” – The Economist
Is housing a luxury?

Warren Buffet’s house; bought in 1957 for $31,500 (now worth 10 times that). His net worth is $47 billion.
NYC Basic Tips and Etiquette

How hard is it to find a place to live?

Your standards

An inverse relationship

Your desperation to live in NYC

Looks good!

Facebook: NYCBSICITIPSANDETIQUETTE
Is there a secular change in housing demand?

Continuous, but slowing, secular decrease in the number of people per household. Fraction living alone continues to rise.
Rising Prices: Is supply keeping up with demand?

A secular decrease in the number of housing starts per household: the “boom” was a return to the 1980s.
Affordability an issue in many other countries, e.g. the U.K.

“being 'housing pinched' is something that predominantly affects working age households in the bottom half of the income distribution.” – Resolution Foundation

Figure 1:
Growth of the proportion of households spending more than 33 per cent of net income on housing costs, UK

“being 'housing pinched' is something that predominantly affects working age households in the bottom half of the income distribution.”

Resolution Foundation
Housing’s Rising Income Share

- Odd if income is growing and housing is a necessity.
  - We do see that the relative price of housing is rising.
- Increasing income shares: greater income from land (& structures)
  - The return of the Ricardian “landlord” class? (Henry George!)
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The “Rental Affordability Crisis” (not just high housing prices)

- Is there a crisis? Is this due to prices, incomes, or what?
- High rents hurt more without good substitutes for housing.
- How should we adjust cost-of-living indices over space and time?
  - Rising prices may hurt poor more than rich.
  - Revisit the role of inequality and how it has changed.
Introduction: Econometric and Modeling Issues

The Demand for Housing

- What is the best data or evidence housing demand?
  - Experimental evidence? rare, narrowly-focused, and short term
  - Observational evidence? Estimation over space and time?
- Observe expenditures = quantity*price, quantity alone hard
- Determinants: price and (permanent) income – how to measure?
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Modeling Housing in Economic Models (Urban, Macro, Public...)

- What is the right model to use for housing demand? Common are:
  - Fixed demand: inelastic to price and income
  - Cobb-Douglas: unit elastic; constant income share
  - Neither seems compelling; is one a better approximation?
- Is there a better specification of preferences?
Introduction and Motivation

Model

Data

Estimation and Results

Putting the Parameter Estimation into Use

Conclusion
Figure 2: Housing Consumption with Production Possibility Expansions

LEGEND
A = original bundle
C = new bundle
A to B = income effect (necessity)
B to C = substitution effect (inelastic)
A to D = income effect (neutral)
D to E = substitution effect (unit)
Figure 2: Housing Consumption with Production Possibility Expansions

An economic model illustrating the effects of production possibility frontier expansions. The graph shows the original and new budget constraints, along with indifference curves. The key points are labeled as follows:

- **A**: Original bundle
- **C**: New bundle
- **A to B**: Income effect (necessity)
- **B to C**: Substitution effect (inelastic)
- **A to D**: Income effect (neutral)
- **D to E**: Substitution effect (unit)

The LEGEND explains the symbols and effects:

- **A** = original bundle
- **C** = new bundle
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Non-housing

New production possibility frontier

Housing

C

A

B

New Indifference Curve

Original production possibility frontier

Equivalent new budget constraint at original relative prices

New Indifference Curve

New budget constraint

Income expansion path

Original budget Constraint

New production possibility frontier

Original production possibility frontier

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Figure 2: Housing Consumption with Production Possibility Expansions
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Non-housing

New production possibility frontier

Housing

Constant expenditure share at changing prices

Constant expenditure share at original prices

Income expansion path

Original production possibility frontier

New production possibility frontier

Original budget Constraint

New budget Constraint

Equivalent new budget constraint at original relative prices

LEGEND

A = original bundle
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Innovations

Theoretical

- Use demand theory to explain variation over time.
  - Time: Uncompensated demand varying income
  - Space: Compensated demand varying utility

- Utility function: Non-homothetic constant & elasticity of substitution

Empirical

- Use data on non-housing prices to test demand restrictions
- Interpret permanent income through utility function (OLS vs IV)
- Decompose change over time into 1) income, 2) compensated price, and 3) time or taste-change effects.

An ideal cost-of-living index to with income & substitution effects better than Consumer Price Index for studying inequality.
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- An ideal cost-of-living index to with income & substitution effects
  - Better than Consumer Price Index for studying inequality.
Housing Demand: An Older Literature

- **Price elasticity**
  - Inelastic (< 1), i.e. Pollinsky and Ellwood (1979): similar empirics; Hanushek and Quigley (1980) use adjustment model.
  - Unit or elastic (≥ 1), i.e. Harmon (1989) using PSID; Davis and Ortalo-Magne (2011), similar empirics, control for distance.

- **Income elasticity**
  - Inelastic (< 1) Engel (1857), Schwabe (1868) – became known as “Schwabe’s Law of Rent” Hansen et al. (1998)
  - Unit or elastic, i.e. Larsen (2002), Cheshire and Sheppard (1998)
Related Literature

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Important modeling uses outside of housing economics

- **Urban:** rent and pop. density gradients; inter-urban density; sorting
- **Public:** demand for public goods through sorting and property taxes
- **Macro:** price-responsiveness to shocks; importance supply vs. credit.
Related Literature

Cost-of-Living Heterogeneity: A New Literature

- Baum-Snow and Pavan (2013) Inequality gaining in big cities.
- Handbury (2013) Non-homothetic demand makes expensive cities cheaper for rich through greater variety of groceries.
- Moretti (2013) Increases in real-income inequality lowered by sorting of skilled to expensive cities.
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Rising income share

- Piketty and Zucman (2013) Housing an increasing share of capital.
- Davis and Palumbo (2009) Land rents may be rising, too.
What Are Cities Worth? Housing Costs versus Wage Levels across Metro Areas, 2000

Log Wage Differential

Log Housing-Cost Differential

METRO POP

- >5.0 Million
- 1.5-5.0 Million
- 0.5-1.5 Million
- <0.5 Million

Avg Mobility Cond: slope = 1.53

Avg Zero-Profit Cond: slope = -7.37
Fraction of Income Willing to Pay to Live in Area, Relative to National Average

0.33 - 0.34
0.31 - 0.32
0.29 - 0.30
0.27 - 0.28
0.25 - 0.26
0.23 - 0.24
0.21 - 0.22
0.19 - 0.20
0.17 - 0.18
0.15 - 0.16
0.13 - 0.14
0.11 - 0.12
0.09 - 0.10
0.07 - 0.08
0.05 - 0.06
0.03 - 0.04
0.01 - 0.02
-0.01 - 0.00
-0.03 - -0.02
-0.05 - -0.04
-0.07 - -0.06
-0.09 - -0.08
-0.11 - -0.10
-0.13 - -0.12
-0.15 - -0.14
-0.17 - -0.16
-0.19 - -0.18

Unaffordabilidity (Quality of Life) across the United States, 2010
Figure 2: Housing Prices vs. Land Values

METRO POP
- <0.5 Million
- 0.5-1.5 Million
- 1.5-5 Million
- >5.0 Million

Linear Fit: Slope = 0.589 (0.045)
Quadratic Fit:
Slope at Zero = 0.575 (0.038),
Elasticity of Sub = 0.184 (0.547)
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Household Budgets and Preferences

Follow the model of housing demand in Rosen (1979) and Albouy (2008):

- Cities are indexed by $j$, with potentially mobile households.
  
  Housing good $y$ with price $p^j$,
  Non-housing good $x$ with price $c^j$,
  Total income $m^j$ varies from wage.
  Quality of life $Q^j$ varies by place, may be vector.
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  - Housing good $y$ with price $p^j$,
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  - Total income $m^j$ varies from wage.
  - Quality of life $Q^j$ varies by place, may be vector.

- From the utility function $U(x, y; Q^j)$ and budget constraint $c^j x + p^j y = m^j$ we obtain
  - Indirect utility function $V(c^j, p^j, m^j; Q^j)$
  - Expenditure function $e(c^j, p^j, u; Q^j)$
Expenditures share on housing in city $j$ is $s^j_Y \equiv \frac{p^j y^j}{m^j}$.

- By definition, log-linearizing the difference in shares:

$$\hat{s}^j_Y = \hat{p}^j + \hat{y}^j - \hat{m}^j$$

(1)

the hat represents log deviations from the national average.
Expenditures share on housing in city $j$ is $s^j_y \equiv \frac{p^j y^j}{m^j}$.

- By definition, log-linearizing the difference in shares:
  \[
  \hat{s}^j_y = \hat{p}^j + \hat{y}^j - \hat{m}^j
  \] (1)

  the hat represents log deviations from the national average.

- Log-linearizing Uncompensated demand $y^j = y(c^j, p^j, m^j; Q^j)$
  \[
  \hat{y}^j = \epsilon_{y,p} \hat{p}^j + \epsilon_{y,c} \hat{c}^j + \epsilon_{y,m} \hat{m}^j + \epsilon_{y,Q} \hat{Q}^j
  \] (2)

  $\epsilon_{y,p}, \epsilon_{y,c}, \epsilon_{y,m}, \epsilon_{y,Q}$: uncompensated own and cross-price elasticity of demand

  the income and amenity elasticity of demand
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- Log-linearizing Uncompensated demand $y^j = y(c^j, p^j, m^j; Q^j)$

$$\hat{y}^j = \epsilon_{y,p} \hat{p}^j + \epsilon_{y,c} \hat{c}^j + \epsilon_{y,m} \hat{m}^j + \epsilon_{y,Q} \hat{Q}^j \quad (2)$$

$\epsilon_{y,p}$, $\epsilon_{y,c}$: uncompensated own and cross-price elasticity of demand
$\epsilon_{y,m}$, $\epsilon_{y,Q}$: the income and amenity elasticity of demand

- Combining (1) and (2) gives the combined identity:

$$\hat{s}^j_y = (1 + \epsilon_{y,p}) \hat{p}^j + \epsilon_{y,c} \hat{c}^j + (\epsilon_{y,m} - 1) \hat{m}^j + \epsilon_{y,Q} \hat{Q}^j \quad (3)$$

- Restriction: demand homogenous of degree zero $\epsilon_{y,p} + \epsilon_{y,c} + \epsilon_{y,m} = 0$
Empirical Model of Uncompensated Demand

Adding a constant to (3) motivates the regression,

\[ \ln s_j^y = \alpha_0 + \alpha_1 \ln p_j^y + \alpha_2 \ln c_j^y + \alpha_3 \ln m_j^y + \alpha_4 q_j^y + e_j \]  

(4)

\[ = \alpha_0 + \alpha_1 (\ln p_j^y - \ln c_j^y) + \alpha_3 (\ln m_j^y - \ln c_j^y) + \alpha_4 q_j^y + e_j \]  

(5)

where \( \alpha_0 = \ln \bar{s}_y \), \( \alpha_1 = 1 + \epsilon_{y,p} \), \( \alpha_2 = \epsilon_{y,c} \), and \( \alpha_3 = \epsilon_{y,m} - 1 \).
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Equation 5 imposes the homogeneity assumption \( \alpha_1 + \alpha_2 + \alpha_3 = 0 \).

- \( \bar{s}_y = e^{\alpha_0} \) = the geometric mean of expenditure shares.
- \( \epsilon_{y,p} = \alpha_1 - 1 \) = own-price uncompensated elasticity.
- \( \epsilon_{y,m} = \alpha_3 + 1 \) = income elasticity.
- \( \epsilon_{y,Q} = \alpha_4 \) complementarity with amenities (e.g., mountains)

Division bias from \( m^j \) a potential issue.

- May be inevitable for time-series; can do better for cross-section.
Compensated Demand and Expenditures

- Log-linearizing the expenditure function $m^j = e(p^j, c^j, u^j; Q^j)$

\[
\hat{m}^j = \bar{s}_y \hat{p}^j + (1 - \bar{s}_y) \hat{c}^j + \epsilon_{m,u} \hat{u}^j + \epsilon_{m,Q} \hat{Q}^j
\]  \hspace{1cm} (6)

- With mobility $u$ should not vary by city, only type $\zeta_i$.

Combining equations 3 and 6 with Slutsky equations, and imposing homogeneity $\epsilon^{H}_{y,p} + \epsilon^{H}_{y,c} = 0$

\[
\hat{s}_y^j = (1 - \bar{s}_y + \epsilon^{H}_{y,p})(\hat{p}^j - \hat{c}^j) + (\epsilon^{H}_{y,u} - \epsilon_{m,u})\hat{u}^j + (\epsilon^{H}_{y,Q} - \epsilon_{m,Q})\hat{Q}^j
\]  \hspace{1cm} (7)
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- Log-linearizing the expenditure function \( m^j = e(p^j, c^j, u^j; Q^j) \)

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\]  

(7)

Using a money-metric utility function, replacing \( \epsilon_{m,u}\hat{u}^j = \hat{\zeta}^j \) as an index of location-invariant skill which increases earnings proportionally

\[
\hat{s}^j_y = (1 - \bar{s}y + \epsilon^H_{y,p})(\hat{p}^j - \hat{c}^j) + (\epsilon_{y,m} - 1)\hat{\zeta}^j + (\epsilon^H_{y,Q} - \epsilon_{m,Q})\hat{Q}^j
\]  

(8)
City A (Columbus) average QOL = 1, and average wage = 1
City B (Chicago) average QOL = 1, and high wage = 2
City C (Honolulu) high QOL = 1, and average wage = 1
Empirical Model of Compensated Demand

Equation 8 motivates a regression of $\ln(s_y)$ on

$$\ln s_y^j = \beta_0 + \beta_1 \hat{p}^j + \beta_2 \hat{c}^j + \beta_3 \hat{\zeta}^j + \beta_4 q^j + e^j$$  \hspace{1cm} (9)

$$= \beta_0 + \beta_1 (\hat{p}^j - \hat{c}^j) + \beta_3 \hat{\zeta}^j + \beta_4 q^j + e^j$$ \hspace{1cm} (10)

Where $\beta_0 = \ln \bar{s}_y$, $\beta_1 = \epsilon_{y,p}^H + 1 - s_y = -\beta_2$ and $\beta_3 = \epsilon_{y,m}$. Permanent income measure used is location independent. Price terms look similar to the uncompensated case, but interpreted differently

- $\bar{s}_y = e^{\beta_0}$
- $\epsilon_{y,m} = \beta_3 + 1$
- $\epsilon_{y,p}^H = \beta_1 - 1 + e^{\beta_0}$.

Elasticity of substitution estimated directly from prices only

$$\sigma_D \equiv - \frac{\hat{y}^j - \hat{x}^j}{\hat{p}^j - \hat{c}^j} = - \frac{\epsilon_{y,p}^H}{1 - \bar{s}_y} = 1 - \frac{\beta_1}{1 - e^{\beta_0}}$$ \hspace{1cm} (11)
We can correct for one-and-a-half of these biases

1. Modeling covariance between housing prices and non-housing prices,

\[ \hat{c}^j = \rho \hat{p}^j + \nu^j \]  

(12)

where \( \rho \) may be non-zero and \( \nu^j \) is white noise. The estimated elasticity of substitution,

\[ \sigma_D = 1 - \frac{\beta_1}{(1 - e^{\beta_0})(1 - \rho)} \]

Greater \( \rho \) biases \( \hat{\beta}_1 \) towards zero and \( \hat{\sigma}_D \) towards one.
Potential Biases: Unobserved Prices, Skills, and Tastes

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3. Weak preference for housing sort into higher-price cities, \( \sigma \) biased up.
   
   Reason to be wary of within-city estimates: greater household sorting!
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3. Weak preference for housing sort into higher-price cities, \( \sigma \) biased up.
   - Reason to be wary of within-city estimates: greater household sorting!

4. Renter-status more likely for higher income in expensive cities, \( \sigma \), biased up.
Non-Homothetic Utility and Expenditure Function

Non-homothetic separable family CES (Sato 1974, 1977), with $Q^j = 1$,

$$U = \left[ \frac{\delta_1 x^{\frac{\sigma-1}{\sigma}} + \theta_1}{\delta_2 y^{\frac{\sigma-1}{\sigma}} + \theta_2} \right]^{\frac{\sigma}{\gamma}} \tag{13}$$

where $\theta_i = (1 - \sigma - \gamma \delta_i)/(\gamma \sigma), i = 1, 2$ and $\delta_1 = \delta_2 + 1$

Log-linear way to identify parameters (full non-linear estimates soon...)

$$\beta_0 = \sigma \ln(-\delta_2) = \ln \bar{s}_y, \beta_1 = (1 - \bar{s}_y)(1 - \sigma), \beta_3 = -\gamma(1 - \bar{s}_y)(1 - \sigma)\epsilon_{u,m}$$
Non-Homothetic Utility and Expenditure Function

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$$U = \left[ \frac{\delta_1 \times \frac{\sigma}{\sigma} - 1 + \theta_1}{\delta_2 y \frac{\sigma}{\sigma} - 1 + \theta_2} \right]^{\frac{\sigma}{\gamma(\sigma-1)}} \tag{13}$$

where $\theta_i = (1 - \sigma - \gamma \delta_i)/(\gamma \sigma)$, $i = 1, 2$ and $\delta_1 = \delta_2 + 1$

Log-linear way to identify parameters (full non-linear estimates soon...)

$\beta_0 = \sigma \ln(-\delta_2) = \ln \bar{s}_y$, $\beta_1 = (1 - \bar{s}_y)(1 - \sigma)$, $\beta_3 = -\gamma(1 - \bar{s}_y)(1 - \sigma)\epsilon_{u,m}$

Inverting, the parameters can be expressed recursively as

$$\sigma = 1 - \beta_1/(1 - e^{\beta_0}), \quad \delta_2 = -e^{\beta_0}/\sigma, \quad \gamma = -\epsilon_{m,u}\beta_3/\beta_1$$

Expenditure function for calculating $\epsilon_{m,u}$ and ideal cost-of-living index

$$e(p, c, u; 1) = \left\{ \frac{c^{1-\sigma} \delta_1^\sigma u^{\gamma(1-\sigma)} - p^{1-\sigma} \delta_2^\sigma}{\theta_2 - \theta_1 u^{\gamma(1-\sigma)/\sigma}} \right\}^{\frac{1}{1-\sigma}} \tag{14}$$
Outline

1 Introduction and Motivation
2 Model
3 Data
4 Estimation and Results
5 Putting the Parameter Estimation into Use
6 Conclusion
Data for Metro-Level Indices

  - median and aggregate rental or housing expenditure
  - rental price index, controlling for housing structure
  - predicted wage differentials based on worker characteristics (skill index)
  - focus on renters rather than home-owners.
  - Using grouped data: less transitory and measurement error
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- 1982-2012 Panel of Interarea Price Indices (Carrillo et al. 2013)
  - Non-housing goods price index across metro areas (ACCRA)
  - Rental index from Section 8 Consumer Satisfaction Survey (CSS)

- Alternative data for housing and workers.
  - Consumer Expenditure Survey (CEX)
  - American Housing Survey (AHS)
  - Bureau of Economic Analysis (BEA) time series only.
Rental and Worker-Skill Indices

Rental-price index from hedonic regression model:

$$\ln(P_{ij}) = \alpha_P + \beta_{PX}X_{ij}^P + \delta^j_P + \epsilon^j_P$$  \hspace{1cm} (15)

- $P_{ij}$ gross rent for unit $i$ in area $j$ (or user-cost for housing)
- $X_{ij}^P$ a vector of unit characteristics
- $\delta^j_P$ area fixed effects: use for rental index.
Rental and Worker-Skill Indices

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- \( \delta_P^j \) area fixed effects: use for rental index.

Skill or human-capital index, running regressions of the form:

\[ \ln(W_{ij}) = \alpha_W + \beta_{WX} X_{ij}^W + \delta_W^j + \epsilon_W^i \]  \hspace{1cm} (16)

- \( W_{ij} \) hourly nominal wage for person \( i \) in area \( j \)
- \( X_{ij}^W \) is a vector of personal characteristics, use mean value for worker-skill index
- \( \delta_W^j \) is a set of area fixed effects
Data and Aggregation Issues

- Micro versus macro elasticities.
  - Micro tends to produce lower elasticities (same here).
  - Is the difference due to sorting or data issues?
  - Some advantages in averaging.

Price and income elasticity may change with income. Utility: function predicts better off more price elastic.
Can use other percentiles (e.g. 25th or 75th)
Under-reporting of income v. important for poor.
Demographic characteristics: levels, interactions
Household size and number of children (higher)
Race, ethnicity (lower)
How to incorporate home-owners/owner-occupiers?
Imputed rent on equity both income and consumption.
Spending may represent savings/investments.
User-cost depends on capital gains, taxes, etc.
Better to invest in a growing (high-price?) area.
Data and Aggregation Issues

- Micro versus macro elasticities.
  - Micro tends to produce lower elasticities (same here).
  - Is the difference due to sorting or data issues?
  - Some advantages in averaging.

- Price and income elasticity may change with income
  - Utility: function predicts better off more price elastic
  - Can use other percentiles (e.g. 25th or 75th)
  - Under-reporting of income v. important for poor.
Data and Aggregation Issues

- Micro versus macro elasticities.
  - Micro tends to produce lower elasticities (same here).
  - Is the difference due to sorting or data issues?
  - Some advantages in averaging.

- Price and income elasticity may change with income
  - Utility: function predicts better off more price elastic
  - Can use other percentiles (e.g. 25th or 75th)
  - Under-reporting of income v. important for poor.

- Demographic characteristics: levels, interactions
  - Household size and number of children (higher)
  - Race, ethnicity (lower)
Data and Aggregation Issues

- Micro versus macro elasticities.
  - Micro tends to produce lower elasticities (same here).
  - Is the difference due to sorting or data issues?
  - Some advantages in averaging.

- Price and income elasticity may change with income
  - Utility: function predicts better off more price elastic
  - Can use other percentiles (e.g. 25th or 75th)
  - Under-reporting of income v. important for poor.

- Demographic characteristics: levels, interactions
  - Household size and number of children (higher)
  - Race, ethnicity (lower)

- How to incorporate home-owners/owner-occupiers?
  - Imputed rent on equity both income and consumption.
  - Spending may represent savings/investments.
  - User-cost depends on capital gains, taxes, etc.
  - Better to invest in a growing (high-price?) area.
Outline

1. Introduction and Motivation
2. Model
3. Data
4. Estimation and Results
5. Putting the Parameter Estimation into Use
6. Conclusion
Figure 3A: Median Share of Income Spent on Rent and the Relative Price of Housing, Renters Only 2000

Figure 3B: Median Share of Income Spent on Housing and the Relative Price of Housing, All Households 2000

Median Expenditures on Gross Rent as a Share of Income

Median Expenditures on Housing as a Share of Income
<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(8)</th>
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</thead>
<tbody>
<tr>
<td>Rental/Housing Price Index</td>
<td>Log Median Rental Share</td>
<td>Log Median Rental Share</td>
<td>Log Median Rental Share</td>
<td>Log Median Housing Share</td>
<td>Log Median Rental Share (w/in PMSA)</td>
</tr>
<tr>
<td></td>
<td>0.234</td>
<td>0.231</td>
<td>0.223</td>
<td>0.489</td>
<td>0.192</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.020)</td>
<td>-(0.034)</td>
<td>(0.023)</td>
<td>(0.015)</td>
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<td>Non-Housing Price Index</td>
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<td>-0.223</td>
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<td>0.161</td>
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<td>-(0.034)</td>
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<td>-0.490</td>
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<td>(0.107)</td>
<td>(0.103)</td>
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<td>Homeownership</td>
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<td>Yes</td>
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<td>P-value of Test of Homogeneity of Demand</td>
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<td>0.844</td>
<td>0.824</td>
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<td>Renters Only</td>
<td>Renters Only</td>
<td>Owners Only</td>
<td>Renters Only</td>
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<tr>
<td>Unit of Observation</td>
<td>MSA</td>
<td>MSA</td>
<td>MSA</td>
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<td>PUMA</td>
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<td>Implied Demand Parameters:</td>
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<td></td>
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<tr>
<td>Geometric Mean Expenditure Share</td>
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<td>0.223</td>
<td>0.223</td>
<td>0.161</td>
<td>0.223</td>
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<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Uncompensated Own Price Elasticity of Housing Demand</td>
<td>-0.687</td>
<td>-0.689</td>
<td>-0.699</td>
<td>-0.444</td>
<td>-0.699</td>
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<td>Income Elasticity of Housing Demand</td>
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<td>0.639</td>
<td>0.649</td>
<td>0.580</td>
<td>0.510</td>
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<td>(0.100)</td>
<td>(0.107)</td>
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<tr>
<td>Elasticity of Substitution Between Housing and Consumption Goods</td>
<td>0.703</td>
<td>0.713</td>
<td>0.418</td>
<td>0.753</td>
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<tr>
<td></td>
<td>(0.026)</td>
<td>(0.034)</td>
<td>(0.027)</td>
<td>(0.019)</td>
<td></td>
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</tbody>
</table>

Robust standard errors in parentheses. Test of homogeneity of demand is that the coefficient on both price indices sum to zero.
# TABLE 2: COMPENSATED DEMAND FUNCTIONS ESTIMATED CROSS-SECTIONALLY - ADDITIONAL YEARS, DATASETS, AND PRICE INDICES

<table>
<thead>
<tr>
<th>Dataset/Price Index:</th>
<th>Census 1980</th>
<th>Census 1990</th>
<th>ACS 2007-11</th>
<th>Alt Housing Price Index</th>
<th>CEO Housing Price Index</th>
<th>AHS</th>
<th>CEX</th>
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<td>Dependent Variable:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Rental/Housing Price Index</td>
<td>0.154</td>
<td>0.220</td>
<td>0.196</td>
<td>0.220</td>
<td>0.178</td>
<td>0.182</td>
<td>0.394</td>
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<td>(0.038)</td>
<td>(0.024)</td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.034)</td>
<td>(0.038)</td>
<td>(0.056)</td>
</tr>
<tr>
<td>Predicted Wage Index</td>
<td>-0.222</td>
<td>-0.258</td>
<td>-0.431</td>
<td>-0.359</td>
<td>-0.331</td>
<td>-0.812</td>
<td>0.131</td>
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<tr>
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<td>(0.132)</td>
<td>(0.132)</td>
<td>(0.058)</td>
<td>(0.104)</td>
<td>(0.138)</td>
<td>(0.269)</td>
<td>(0.145)</td>
</tr>
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<td>380</td>
<td>380</td>
<td>380</td>
<td>135</td>
<td>165</td>
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<td>Adjusted R-squared</td>
<td>0.134</td>
<td>0.453</td>
<td>0.402</td>
<td>0.364</td>
<td>0.212</td>
<td>0.169</td>
<td>0.336</td>
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<td>0.001</td>
<td>0.128</td>
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<td><strong>Implied Demand Parameters:</strong></td>
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<tr>
<td>Geometric Mean Expenditure Share</td>
<td>0.219</td>
<td>0.228</td>
<td>0.300</td>
<td>0.223</td>
<td>0.223</td>
<td>0.270</td>
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<td>(0.001)</td>
<td>(0.001)</td>
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<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.003)</td>
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<td>Uncompensated Own Price Elasticity of Housing Demand</td>
<td>-0.797</td>
<td>-0.721</td>
<td>-0.675</td>
<td>-0.700</td>
<td>-0.749</td>
<td>-0.599</td>
<td>-0.653</td>
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<tr>
<td></td>
<td>(0.060)</td>
<td>(0.049)</td>
<td>(0.030)</td>
<td>(0.035)</td>
<td>(0.055)</td>
<td>(0.089)</td>
<td>(0.082)</td>
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<tr>
<td>Income Elasticity of Housing Demand</td>
<td>0.778</td>
<td>0.742</td>
<td>0.569</td>
<td>0.641</td>
<td>0.669</td>
<td>0.188</td>
<td>1.131</td>
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<tr>
<td></td>
<td>(0.132)</td>
<td>(0.132)</td>
<td>(0.058)</td>
<td>(0.104)</td>
<td>(0.138)</td>
<td>(0.269)</td>
<td>(0.145)</td>
</tr>
<tr>
<td>Elasticity of Substitution Between Housing and Consumption Goods</td>
<td>0.802</td>
<td>0.715</td>
<td>0.721</td>
<td>0.717</td>
<td>0.772</td>
<td>0.751</td>
<td>0.384</td>
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<td>(0.049)</td>
<td>(0.032)</td>
<td>(0.027)</td>
<td>(0.026)</td>
<td>(0.044)</td>
<td>(0.053)</td>
<td>(0.088)</td>
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</table>

All specifications includes renters only.
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<th>Hicksian Demand</th>
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<th>Marshallian Demand</th>
<th>Marshallian Demand</th>
<th>Marshallian Demand</th>
<th>Log Median Expenditure Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing Price Index</td>
<td>0.597</td>
<td>0.580</td>
<td>0.242</td>
<td>0.574</td>
<td>0.551</td>
<td>0.563</td>
<td>0.475</td>
<td>(0.034)</td>
</tr>
<tr>
<td>Non-Housing Price Index</td>
<td>-0.006</td>
<td>-0.024</td>
<td>-0.242</td>
<td>-0.574</td>
<td>-0.551</td>
<td>-0.563</td>
<td>-0.475</td>
<td>(0.088)</td>
</tr>
<tr>
<td>Household Income Index</td>
<td>-0.687</td>
<td>-0.686</td>
<td>-0.410</td>
<td>-0.664</td>
<td>-0.207</td>
<td>-0.180</td>
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</tr>
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<td>Inverse Distance to Coast</td>
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<td></td>
<td>(0.048)</td>
</tr>
<tr>
<td>Average Slope of Land</td>
<td></td>
<td></td>
<td>0.012</td>
<td></td>
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<td>(0.048)</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>Renters Only</td>
<td>Renters Only</td>
<td>Renters Only</td>
<td>Renters Only</td>
<td>Owners Only</td>
<td>Owners Only</td>
<td>Owners Only</td>
<td>Renters and Owners</td>
</tr>
<tr>
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<td>380</td>
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<td>376</td>
<td>165</td>
<td>165</td>
<td>165</td>
<td>165</td>
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<tr>
<td>Adjusted R-squared</td>
<td>0.803</td>
<td>0.801</td>
<td>0.455</td>
<td>0.808</td>
<td>0.606</td>
<td>0.766</td>
<td>0.694</td>
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<tr>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
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<tr>
<td>P-value of Test of Homogeneity of Demand</td>
<td>0.190</td>
<td>0.190</td>
<td>0.318</td>
<td>0.136</td>
<td>0.176</td>
<td>0.077</td>
<td>0.288</td>
<td></td>
</tr>
</tbody>
</table>

**Implied Demand Parameters:**

|                                | Marshallian Demand | Marshallian Demand | Marshallian Demand | Marshallian Demand | Marshallian Demand | Marshallian Demand | Marshallian Demand | Marshallian Demand | Marshallian Demand |
|--------------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Geometric Mean Expenditure Share | 0.223              | 0.223              | 0.223              | 0.223              | 0.360              | 0.417              | 0.393              |                    |                    |
| Uncompensated Own Price Elasticity of Housing Demand | -0.403             | -0.420             | -0.667             | -0.426             | -0.449             | -0.437             | -0.525             |                    |                    |
| Income Elasticity of Housing Demand | 0.313              | 0.314              | 0.590              | 0.336              | 0.793              | 0.820              | 0.807              |                    |                    |
| Elasticity of Substitution Between Housing and Consumption Goods | 0.451              | 0.689              | 0.451              | 0.255              | 0.163              | 0.163              | 0.342              |                    |                    |

All specifications include renters only. Robust standard errors in parentheses. Test of homogeneity of demand is that the coefficients on both price indices and income sum to zero.
<table>
<thead>
<tr>
<th></th>
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<th>(2)</th>
<th>(3)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Main Effects</td>
<td>Interacted with Household Size</td>
<td>Interacted with Non-White</td>
</tr>
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<td>Constant</td>
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<td>-0.015</td>
<td>-0.062</td>
</tr>
<tr>
<td></td>
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<td>(0.001)</td>
<td>(0.005)</td>
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<td>Housing Price Index</td>
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<td>-0.036</td>
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<td></td>
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<td>(0.004)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Non-Housing Price Index</td>
<td>-0.288</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.004)</td>
<td>(0.022)</td>
</tr>
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<td>Predicted Wage Index</td>
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<td>-0.028</td>
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<td>(0.001)</td>
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<td>Adjusted R-squared</td>
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</tr>
<tr>
<td>P-value of Test of Homogeneity of Demand</td>
<td></td>
<td>0.303</td>
<td></td>
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</table>

Regressions constrained to have homogeneous demand.
Figure 1: Expenditure Share on Housing 1970-2013

Note: For non-BEA series, a moving average with weight of 0.5 for the year after and the year before is shown in the curve.
TABLE 5: PRICES, INCOMES, AND HOUSING EXPENDITURE SHARES - TIME SERIES DATA

<table>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Log Aggregate Housing Share</td>
<td>Log Average Rental Share</td>
<td>Log Aggregate Housing Share</td>
<td>Log Average Rental Share</td>
<td>Log Median Housing Share</td>
<td>Log Median Rental Share</td>
</tr>
<tr>
<td>Data Source:</td>
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<td>CEX</td>
<td>CEX</td>
<td>AHS</td>
<td>AHS</td>
<td>Census</td>
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<tr>
<td>P-value of Test of Homogeneity of Demand</td>
<td>0.737</td>
<td>0.735</td>
<td>0.342</td>
<td>0.506</td>
<td>0.573</td>
<td>0.546</td>
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<tr>
<td>Restricted Regression Results:</td>
<td>0.368</td>
<td>0.248</td>
<td>0.153</td>
<td>0.306</td>
<td>0.131</td>
<td>0.493</td>
</tr>
<tr>
<td>Log CPI-U: Shelter minus Log CPI-U: All Items Less Shelter</td>
<td>(0.055)</td>
<td>(0.274)</td>
<td>(0.080)</td>
<td>(0.146)</td>
<td>(0.161)</td>
<td>(0.300)</td>
</tr>
<tr>
<td>Log Average Household Income minus Log CPI-U: All Items Less Shelter</td>
<td>-0.429</td>
<td>-0.231</td>
<td>-0.133</td>
<td>-0.677</td>
<td>-0.937</td>
<td>-0.644</td>
</tr>
<tr>
<td>Linear Time Trend (Per Decade)</td>
<td>(0.050)</td>
<td>(0.121)</td>
<td>(0.031)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.183)</td>
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<tr>
<td>N (years)</td>
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<td>28</td>
<td>29</td>
<td>21</td>
<td>23</td>
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Implied Demand Parameters from Restricted Regressions:

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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Mean Expenditure Share</td>
<td>0.18</td>
<td>0.292</td>
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<td>0.27</td>
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<td>Uncompensated Own-Price Elasticity of Housing Demand</td>
<td>-0.632</td>
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<td>Income Elasticity of Housing Demand</td>
<td>0.571</td>
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<td>0.323</td>
<td>0.063</td>
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<td>(0.050)</td>
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<td>(0.031)</td>
<td>(0.090)</td>
<td>(0.185)</td>
<td>(0.183)</td>
</tr>
</tbody>
</table>

Decomposition of Long-run Change in Expenditure Share on Housing:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
<tr>
<td>Total Change in Log Share</td>
<td>0.070</td>
<td>0.262</td>
<td>0.271</td>
<td>0.369</td>
<td>0.414</td>
<td>0.321</td>
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<tr>
<td>Change Attributable to Time Trend</td>
<td>0.122</td>
<td>0.285</td>
<td>0.281</td>
<td>0.244</td>
<td>0.461</td>
<td>0.202</td>
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<td>(0.034)</td>
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<td>Change Attributable to Compensated Relative Price Effect</td>
<td>0.084</td>
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<td>(0.037)</td>
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<td>Change Attributable to Income Effect</td>
<td>-0.139</td>
<td>-0.018</td>
<td>-0.029</td>
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<td>0.040</td>
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<tr>
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<td>(0.016)</td>
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<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.004)</td>
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<tr>
<td>Residual</td>
<td>0.002</td>
<td>-0.057</td>
<td>-0.013</td>
<td>0.049</td>
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<td>(0.012)</td>
<td>(0.022)</td>
<td>(0.028)</td>
<td>(0.036)</td>
</tr>
</tbody>
</table>

Homogeneity of demand requires that the coefficients on log CPI-U for shelter, log CPI-U for all items less shelter, and log real household income sum to zero. The restricted regressions impose this constraint. Newey-West standard errors reported in parentheses. For non-BEA series, a moving average with weight of 0.5 for the year after and the year before is used.
<table>
<thead>
<tr>
<th>Corresponding Regression Column</th>
<th>Table 1, Column 3</th>
<th>Table 1, Column 6</th>
<th>Table 4, Column 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Substitution (Sigma)</td>
<td>0.696 (0.019)</td>
<td>0.757 (0.018)</td>
<td>0.629 (0.067)</td>
</tr>
<tr>
<td>Distribution Parameter (Delta)</td>
<td>-0.182 (0.010)</td>
<td>-0.157 (0.010)</td>
<td>-0.111 (0.026)</td>
</tr>
<tr>
<td>Non-Homotheticitiy Parameter (Gamma)</td>
<td>0.401 (0.091)</td>
<td>0.196 (0.100)</td>
<td>0.469 (0.102)</td>
</tr>
<tr>
<td>Time Trend</td>
<td>-0.001 (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regression Type</td>
<td>Cross-Sectional</td>
<td>Cross-Sectional</td>
<td>Time Series</td>
</tr>
<tr>
<td>Dependent Variable</td>
<td>Log Median Rental Share</td>
<td>Log Aggregate Rental Share</td>
<td>Log Aggregate Housing Share</td>
</tr>
<tr>
<td>Data Source</td>
<td>2000 Census</td>
<td>2000 Census</td>
<td>BEA</td>
</tr>
</tbody>
</table>
Outline

1 Introduction and Motivation

2 Model

3 Data

4 Estimation and Results

5 Putting the Parameter Estimation into Use

6 Conclusion
Decomposition of housing share changes over time

What role do the following play for the representative household?

1. Compensated-price effect: change in relative price
2. Income effect: accounts for welfare change
3. Time trend: taste change, amenities, measurement
4. Residual: everything else

\[
\hat{s}_y^t = (1 - \tilde{s}_y + \epsilon_{y,p}^H)(\hat{p}^t - \hat{c}^t)
+ (\epsilon_{y,m} - 1)[(\hat{m}^t - \hat{c}^t) - \tilde{s}_y(\hat{p}^t - \hat{c}^t)]
+ \alpha_t t
+ e_t
\]

For some groups real income may have fallen. (We are estimating the proper deflator empirically!)
<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Source:</td>
<td>Log Aggregate Housing Share</td>
<td>Log Average Rental Share</td>
<td>Log Aggregate Housing Share</td>
<td>Log Average Rental Share</td>
<td>Log Median Housing Share</td>
<td>Log Median Rental Share</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BEA</td>
<td>CEX</td>
<td>CEX</td>
<td>AHS</td>
<td>AHS</td>
<td>Census</td>
<td></td>
</tr>
<tr>
<td>P-value of Test of Homogeneity of Demand</td>
<td>0.737</td>
<td>0.735</td>
<td>0.342</td>
<td>0.506</td>
<td>0.573</td>
<td>0.546</td>
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<tr>
<td><strong>Restricted Regression Results:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log CPI-U: Shelter minus Log CPI-U: All Items Less Shelter</td>
<td>0.368</td>
<td>0.248</td>
<td>0.153</td>
<td>0.306</td>
<td>0.131</td>
<td>0.493</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.274)</td>
<td>(0.080)</td>
<td>(0.146)</td>
<td>(0.161)</td>
<td>(0.300)</td>
<td></td>
</tr>
<tr>
<td>Log Average Household Income minus Log CPI-U: All Items Less Shelter</td>
<td>-0.429</td>
<td>-0.231</td>
<td>-0.133</td>
<td>-0.677</td>
<td>-0.937</td>
<td>-0.644</td>
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<td>(0.121)</td>
<td>(0.031)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.183)</td>
<td></td>
</tr>
<tr>
<td>Linear Time Trend (Per Decade)</td>
<td>0.003</td>
<td>0.008</td>
<td>0.007</td>
<td>0.007</td>
<td>0.012</td>
<td>0.006</td>
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<tr>
<td></td>
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<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
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<td>N (years)</td>
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<td>28</td>
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<td>21</td>
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<td>15</td>
<td></td>
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<tr>
<td><strong>Implied Demand Parameters from Restricted Regressions:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometric Mean Expenditure Share</td>
<td>0.18</td>
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<td>-0.012</td>
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Homogeneity of demand requires that the coefficients on log CPI-U for shelter, log CPI-U for all items less shelter, and log real household income sum to zero. The restricted regressions impose this constraint. Newey-West standard errors reported in parentheses. For non-BEA series, a moving average with weight of 0.5 for the year after and the year before is used.
Figure 2: Changes in Housing Consumption with Rising Income and Housing Prices

Figure 2A: Consumption in Levels
Slope = Negative Relative Price of Housing to non-Housing

- A = original bundle
- F = new bundle
- A to B = income effect (necessity)
- B to C = substitution effect (inelastic)
- C to F = time trend/taste change
- A to D = income effect (neutral/unit)
- D to E = substitution effect (unit)

Figure 2B: Consumption in Logarithms
Slope = Negative Relative Expenditure Share of Housing to non-Housing

- A = original bundle
- F = new bundle
- A to B = income effect (necessity)
- B to C = substitution effect (inelastic)
- C to F = time trend/taste change
- A to D = income effect (neutral/unit)
- D to E = substitution effect (unit)
Costs-of-Living Indices

- No substitution fixed weight Lespeyres index, where $\delta = 1 - \bar{s}_y$:

$$COL_1 = \frac{\delta c_j + (1 - \delta)p_j}{\delta \bar{c} + (1 - \delta)\bar{p}}$$ (17)

- When demand is unit elastic, Cobb-Douglas index:

$$COL_2 = \frac{(c^j)^\delta (p^j)^{1-\delta}}{(\bar{c})^\delta (\bar{p})^{1-\delta}}$$ (18)

- For arbitrary $\sigma$ where $\delta = [1 + (s_y/(1 - s_y))(1/\sigma)]^{-1}$

$$COL_3 = \frac{\left[\delta^\sigma c_j^{1-\sigma} + (1 - \delta)^\sigma p_j^{1-\sigma}\right]^{1/(1-\sigma)}}{\left[\delta^\sigma \bar{c}^{1-\sigma} + (1 - \delta)^\sigma \bar{p}^{1-\sigma}\right]}$$ (19)

- For non-homothetic CES

$$COL_4 = \frac{\left[\delta^\sigma u^{\gamma(1-\sigma)} c_j^{1-\sigma} + (1 - \delta)^\sigma p_j^{1-\sigma}\right]^{1/(1-\sigma)}}{\left[\delta^\sigma u^{\gamma(1-\sigma)} \bar{c}^{1-\sigma} + (1 - \delta)^\sigma \bar{p}^{1-\sigma}\right]}$$ (20)
Figure 4A: Comparison of Cost-of-living Indices at median household income

Figure 4B: Comparison of Cost-of-living Indices at one half of median household income
How has Housing Price-Inflation affected Inequality?

Take into account both substitution and income effects

<table>
<thead>
<tr>
<th>Household Position</th>
<th>Income Ratio 2009/1970</th>
<th>Ideal Deflator</th>
<th>Ideal Deflated Income</th>
<th>Deflated Fixed Bundle</th>
<th>Ideal Correction to Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th Percentile</td>
<td>6.103</td>
<td>5.480</td>
<td>1.114</td>
<td>1.121</td>
<td>-0.008</td>
</tr>
<tr>
<td>50th Percentile</td>
<td>6.002</td>
<td>5.404</td>
<td>1.111</td>
<td>1.103</td>
<td>0.008</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>7.869</td>
<td>5.310</td>
<td>1.482</td>
<td>1.446</td>
<td>0.036</td>
</tr>
</tbody>
</table>

- Higher housing share lowers income gain for poorest.
- For other households, substitution effects lower inflation impact
- Add another 5 percentage points to the 90-10 real income differential!
What Do Housing Affordability Measures Mean?

- Moderate ($\geq 30\%$) and Extreme ($\geq 50\%$) Burdens
- Well-being? depends on income elasticity $\leq 1$
- Does it reflect price levels? (Amenities and work-access) depends on price elasticity $\leq 1$
- Taste-based reasons (demographics, privacy)

Recent Increases in Unaffordability

1. Increasing income inequality
2. Increasing rent levels
3. Changes in relative rents
4. Increasing average incomes
### TABLE 7: UNDERSTANDING INCREASES IN HOUSING AFFORDABILITY BURDENS, 1980-2010

<table>
<thead>
<tr>
<th></th>
<th>Share with Moderate Burden</th>
<th>Share with Extreme Burden</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Renter Households in 2010</strong></td>
<td>0.528</td>
<td>0.283</td>
</tr>
<tr>
<td><strong>Counterfactuals for 2010</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Undoing Increases in Income Inequality</td>
<td>0.496</td>
<td>0.267</td>
</tr>
<tr>
<td>2. Undoing Increase in Average Rents</td>
<td>0.468</td>
<td>0.250</td>
</tr>
<tr>
<td>3. Undoing Changes in Relative Rents</td>
<td>0.474</td>
<td>0.253</td>
</tr>
<tr>
<td>4. Undoing Increase in Average Income</td>
<td>0.494</td>
<td>0.265</td>
</tr>
<tr>
<td><strong>Renter Households in 1980</strong></td>
<td>0.391</td>
<td>0.196</td>
</tr>
</tbody>
</table>

Notes: moderate burden is defined as an expenditure share on housing in excess of 30%; extreme burden is defined as expenditure share in excess of 50%. Counterfactual 1 assumes no increase in income inequality 1980-2010. Counterfactual 2 additionally assumes no increase in national rents 1980-2010. Counterfactual 3 additionally assumes no increases in dispersion of rents across metropolitan areas 1980-2010. Counterfactual 4 additionally assumes no change in average incomes 1980-2010.
Outline

1. Introduction and Motivation
2. Model
3. Data
4. Estimation and Results
5. Putting the Parameter Estimation into Use
6. Conclusion
Conclusion

- Interesting souped-up framework for housing demand
- Cross-section and time-series do match up!
- Housing demand is price and income inelastic (surprise!?)
  - Price elasticity: compensated -1/2; uncompensated -2/3.
  - Income elasticity near 2/3 (less robust)
  - Elasticity of substitution near 2/3: expenditure share rises with rent.
- Rising expenditure share since 1970 partly explained by prices
  - Time-trend for housing (measurement error, taste for privacy?) important.
- Ideal local costs-of-living put more weight on housing for poor.
  - Inequality increased by increase in the relative price of housing.
  - Predicts residential sorting of low-skilled out of high-price areas.
  - Reduced housing affordability partly explained by growing income inequality and rising price levels.
Plan for Future Work

- More with owned units and user costs
- Micro vs macro/metro elasticities
  - Resolve differences
  - More on household characteristics, too.
- Quantile regressions: possibly with IV.
  - Robust inference away from mean
- Alternative micro-data sets
  - CEX, AHS
- Calculate time change in housing productivity with PPF
  - As land has increased and labor the same, may have gone down.
- Estimate cross-price elasticities of housing with leisure
  - Should housing be taxed more heavily than other consumption?
Deaton and Muellbauer (1980):

\[ \ln s_{ij} = \alpha + \gamma (\hat{p}^j - \hat{c}^j) + \beta \ln (\hat{m}^j / \hat{P}^j) \]  
\[ \ln (\hat{P}^j) = \alpha \hat{p}^j + (1 - \alpha) \hat{c}^j + 0.5 \gamma (\hat{p}^j - \hat{c}^j)^2 \]

- \( i \)'s are individual households in city \( j \)
- First-order approximation of arbitrary demand system
- We impose homogeneity of demand
- Aggregates over households with few assumptions
TABLE 9: ALMOST IDEAL DEMAND SYSTEM AT INDIVIDUAL LEVEL - 2000 CENSUS DATA

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>(1) Main Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regression Results:</strong></td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td>2.104</td>
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<tr>
<td>(0.002)</td>
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<tr>
<td>Gamma</td>
<td>-0.284</td>
</tr>
<tr>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>-0.182</td>
</tr>
<tr>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Sample Size</td>
<td>1,422,356</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.844</td>
</tr>
<tr>
<td>Constrained Regression</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample</td>
<td>Renters Only</td>
</tr>
</tbody>
</table>

**Implied Demand Parameters:**

- Uncompensated Own Price Elasticity of Housing Demand: 0.352 (0.002)
- Income Elasticity of Housing Demand: 0.316 (0.001)