Price Jumps in Visegrad-Country Stock Markets: An Empirical Analysis

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Abstract

We employ high frequency data to study extreme price changes (i.e., price jumps) in the Prague, Warsaw, Budapest, and Frankfurt stock market indexes from June 2003 to December 2010. Since the descriptive statistics show strong deviation from a Gaussian distribution, we use the price jump index and normalized returns to analyze the distribution of extreme returns. The comparison of jump distributions across different frequencies, periods, up and down moves, and markets suggests that the behavior of the Prague index significantly deviates from the rest of the stock indices—the lower the frequency, the less price jumps are observed. We suggest that it is the relationship with market micro-structure—low turnover and very tolerant margin trading—which makes it deviate significantly. We also show that the recent financial crisis resulted in an overall increase in volatility; however, jump distribution was not significantly changed.

Keywords: Central European stock markets, financial markets, price jumps, market returns, standardized returns
JEL: G15, P59

1. Literature review and motivation

The volatility of financial markets has been a deeply studied phenomenon in the financial literature for more than a century (see, e.g., the original work of Bachelier in Bachelier et al., 2006, or the recent survey of Gatheral, 2006). There exists an enormous stream in the literature that directly separates volatility into two parts: the noise and irregular and extreme price movements known as price jumps. See Merton (1976) for an early reference or the recent discussion of how to decompose volatility by Giot et al. (2010). However, most of the attention has so far been focused on the part of volatility known as regular noise, which can be described by a standard Gaussian distribution. The volatility of real assets, however, does not follow a simple Gaussian motion and the true volatility dynamics is more complex. (There is a wide range of technical literature devoted to non-Gaussian price-generating models, for example based on chaos theory, see the survey in Chorafas, 1994, based on fractal Brownian motion, see the introductory

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paper of Mandelbrot and Van Ness, 1968, or models with positive feedback, see Lin et al., 2009.) In the recent literature, price jumps attract more attention despite the fact that they are difficult to explicitly define or handle mathematically (Broadie and Jain, 2008; Johannes, 2004; Nietert, 2001; Pan, 2002).

Price jumps can be connected to important issues in market micro-structure, such as the efficiency of price formation, the provision of liquidity or the interaction between market players, as can be seen in Madhavan (2000). From a practical point of view, traders and portfolio managers are also interested in analyzing price jumps since they are a part of volatility and therefore associated with potential big losses and gains. Moreover, one see interesting applications of price jumps distributions for financial engineers computing appropriate risk measures, including modified value at risk. Thus, understanding price jumps helps to avoid big losses, improves portfolio performance and better hedges positions. Finally, knowledge of price jumps is needed by financial regulators; see Becketti and Roberts (1990), Tiniç (1995), and Li and Rose (2009).

In this paper, we empirically estimate a broad range of price jump properties for the main stock indexes of the Central and Eastern European (CEE) emerging markets. For our analysis we employ a discrete-time framework, which is suitable for markets with low and irregular frequency of trades. In particular, we are interested in measuring to what extent the jumpiness of the selected CEE markets has been affected by the recent financial crisis and how price jump indexes depend on a chosen frequency, as well as analyzing the (a)symmetry of the underlying jump distribution. In such a setup, the exact prediction of a price jump is not the primary concern, rather we want to compare the propensity of indexes to jump using a non-parametric framework.

We use high-frequency (five-minute) data on the CEE stock market indexes covering the Czech Republic, Poland, and Hungary. These countries belong to the Visegrad region—small emerging economies regionally and culturally close to each other.[1] An analogous country setup was used by Jayasuriya (2011), who studied the effect of the Chinese market on its emerging market neighbors. In our country group we employed the German DAX index from the Frankfurt stock exchange as a benchmark for two main reasons. First, Germany is by far the most important foreign trade partner for all CEE firms, and second, the German stock market is geographically the closest mature market and a very good proxy for Eurozone financial markets.

The data spans from June 2003 to the end of 2010 and thus covers the period before the recent financial crisis, the phase before the crisis, as well as part of the recovery phase. To our knowledge, this is the first study of price jumps for small emerging markets that includes economic and financial interference and discusses the impact of a financial crisis on extreme price movements. Moreover, it is the first study suggesting comparisons of jump propensities across markets and time periods.

The rest of the paper is structured as follows. Section 2 gives a short overview and classification of various price jump indicators. Section 3 briefly describes our methodology, in particular how we use non-parametric measures to compare stock market jumpiness across time and markets. Section 4 describes the data, Section 5 is devoted to our results, and finally Section 6 concludes the paper.

[1]The Visegrad region actually consists of four countries: the Czech Republic, Poland, Hungary and Slovakia. The sample does not include Slovakia since its financial market has very low capitalization and extremely low turnover, and therefore it is not suitable for any high-frequency statistical analysis.

2. Measuring and identifying price jumps: Price jumps indicators

One of the major problems associated with price jumps is the lack of theoretical foundations and very different views about their origins presented in the literature. The lack of a theoretical explanation of price jumps means that empirical analysis is currently the only tool one can employ. The appearance of price jumps is associated with several explanations running from changes in market mood to flows of new information (both macro and firm announcements) and insider trading to herd behavior. Basically, most of the factors used for the determinants of market volatility can also be used as a potential source of price jumps.

The literature contains a variety of different jump indicators based on different assumptions. Generally, we can divide the price jump indicators into two main categories: indicators aiming to exactly identify the moments or periods when the price jump occurred and indicators evaluating the statistical propensity of a given time series to undergo price jumps (see, e.g., Hanousek et al., 2011 for a simulation study comparing the use of jump indicators).

The first group of indicators are those employing higher moments like bipower variance and swap variance. Bipower variance-based indicators deal with statistics coming from the difference between the realized variance and the bipower variance: the measure of variance, which, as opposed to the realized variance, is not sensitive to non-normal price movements (see, e.g., Barndorff-Nielsen and Shephard, 2004, Barndorff-Nielsen et al., 2006, Barndorff-Nielsen and Shephard, 2006, and Barndorff-Nielsen et al., 2008 among others). Lee and Mykland (2008) further elaborated this method and derived an indicator that tests for the exact moments when a price jump occurred. Swap variance-based indicators were developed by Jiang and Oomen (2008) and rely on the difference between the realized variance and the swap variance. The authors claim that that swap variance is more sensitive to price jumps than bi-power variance and thus the indicator is more efficient, which they support by a Monte Carlo simulation study.

The second group of price jump indicators targets a measure of jumpiness, not the individual jumps. One of the streams of indicators belonging to this group are those developed by Aït-Sahalia along with various co-authors (see the papers by Aït-Sahalia, 2004, Aït-Sahalia and Jacod, 2009a, Aït-Sahalia et al., 2009, and Aït-Sahalia and Jacod, 2009b). Let us note that these indicators are by construction suited for the investigation of the ultra-high-frequency properties of price time series.

The stream of indicators measuring price jumpiness that will be employed in this paper use the price jump index and normalized returns. These indicators belong to the field of statistical finance, see e.g. Mantegna and Stanley (2000) for further references.

The price jump index was introduced by Joulin et al. (2008) and is defined as the absolute value of a return normalized by the moving average of the same quantity over a given window:

$$j_T(t) = \frac{|r(t)|}{< |r(t)| >_T} , \tag{1}$$

where $< |r(t)| >_T$ denotes the equally weighted moving average of T values of absolute log-returns, including the current value, i.e.,

$$< |r(t)| >_T = \frac{1}{N_T} \sum_{i=0}^{T-1} |r(t-i)| , \tag{2}$$

where N_T stands for the actual number of observations in the time window to control for missing observations.

The authors also employed a large data set of US stocks and showed that the price jump index has a distribution with tails following power-law behavior with a characteristic coefficient corresponding to an inverse cubic distribution. The prominent role of the inverse cubic distribution in the financial high-frequency data was confirmed by other studies. For example, Plerou et al. (1999) and Gopikrishnan et al. (1999) employed normalized returns to study price jumpiness for US stocks and the S&P 500 index using high-frequency data. They confirm that the tail distribution of normalized returns for US stocks follows a distribution close to the inverse of the cubic one, i.e., they do not behave either as a pure Gaussian or as a Levy-like distribution with infinite variance.

Normalized returns, on the other hand, are defined as centered returns normalized by their standard deviation:

$$r_T^n(t) = \frac{r(t) - < r(t) >_T}{\sigma_T(t)}\,, \tag{3}$$

where $< r(t) >_T$ is defined in a similar way as in equation (2), and $\sigma_T(t)$ is the standard deviation—or the realized volatility with respect to the L_2 measure—calculated from the same set of T returns. Such a definition allows us both to compare the different price time series and to compare periods with different market volatility.

Eryigit et al. (2009) study price jumps for a broad range of stock market indexes with the help of normalized returns. They focus mainly on the functional form of the tail distribution and test a broad range of possible tails of price jump distributions. The big Chinese emerging markets are studied by Jiang et al. (2009) using normalized returns, where the authors show that power-law behavior is valid only for long-term moving averages, while for a short-term history, the tail behavior is more exponential-like.

In this paper we are interested in analyzing how jumps (and their distributions) have been affected by the recent financial crisis, how they depend on a chosen frequency, etc. In such a setup, the exact prediction of a price jump is not the primary concern, rather we want to compare the propensity of indexes to jump. Hence, we will use in our comparison two main price jump indicators based on the price jump index and normalized returns.

3. Methodology

We employ two measures for price jumps: the price jump index introduced in equation (1) and the normalized returns defined by equation (3). The two definitions of price jump indicators are close to each other but still they are significantly different. The similarity lies in the normalization with respect to recent history expressed by dividing the returns by the realized volatility. The moving average of the absolute returns and standard deviation represent two different definitions of realized volatility. It is easy to show that standard deviation is relatively more sensitive to extreme returns compared to the average of absolute returns. Hence, normalized returns are on average more suppressed.

In this section we will show what parameters we opted to use for particular price jump indicators (and why), how we estimated the characteristic coefficient α describing a parameter for the tail behavior, and how the whole comparative analysis was conducted.

3.1. Memory of market history

Both definitions of price jump indicators require to choose a time window within which we compare the recent returns. The length of the time window is set by parameter T, which is defined as a number of time steps. This means that the same value of T has a different absolute length in minutes for different frequencies. The length of the time window defines the filtering property of the price jump indicator, i.e., it is related to the frequency of the processes that will be captured by the chosen indicator. Generally, the longer the time window, the more sensitive the indicator is with respect to low frequency processes (cycles), and similarly the less sensitive the indicator would be with respect to high-frequency events. On the other hand, a very short time window does not take into account slowly varying processes and considers only fast and abrupt changes.

When using the price jump indicator we will, therefore, employ a wide range of values for T, which allows us to capture processes at all time scales. It is known that price jump properties are filter (time) dependent. For example, Plerou et al. (1999) show that the longer the time window, the higher the probability of the occurrence of extreme events. This is also supported by the theoretical and empirical evidence presented in Kleinert (2009). In addition, the authors also provide empirical evidence that for a short time window, the behavior of the tail distribution for normalized returns is exponential and thus different from power-law behavior, which was observed for a long time window by many authors (see Joulin et al., 2008), also confirming the presence of power-law behavior.

To capture the views and recommendations made by various authors we therefore employ the following sequence of time windows: $T = 12$, 24, 100, 1000, 2000, and 5000 time steps. Naturally, the length in minutes also depends on the frequency used. We applied steps $T = 12$ and 24 to focus on the immediate effects during the same trading day. Time widows $T = 100$ and longer are taken to study the behavior of long-term averages, as suggested in the above-mentioned literature.

3.2. Estimating tail behavior

Joulin et al. (2008), among others, show that the tail distribution of the price jump index $j_T(t)$ for various financial assets should behave similarly as $\propto s^{-\alpha_T^{(f)}}$, i.e.,

$$P(j_T > s) \sim s^{-\alpha_T^{(f)}}\,, \tag{4}$$

where $\alpha_T^{(f)}$ depends both on the frequency of the data and the length of the time window T. For the sake of simplicity, in the following formulas and expressions, we will omit the frequency index. The index for the time window T will be kept explicit in all expressions. Let us note that the values of the estimated parameter $\alpha_T^{(f)}$, or α, are associated with certain distributions. For example several authors claim that the characteristic parameter α tends to be around 4 for a large set of US stocks, which corresponds to an inverse cubic distribution. Such power-law behavior was also confirmed by Plerou et al. (1999) and Gopikrishnan et al. (1999). On the other hand, a value of $\alpha \leq 3$ indicates Levy-like behavior with infinite volatility, see Kleinert (2009). It is clear that the following rule holds: the lower α is, the more likely extreme price jumps will be observed.

The characteristic coefficient α, as defined in equation (4), is estimated for both the price jump index and normalized returns. We first linearize equation (4) by recasting it into the form

$$\ln P(j > s) \propto -\alpha \ln s\,. \tag{5}$$

In the next step, we employ the following algorithm to estimate the characteristic coefficient: Using OLS, we estimate α in equation (5) for various tail intervals and the resulting value of α is from the OLS regression with the highest R^2. Such an algorithm is both simple and corresponds to the linearization of the tail distribution. It is worth noting that there exist alternative approaches, such as those based on MLE or Principal Component Analysis, as in Vaglica et al. (2008).

3.3. Comparative analysis

The idea of our comparative analysis is based on a very simple and intuitive approach. We aim to estimate the characteristic coefficients α over various sub-samples representing different phases of the market, different time frequencies, etc. Associated non-parametric tests comparing the underlying distributions (or the equality of α coefficients) would actually test if the propensity to jump of the price indexes has changed. We can therefore test if the jumpiness of stock market indexes is different in different time periods like during the recent financial crisis; we can test the (a)symmetry of jumps up and down, as well as the sensitivity of α to particular high frequencies. This relatively simple setup allows us to study the jump component of the price generating process in more detail.

For the above-mentioned comparison we need to use a test that will be distribution-free, i.e., a non-parametric test with a null hypothesis that two or more sets of estimated parameters have the same distribution. The non-parametric feature of the test is necessary, since we cannot assume any underlying distribution of the estimated parameters. Moreover, we also require that the chosen test(s) would have good final sample properties even for small samples. Taking all of these considerations together, we opted for two non-parametric tests: the Wilcoxon rank-sum test and the Kruskal-Wallis test.

The Wilcoxon rank-sum test. is a non-parametric statistical test used to compare whether two independent samples have equally large values. (For the original reference, see Wilcoxon, 1945; a modification of the test is known as the Mann-Whitney test, see Mann and Whitney, 1947.) The test itself goes in two steps. In the first step, all observations are ranked together according to their value no matter what sample they belong to. The sum of all the ranks assigned to all observations is equal to $N\left(N+1\right)/2$, where N is the number of observations in both samples together. When both samples are equally distributed, the sum of ranks tends to be equal. Therefore, in the second step, two statistics are constructed:

$$U_i = \sum_{i \in Sample_i} rank_i - \frac{n_i\left(n_i+1\right)}{2}\,, \qquad (6)$$

where n_i is the number of observations in sample i and $rank_i$ is the rank of observation i.

For large samples, the sum of the two later statistics, $U = U_1 + U_2$, is asymptotically equal to a normal distribution (Asymptotic distribution can be used for $n > 30$, for smaller n special critical values have to be used). For convenience, we employ standardized statistics

$$z = \frac{U - m_U}{\sigma_U}\,, \qquad (7)$$

with mean $m_U = \frac{n_1 n_2}{2}$ and standard deviation $\sigma_U = \sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}$, which is asymptotically equal to $z \sim N\left(0,1\right)$.

The Kruskal-Wallis test. is a non-parametric statistical test used to test the null hypothesis that K independent samples were drawn from distributions with the same median (for the reference see Kruskal and Wallis, 1952). The test is a direct generalization of the Mann-Whitney U test for two samples. The test follows a similar strategy to the later test of Wilcoxon and goes in two steps. In the first step, all observations are ranked together according to their value no matter what sample they belong to. In the second step, the Kruskal-Wallis statistics is calculated according to the following formula:

$$KW = (N-1) \frac{\sum_{i=1}^{K} n_i \left(\bar{r}_i - \bar{r}\right)^2}{\sum_{i=1}^{K} \sum_{j \in Sample_i} \left(r_j - \bar{r}\right)^2}, \tag{8}$$

where n_i is the number of observations in sample i, $\bar{r} = (N+1)/2$ is the average rank of the entire sample, $\bar{r}_i = \left(\sum_{j \in Sample_i} r_j\right)/n_i$, N is total number of observations, and K is the number of independent samples. The KW statistics is for large values of all n_i asymptotically equal to $KW \sim \chi^2_{K-1}$.

4. Data and Descriptive Statistics

In our analysis we use 5-minute-frequency data from the Prague Stock Exchange (PSE, the PX index), the Budapest Stock Exchange (BSE, the BUX index), the Warsaw Stock Exchange (WSE, the WIG20 index), and the Frankfurt Stock Exchange (FSE, the DAX index); the data spans from June 2003 to December 2010. The sample starts just before the three emerging markets joined the European Union and covers the period before the recent financial crisis, the early phases of the crisis at the end of 2007 and beginning of 2008, the full emergence of the financial crisis in 2008 and 2009 and, finally, the start of the recovery until the end of 2010. It thus describes the evolution of three CEE emerging financial markets. The Czech Republic, Hungary, and Poland are culturally, historically, geographically, and economically connected countries. In our analysis we add a German stock market index (DAX) as a Eurozone benchmark to compare the CEE jump distributions in the EU context. Moreover, this choice is supported by the fact that Germany is the most important foreign trade partner for CEE firms, hence there exists a link between CEE firm fundamentals and German economic growth. Besides, the German stock market is the geographically closest mature market.

Since the main purpose of this paper is to study market dynamics and especially the propensity of each particular market for extreme price changes (i.e. jumps), we have cut off the very beginning and the very end of each trading day (we used various cuts from 10 to 30 minutes). The cut-off at the beginning of the trading day is performed due to the different construction of the market indexes. Generally, a market index can be either dividend-included or dividend-excluded. In our case, the DAX and BUX indexes are dividend-included, while the PX and WIG indexes are dividend-excluded. This causes different behavior at market opening due to the ex-dividend day effects.[2] Obviously, the beginning-of-day cut-off could have a negative effect on the observed propensity for

[2]Dividend-excluded market indexes measure the price performance of markets without including dividends. This means that on any given day, the price return of an index captures the sum of its constituents' free-float-weighted market capitalization returns. A description of the dividend structures and the particular composition of the stock market indexes can be found at: www.bcpp.cz, www.bse.hu, www.gpw.pl, and www.deutsche-boerse.com. For an illustration of how the dividend process influences the price process (for the dividend-included DAX index), see Fengler et al. (2007).

sizable price moves. This is, first, because we remove the period close to the opening, when markets react to overnight events. Second, the cut-off data could show even an opposite and smoother reaction to the (overnight) events since markets could in the very first moments over-react negatively to negative events, and after this abrupt overshooting, they could positively and smoothly move up to adjust the previous (cut-off) price change. The cut-off at the end of the trading day is performed for similar reasons: some markets have a different final stage (such as a final auction), so cutting off the very end also avoids a possible bias in the data. For example, the Prague Stock Exchange has a closing auction from 4:20 p.m. to 4:27 p.m. Traders may postpone some trades from the trading hours to the closing auction and thus the trading hours may not fully capture the trading activity. Nevertheless, the cut-off occurs long enough after US markets open, therefore we do not lose significant market moves associated with the start of US trading.

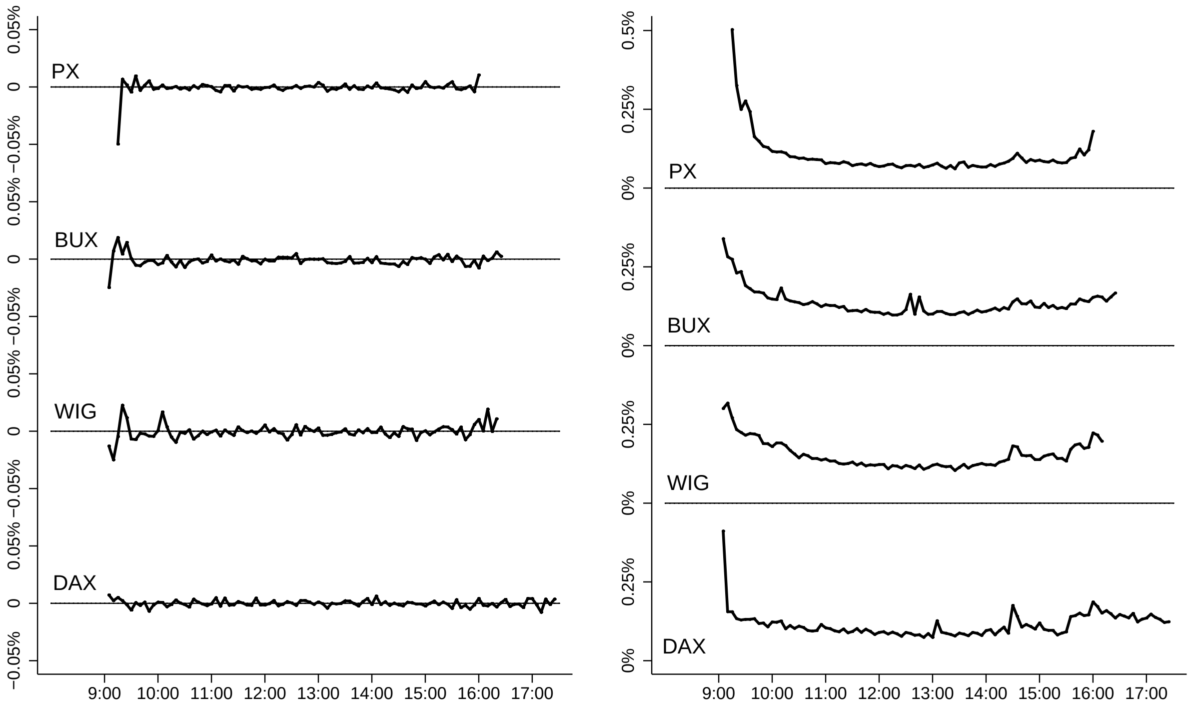
Cutting off the initial phase of the market would be also necessary when one wants to treat markets in a panel-like manner. Nevertheless we do not use a panel-like approach for these CEE markets. The main reason stems from the fact that these stock exchanges open and close at different times. When markets open they usually accommodate information that happened overnight. Thus, comparing these markets at the same time of the day can result in a situation where one of the markets is just in the opening/closing stage while the others are relatively far from the boundaries of their trading days. This could produce some false signals and lower market correlation.

In order to provide a valid inference, we first summarize in Figure 1 the distribution of returns and the standard deviation over the entire trading day (without any cut-off) for all four indexes at a 5-minute frequency. In addition, for sensitivity tests, we have constructed all the relevant variables also on three lower frequencies: 10, 15, and 30 minutes. The figure on the left side shows that the three emerging markets have on average negative returns during the opening period, i.e., all three markets drop during the opening phase. On the contrary, the mature market does the opposite, i.e., it slightly increases in value. In addition, PX has the most abrupt changes (likely also fueled by the fact that PX is a dividend-excluded index), which is further supported by the figure on the right, where we present the distribution of the standard deviation. Besides the well-known U-shaped distribution during the trading day, which is again the strongest for the PX index and less pronounced for the German market, we can see a small increase in volatility during the lunch period and during the opening of US markets.

More specifically, the regular trading hours of the studied stock exchanges are as follows: the Czech Republic (PX) from 9:15 to 16:00, Germany (DAX) from 9:00 to 17:30, Hungary (BUX) from 9:00 to 17:00, and Poland (WIG) from 9:00 to 16:20. PX was originally available from 9:30 to 16:00, which changed on June 30, 2008 to 9:15 to 16:00. The BUX index started from 9:00 to 16:30 and on December 2, 2010, the close of trading hours was changed to 17:00. The WSE index originally covered the period from 10:00 to 16:00. From October 3, 2005, exchange trading opened at 9:30 and closed at 16:10 and this was further modified on September 1, 2008, when market operations were changed to 9:00 to 16:10.

Because of possible sensitivity to (the size of) the cut-off trading periods at the beginning and end of the trading day, we consider various cut-offs running from 10 to 30 minutes. Since we want to present here how the data frequency effects the jump distribution, we present in the paper results associated with the largest cut-off (30 minutes). Shorter cut-offs provide similar (yet a bit stronger) results which are available upon request. In the following pictures, graphs and computations, we distinguish between two

Figure 1: Distribution of returns (left panel) and standard deviation of returns (right panel) over the trading day.

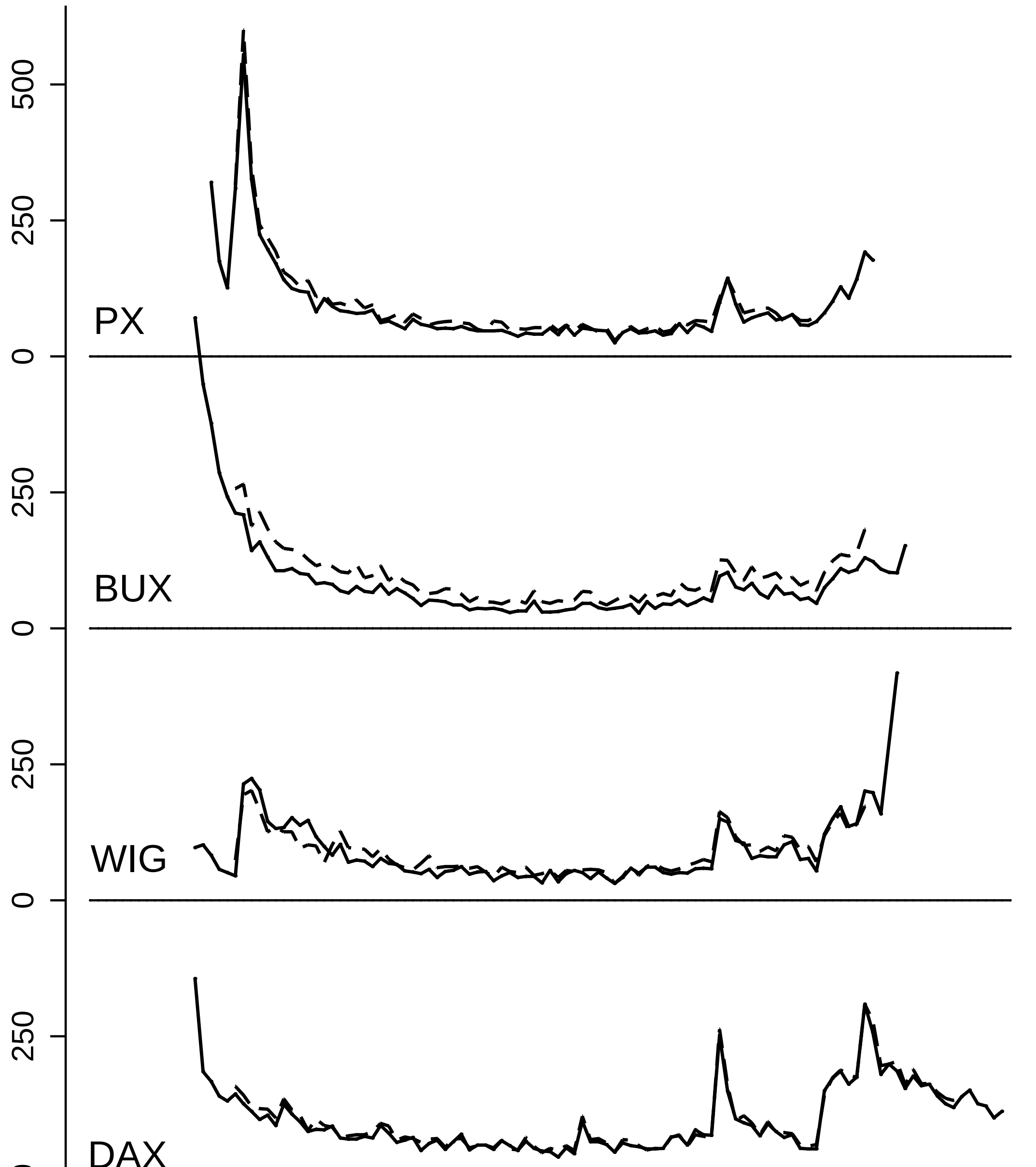


Note: The left panel of the figure describes the distribution of returns over a trading day using a 5-minute frequency. Plotted are distributions for all four indexes: PX, BUX, WIG, and DAX. The right panel of the figure captures the distribution of the standard deviation over a trading day using the same 5-minute frequency for the four indexes depicted in the same order. The initial double peak for the WIG index is caused by the fact that the stock exchange changed its operating hours in the middle of the sample from 10:00 to 9:00.

types of time: clock time and trading time. Trading time skips the cut-off early morning and late afternoon phases and the relevant variables are stacked into one time series with no gaps, i.e., the last minute at the end of the trading period is followed by the first minute of the next trading period.

The effect of cutting off the very first and very last moments of the trading period on the distribution of extreme movements is depicted in Figure 2. The figure shows the distribution of the number of extreme returns over the trading day for both the entire trading day without any cut-off (solid line), and for the day with the cut-off (dashed line). Extreme returns are defined as those that are below the 2.5-th centile or above the 97.5-th centile, calculated over the entire sample. The two lines tend to coincide for all four indexes, except for a small deviation for the BUX index. The coincidence of the two lines combined with the information in Figure 1 means that the initial and/or final periods do not contain significantly more price jumps. The overall pattern of the data depicted in the left panel of Figure 1 suggests that the initial moments consist of returns with the same sign rather than being dominated by extreme downward movements. However, the right panel of Figure 1 still shows that the spread of returns tends to be higher in the initial period.

Figure 2: Distribution of extreme returns over the trading day.



Note: Shown is the distribution of extreme returns over a trading day using a 5-minute frequency. Extreme returns are defined as returns below the 2.5-th centile or above the 97.5-th centile, calculated over the entire period. The solid line takes into account the entire trading day, while the dashed line refers to the trading day with the beginning and end cut off. The double peak for the WIG index is caused by the fact that the stock exchange changed its opening hours in the middle of the sample from 10:00 to 9:00.

4.1. Descriptive Statistics

The descriptive statistics of returns provides the first hints about the possible properties of price jumps. The first four centered moments can be found in Table 1.

Table 1: Basic statistics of returns.

Index	f	N	μ	σ	S	K
PX	5	135 893	-2.52e-06	0.93e-03	-0.284	31.13
	10	68 100	-4.36e-06	1.44e-03	-0.493	26.12
	15	45 693	-5.08e-06	1.89e-03	-0.550	26.95
	30	22 517	-1.19e-05	2.74e-03	-0.580	19.45
BUX	5	148 248	-1.55e-05	1.28e-03	0.025	63.45
	10	75 057	-2.68e-05	1.87e-03	-0.398	39.66
	15	50 668	-3.80e-05	2.33e-03	-0.888	34.20
	30	26 271	-7.28e-05	3.47e-03	-0.560	18.57
WIG	5	137 238	-7.65e-06	1.43e-03	0.040	12.74
	10	69 218	-1.37e-05	2.02e-03	-0.025	11.67
	15	46 509	-1.87e-05	2.45e-03	0.070	11.34
	30	23 765	-4.08e-05	3.52e-03	0.047	12.26
DAX	5	173 382	-1.89e-06	1.11e-03	-0.129	24.52
	10	87 632	-3.98e-06	1.58e-03	-0.111	22.66
	15	59 055	-5.02e-06	1.92e-03	-0.106	17.83
	30	30 460	-6.16e-06	2.84e-03	-0.211	25.12

Note: The table summarizes the standard statistics of log-returns $r(t)$ for all four market indexes used in this study. In brackets is the corresponding stock exchange: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange), and DAX (Frankfurt Stock Exchange). All four frequencies were used: 5-, 10-, 15-, and 30-minutes. The table shows: frequency (f), the number of observations (N), mean returns (μ), standard deviations (σ), skewness (S), and kurtosis (K). We have employed Jarque-Bera statistics to test the deviation from normality, where in all cases the Jarque-Bera statistics had a p-value < 0.0001. We reject a normal distribution of returns for all indexes and frequencies without reporting the statistics.

Table 1 shows that the means of returns are shifted toward negative values. The standard deviation is increasing with decreasing frequency, or with increasing sampling intervals Δt, and is roughly in agreement with the known scaling law, see Mantegna and Stanley (2000),

$$\sigma_{\Delta t} \propto \sqrt{\Delta t}\,. \tag{9}$$

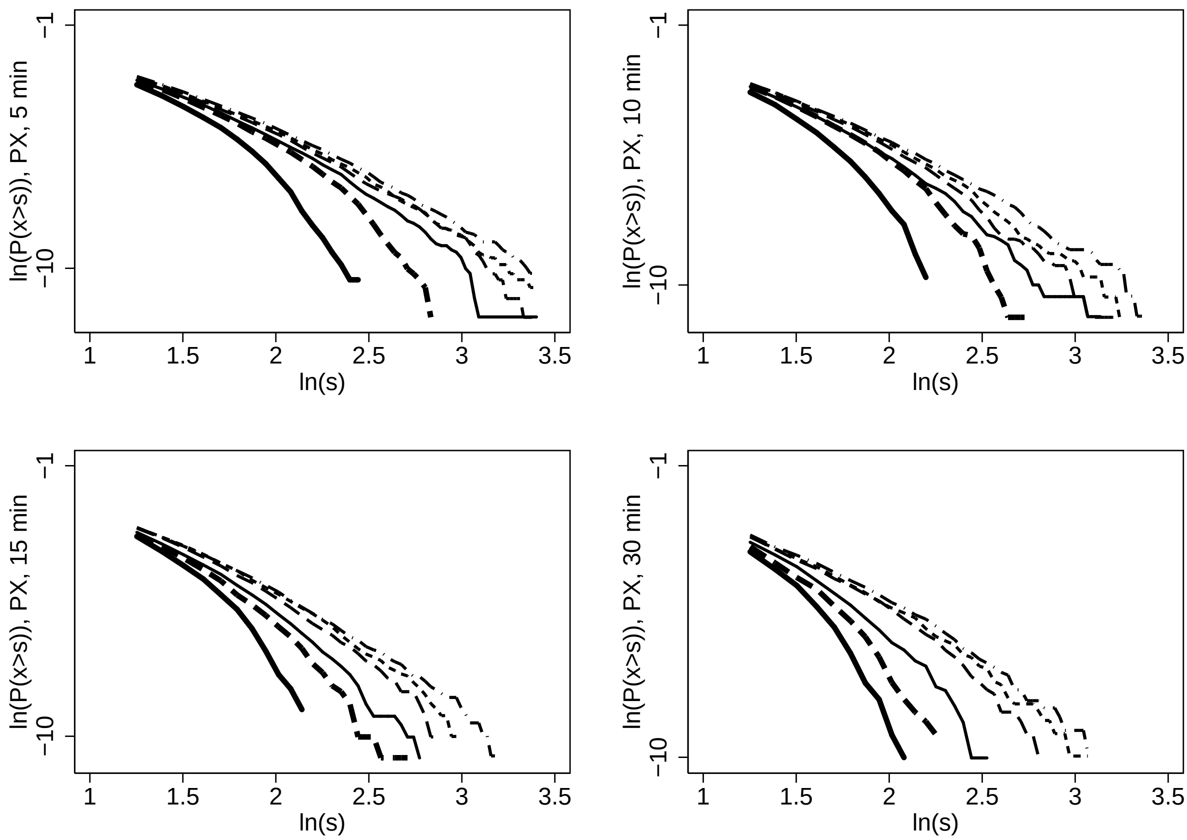
A further measure reported in Table 1 is skewness, which is a measure of the asymmetry of the distribution. The results show that the PX, BUX and DAX indexes have negative skewness and thus their distributions have longer negative tails. On the other hand the WIG index shows positive skewness over all the frequencies, which supports the claim that the WIG index is dominated by positive jumps. The next column with reported kurtosis shows that all time series are leptokurtic, which supports the presence of a fat-tail distribution, or in other words, the presence of extreme price jumps not coming from a Gaussian distribution. This fact is also verified by very low p-values for the Jarque-Bera statistics, which rejects for all reported cases the null hypothesis that data come from an i.i.d. Gaussian distribution.

5. Results

We employ two price jump indicators to assess the price jump properties of four stock market indexes from the Visegrad region and Germany using high-frequency data. We employ returns at a 5-minute frequency, accompanied by returns at three lower frequencies (10, 15, and 30 minutes) for the sake of robustness. To fully explore the filtering properties, we take time windows equal to T = 12, 24, 100, 1000, 2000, and 5000 time steps and plot the distribution of the price jump index for all six time windows and all four frequencies. Since the literature suggests a deviation from power-law behavior (Plerou et al., 1999, and Kleinert, 2009), in Figure 3 we plot the linearized version of equation (5) for the PX index. The other indexes show, though, similar patterns.

The figure clearly shows that the longer the time window T, the higher the probability of the occurrence of extreme events and that a short time window produces a non-linear distribution. Both observations are in agreement with the literature. Since the main scope of this paper is the domain of power-law behavior, we employ in the following the longest time window $T = 5000$ and estimate and present here characteristic coefficients α solely for this filter.

Figure 3: Log-transformed version of the tail part of the price jump index distribution for the PX index.



Note: The distribution was calculated using all four frequencies and six different time windows T. The two short-term windows have a more suppressed occurrence of extreme events compared to the four long-term windows. The symbols used in this table are: $T = 12$ (thick solid), $T = 24$ (thick dash), $T = 100$ (solid), $T = 1000$ (dash), $T = 2000$ (short dash), and $T = 5000$ (dash dot).

5.1. Scaling of the price jumps

We estimate the characteristic coefficients α for both price jump indicators using the linearized equation (5) and the algorithm in which we use the OLS regression maximizing the R^2. First, we report in Table 2 the estimated characteristic coefficients for the price jump index using all four indexes and all four frequencies. Comparing the characteristic coefficients for the highest frequency, the PX index has the lowest significant value of all α among all indexes. This suggests a more frequent presence of extreme price jumps on the PX index. At the other pole stands the WIG index.

Table 2: Estimated characteristic coefficient α_T for the price jump index.

Index	T	5-minute $\alpha_T(\sigma_\alpha)$	t	10-minute $\alpha_T(\sigma_\alpha)$	t	15-minute $\alpha_T(\sigma_\alpha)$	t	30-minute $\alpha_T(\sigma_\alpha)$
PX	5000	3.716 (0.035)	$\overset{a}{>}$	3.408 (0.030)	$\overset{a}{<}$	3.654 (0.030)	$\overset{a}{<}$	3.939 (0.077)
	2000	4.021 (0.043)	$\overset{a}{<}$	4.337 (0.067)	$\overset{a}{>}$	4.085 (0.055)	$\overset{a}{<}$	4.411 (0.111)
	1000	3.640 (0.035)	$\overset{a}{<}$	4.712 (0.097	$\overset{a}{>}$	4.372 (0.067)	$\overset{a}{<}$	5.259 (0.129)
BUX	5000	3.879 (0.032)	$\overset{a}{>}$	3.341 (0.028)	$\approx$	3.407 (0.030)	$\approx$	3.503 (0.062)
	2000	4.814 (0.075)	$\overset{a}{>}$	3.739 (0.034)	$\approx$	3.692 (0.041)	$\approx$	3.678 (0.057)
	1000	4.757 (0.066)	$\overset{a}{>}$	3.925 (0.075)	$\approx$	3.875 (0.059)	$\overset{b}{>}$	4.039 (0.050)
WIG	5000	5.236 (0.087)	$\overset{a}{>}$	4.776 (0.092)	$\overset{a}{>}$	4.489 (0.097)	$\overset{a}{>}$	3.913 (0.067)
	2000	5.202 (0.124)	$\approx$	5.405 (0.228)	$\overset{b}{>}$	5.395 (0.133)	$\overset{a}{>}$	4.009 (0.070)
	1000	6.683 (0.248)	$\overset{c}{<}$	7.612 (0.415)	$\approx$	6.052 (0.179)	$\overset{a}{>}$	4.232 (0.124)
DAX	5000	4.221 (0.043)	$\overset{a}{>}$	3.906 (0.030)	$\overset{a}{>}$	3.617 (0.054)	$\overset{a}{>}$	3.091 (0.027)
	2000	4.723 (0.059)	$\overset{a}{>}$	4.285 (0.043)	$\overset{a}{>}$	3.718 (0.067)	$\overset{a}{>}$	3.126 (0.044)
	1000	4.526 (0.044)	$\overset{a}{<}$	4.738 (0.048)	$\overset{a}{>}$	4.077 (0.080)	$\overset{a}{>}$	2.583 (0.068)

Note: The estimation was done for all four indexes—PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange), and DAX (Frankfurt Stock Exchange)—all four frequencies—5-, 10-, 15-, and 30-minute—and for time windows $T = 5000$, $T = 2000$, and $T = 1000$. The value in the brackets is the standard deviation. The higher the standard deviation, the worse the estimation of the characteristic coefficient. Column t denotes the result of the t-test with the null hypothesis that the two estimated coefficients are equal. The inequality sign illustrates the relation between the estimated coefficients, when we can reject the null hypothesis of no difference between them. Superscripts a , b, and c denote the significance level at which we reject the null hypothesis: a for 99%, b for 95%, and c for 90%. Asymptotic normal distributions were used.

The smaller frequencies also reveal another important pattern that further distinguishes the behavior of the PX index compared to the other three indexes. For the other three indexes, the characteristic coefficient decreases with decreasing frequency. This implies that more extreme events are present for lower frequencies, which is in agreement with general understanding. In the case of the PX index, however, we observe significantly different behavior. Namely, the characteristic coefficient for the 30-minute frequency is higher than the characteristic coefficient for the 5-minute frequency, even exceeding the 95% confidence interval. This suggests the presence of specific determinants on the Prague Stock Exchange that cause a deviation from the rest of the group. We denote this deviation from the standard behavior the "PX Puzzle".

There are several possible market-specific explanations for the PX Puzzle. First, it could be the role of dividends, since the PX index is a dividend-excluded index and a number of the major components of the PX index offer a dividend yield well above 5%. Therefore, the decline of the PX index due to the ex-dividend day could send false signals, which are initially followed by extreme price movements; however; they are smoothed away very soon. If this were so, we would also observe a "WIG puzzle", but we do not. Second, one can further argue that the explanation of the PX Puzzle lies in the small turnover and liquidity of the exchange itself. Several of the major stocks listed at the PSE are actually cross-listed abroad at more mature and bigger stock exchanges. Local prices could for a short period depart significantly from cross-listed counterparts and then—likely using arbitrage trading—the prices are quickly driven closer to the stock prices on the main (abroad) stock exchange. The price of stocks traded at a smaller exchange could easily be influenced, especially over a short period of time. This explanation uses the parallel between the volume of traded assets and the mass in dynamics. The heavier an object is (i.e., more liquid trading), the more effort has to be expended to make it move. Consequently, fast movements, when viewed from a longer perspective, are averaged out and the movements are not so jumpy. The capitalization of the PSE is not extremely high; some stocks have a very low free-float and a low frequency of trades. Such a combination can contribute to this phenomenon as well.

Finally, the market micro-structure point of view offers a complementary argument for the presence of the PX Puzzle. Namely, the representative investor at the Prague Stock Exchange is different from the one in Warsaw. The PSE is dominated by foreign investors, while WSE possesses a large number of domestic institutional investors, namely pension funds, which are obliged to invest in domestic assets. Further, the regulatory differences may also explain the observed PX Puzzle. The PSE has much weaker regulatory requirements especially on the side of margins, where investors may enjoy much higher leverage when compared to the other three exchanges.[3]

5.2. Is there an up/down asymmetry?

Intuitively, the distribution of extreme positive and negative price movements can be different. To assess this intuition on quantitative grounds, we estimate the characteristic coefficients for positive and negative price movements separately. For the normalized returns this modification comes naturally from the definition. In the case of the price jump index, we estimate the characteristic coefficients separately for positive and negative movements, while the average of absolute returns is composed of a given history no matter what the sign of the returns was. We focus on the quantitative comparison between price jumps up and down. Table 3 summarizes the results of a battery of pair-wise comparisons, with the null hypothesis H_0 that the mean of positive and negative jumps are same, i.e., $\alpha_T^{(+)} = \alpha_T^{(-)}$. For every index and every frequency, we calculated the characteristic coefficient separately for negative and positive movements, then we conducted pairwise tests using the mean and standard errors $\alpha_T^{(+)}$ and $\alpha_T^{(-)}$. A positive or negative mark in the table cells denotes whether we observe more price jumps up or down for a given indicator, index, and frequency, i.e., if a given cell contains the + symbol, the coefficient $\alpha_T^{(+)}$ is smaller than $\alpha_T^{(-)}$ and, thus, more extreme price jumps occur in the upward direction. In addition, the significance level of the test is denoted using superscripts a, b, and c (1%, 5%, and 10%); no superscript means that the difference is not statistically significant.

In general, we observe larger negative extreme price movements than positive ones. In terms of the symbols used in Table 3, we should observe substantially more $-$ than $+$. However, the reverse is true: $+$ dominates in the table in all rows and columns with the exception of the BUX index. If we can say that we observe an asymmetry, then CEE markets (with the exception of BUX) show significantly larger positive than negative extreme price movements. This result is robust since both price jump indicators show very similar patterns.

[3]Fortune (2001) discusses the positive correlation between the rate of margin lending and market volatility.

Table 3: Up/down asymmetry for the price jump index and normalized returns.

	5-minute		10-minute		15-minute		30-minute		
	PJI	NR	PJI	NR	PJI	NR	PJI	NR	$\sum$
PX	$-^b$	$+^a$	$-^a$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$
BUX	-	$+^a$	-	$-^a$	$-^a$	$-^a$	$-^a$	$-^a$	$-^a$
WIG	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$
DAX	+	$+^a$	$+^c$	$+^a$	+	$+^a$	+	$+^a$	$+^a$
$\sum$	0	$+^a$	$+^c$	$+^a$	$+^a$	$+^a$	$+^a$	$+^a$	

Note: The estimation was done for all four indexes—PX, BUX, WIG, and DAX—and all four frequencies—5-, 10-, 15-, and 30-minute. The length of the time window is $T = 5000$. In each entry, the characteristic coefficients for positive and negative price jumps were compared. When computing the actual significance level we used the asymptotic normality of estimated coefficients α. The symbol $+$ means that α_T^+ is lower than α_T^-, i.e., more price jumps are observed in the upward direction and similarly for the symbol $-$. Superscripts a , b, and c denote the significance level at which we reject the null hypothesis: a for 99%, b for 95%, and c for 90%. In addition, table presents the marginal effects with respect to frequencies and stock market indicies.

One can speculate that the fact that we observe larger positive extreme price movements can be caused by the data cut-offs at the beginning and end of the trading days. In other words, returns driving the intuitive asymmetry should occur mainly in the truncated period, i.e., shortly after the open and/or shortly before the close we should see significant drops in returns. Although the data shows some negative trends in the cut-off periods (see Figure 1) the downward movements at the beginning of trading days are not dominated by extreme price jumps but rather smooth adjustments with the same downward orientation (see Figure 2).

5.3. Stability of Results – Analysis by Quarters

The previous results were produced using the entire sample. However, the presence of business cycles with repeating peaks and troughs or the recent financial crisis may suggest that the price generating process is not stable over time and we can thus expect a variation in extreme price movements. That is, α coefficients may not be stable over time. Therefore, we divide the data set into smaller sub-samples, repeat the computations on sub-samples, and test the stability of the characteristic coefficients α over time. Since the estimation of the characteristic coefficient requires a large amount of data, the shortest time period for computing α coefficients and therefore for testing their stability is three months. For all stock market indexes, frequencies, up and down movements, and price jump indicators, we estimate the characteristic coefficients and perform a battery of tests, comparing the obtained results.

First, we repeat the analysis of the asymmetry between movements up and down using the quarterly estimated characteristic coefficients. We run the Wilcoxon rank-sum test described above to compare the medians of the characteristic coefficients up and down for every stock market index and both price jump indicators (i.e., H_0: Median of positive and negative jumps are the same, so $\tilde{\alpha}_T^{(+)} = \tilde{\alpha}_T^{(-)}$). We were not able to reject the hypothesis of the stability of the characteristic coefficients in a single case. Therefore, we conclude that the propensity of the studied price indexes to jump has been stable over the whole period studied.

Table 4: Pair-wise comparison of indexes and ordering by α_T.

F	D	PJI	NR
5	$\uparrow$	PX$\overset{c}{<}$DAX$\approx$WIG$\approx$BUX	PX$\approx$DAX$\approx$WIG$\approx$BUX
		PX$\overset{a}{<}$WIG; DAX$\overset{c}{<}$BUX; PX$\overset{a}{<}$BUX	PX$\overset{c}{<}$WIG; DAX$\overset{c}{<}$BUX; PX$\overset{b}{<}$BUX
	$\downarrow$	PX$\overset{b}{<}$DAX$\approx$BUX$\overset{b}{<}$WIG	PX$\overset{c}{<}$BUX$\approx$DAX$\approx$WIG
		PX$\overset{b}{<}$BUX; DAX$\overset{a}{<}$WIG; PX$\overset{a}{<}$WIG	PX$\overset{b}{<}$DAX; BUX$\overset{c}{<}$WIG; PX$\overset{a}{<}$WIG
10	$\uparrow$	PX$\approx$BUX$\approx$WIG$\approx$DAX	PX$\approx$BUX$\approx$WIG$\approx$DAX
		PX$\overset{c}{<}$WIG; PX$<^{b}$DAX	PX$\overset{c}{<}$WIG; PX$\overset{b}{<}$DAX
	$\downarrow$	PX$\overset{b}{<}$BUX$\approx$DAX$\approx$WIG	PX$\approx$ BUX$\approx$DAX$\approx$WIG
		PX$\overset{a}{<}$DAX; PX$\overset{a}{<}$WIG	PX$\overset{b}{<}$DAX; BUX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG
15	$\uparrow$	PX$\approx$BUX$\approx$WIG$\approx$DAX	PX$\approx$BUX$\approx$WIG$\approx$DAX
		PX$\overset{c}{<}$DAX	
	$\downarrow$	PX$\overset{c}{<}$WIG$\approx$DAX$\approx$BUX	PX$\approx$BUX$\approx$DAX$\approx$WIG
		PX$\overset{c}{<}$DAX; PX$\overset{c}{<}$BUX;	PX$\overset{b}{<}$DAX; PX$\overset{b}{<}$WIG
30	$\uparrow$	PX$\approx$BUX$\approx$WIG$\approx$DAX	PX$\approx$WIG$\approx$BUX$\approx$DAX
		PX$\overset{b}{<}$DAX	PX$\overset{c}{<}$BUX; PX$\overset{b}{<}$DAX
	$\downarrow$	PX$\approx$DAX$\approx$BUX$\approx$WIG	PX$\approx$DAX$\approx$WIG$\approx$BUX
		PX$\overset{b}{<}$WIG	

Note: We estimated the characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. We compared the order of indexes using the (pair-wise) Wilcoxon rank-sum test. The test was performed for every frequency (5 to 30 minutes) and both directions up $\uparrow$ and down $\downarrow$. The letter in the superscript of the inequality mark denotes the significance level at which we reject the null hypothesis (i.e., the equality of coefficients): a stands for 99%, b for 95%, and c for 90%. Let us note that smaller coefficients α_T means a higher probability of extreme price movements.

Then, we study up and down jumps separately on different frequencies and compare $\alpha_T^{(+)}$ and $\alpha_T^{(-)}$ across markets. Table 4 depicts the pair-wise mutual ordering of characteristic coefficients α_T for every frequency, and for up and down moves. The first column contains results for the price jump index (PJI), and the second column for normalized returns (NR). Let us note that smaller coefficients α_T means a higher probability of extreme price moves. The pair-wise comparison was conducted using a battery of Wilcoxon rank-sum tests (see Mann and Whitney, 1947, detailed results available upon request). The first row shows the overall ordering; the second row provides tests on distant pairs. Let us note that in the case when the first row shows an overall ordering that is indistinguishable (all symbols are $\approx$), the difference between, say, the first and third index can be very significant, which can be seen in the second row. If the difference is significant, we denote the significance level as follows: a for 99% , b for 95%, and c for 90% . Otherwise, we use the symbol $\approx$ for indexes with statistically equal α (i.e., where we do not reject the null hypothesis of equality).

The results show that major differences are observed on higher frequencies, something we would expect intuitively. Another pattern, which is consistent across all frequencies, both jump price indexes, and up and down moves, is the fact that PX was clearly the most jumpy index (i.e., smallest α) while WIG presented the smallest propensity to jump.

Table 5: Comparing indexes pair-wise: Before and during the crisis.

F	P	D	PJI	NR
5	B	↑	PX≈DAX$\overset{c}{<}$BUX≈WIG PX$\overset{a}{<}$BUX; DAX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG	PX≈DAX≈BUX≈WIG PX$\overset{b}{<}$BUX; DAX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG
5	B	↓	PX$\overset{b}{<}$DAX≈BUX$\overset{c}{<}$WIG PX$\overset{b}{<}$BUX; DAX$\overset{a}{<}$WIG; PX$\overset{a}{<}$WIG	PX$\overset{c}{<}$DAX≈BUX$\overset{c}{<}$WIG PX$\overset{c}{<}$BUX; DAX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG
5	C	↑	WIG≈PX≈DAX≈BUX WIG$\overset{c}{<}$DAX; WIG$\overset{b}{<}$BUX	WIG≈PX≈DAX≈BUX WIG$\overset{a}{<}$BUX
5	C	↓	PX≈BUX≈DAX≈WIG PX$\overset{c}{<}$WIG	PX≈BUX≈WIG≈DAX PX$\overset{c}{<}$DAX
10	B	↑	PX≈BUX≈DAX≈WIG PX$\overset{b}{<}$DAX; PX$\overset{b}{<}$WIG	PX≈BUX≈DAX≈WIG PX$\overset{b}{<}$DAX; PX$\overset{b}{<}$WIG
10	B	↓	PX$\overset{b}{<}$DAX≈BUX≈WIG PX$\overset{b}{<}$BUX; PX$\overset{a}{<}$WIG	PX≈BUX≈DAX≈WIG PX$\overset{c}{<}$DAX; PX$\overset{a}{<}$WIG
10	C	↑	WIG≈PX≈BUX≈DAX WIG$\overset{b}{<}$DAX	BUX≈PX≈WIG≈DAX
10	C	↓	PX≈BUX≈WIG≈DAX PX$\overset{c}{<}$WIG; BUX$\overset{b}{<}$DAX; PX$\overset{b}{<}$DAX	BUX≈PX≈WIG≈DAX PX$\overset{c}{<}$DAX; BUX$\overset{b}{<}$DAX
15	B	↑	PX≈BUX≈DAX≈WIG	PX≈BUX≈DAX≈WIG
15	B	↓	PX≈DAX≈WIG≈BUX PX$\overset{c}{<}$WIG; PX$\overset{b}{<}$BUX	PX$\overset{c}{<}$BUX≈WIG≈DAX PX$\overset{c}{<}$WIG; PX$\overset{b}{<}$DAX
15	C	↑	PX≈WIG≈BUX≈DAX PX$\overset{c}{<}$DAX	PX≈WIG≈BUX≈DAX
15	C	↓	BUX≈PX≈WIG≈DAX	BUX≈PX≈DAX≈WIG
30	B	↑	PX≈ BUX≈DAX≈WIG PX$\overset{c}{<}$DAX; PX$\overset{c}{<}$WIG	PX≈DAX≈WIG≈BUX
30	B	↓	PX≈DAX≈BUX≈WIG PX$\overset{c}{<}$WIG	PX≈WIG≈DAX≈BUX
30	C	↑	BUX≈PX≈WIG≈DAX	PX≈WIG≈BUX≈DAX PX$\overset{c}{<}$DAX
30	C	↓	BUX≈DAX≈PX≈WIG	DAX≈WIG≈PX≈BUX

Note: We estimated the characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. We have employed the Wilcoxon test to compare the order of indexes pair-wise. The test was performed for every frequency, both directions up ↑ and down ↓, and both phases P: before the crisis B and during the crisis C. The financial crisis is defined as the period starting at 2009/Q1 and lasting until the end of the sample at 2010/Q4. A letter denotes the significance level at which we reject the null hypothesis: a for 99%, b for 95%, and c for 90%. This is the significance at which we can say that the indexes are different. Let us note that smaller coefficients α_T means a higher probability of extreme price moves.

Table 6: Financial crisis defined as 2009/Q1 – 2010/Q4.

Index	D	5-minutes PJI	5-minutes NR	10-minutes PJI	10-minutes NR	15-minutes PJI	15-minutes NR	30-minutes PJI	30-minutes NR
PX	$\uparrow$	-2.38^b	-2.00^b	-1.38	-1.05	0.44	-0.11	-1.16	-0.22
	$\downarrow$	-2.22^b	-1.33	-1.27	-1.55	-1.22	-1.22	-1.38	-1.05
	$(\uparrow - \downarrow)$	-0.83	-1.69^c	-0.56	-0.72	1.10	0.24	-0.13	0.78
BUX	$\uparrow$	-1.55	-0.50	-1.33	-0.44	-0.55	-0.33	-0.44	-0.44
	$\downarrow$	-0.50	-0.94	0.50	0.44	-0.16	0.05	-0.11	-0.94
	$(\uparrow - \downarrow)$	-0.61	-0.40	-1.53	-1.10	-0.67	-1.05	-0.08	0.29
WIG	$\uparrow$	2.11^b	1.88^c	1.77^c	0.22	0.66	0.44	-0.33	-0.11
	$\downarrow$	0.22	0.38	0.16	0.33	-1.44	-1.22	-1.16	-0.38
	$(\uparrow - \downarrow)$	0.72	1.21	1.15	-0.83	1.26	1.80^c	0.35	-0.45
DAX	$\uparrow$	-1.61	-1.72^c	-1.88^c	-1.61	-1.33	-1.38	-1.00	-2.27^b
	$\downarrow$	-1.50	-2.00^b	-1.55	-1.27	-1.44	-1.50	0.16	0.00
	$(\uparrow - \downarrow)$	1.26	1.42	-0.51	-0.18	0.24	0.40	-1.42	-1.96^b

Note: We estimated the characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. We employed the Wilcoxon test to compare the characteristic coefficients for the periods before the financial crisis and during the financial crisis. The crisis is defined as the period starting at 2009/Q1 and lasting until 2010/Q4. The directions of the price jumps D are as follows: symbol $\uparrow$ stands for price movements up, the symbol $\downarrow$ for movements down, and symbol $(\uparrow - \downarrow)$ stands for the difference between them. A letter denotes the significance level at which we reject the null hypothesis: a for 99%, b for 95%, and c for 90%. That is the significance at which we can say that the indexes are different. A positive (negative) value in a cell means that the median of the distribution for the characteristic coefficients α before the crisis is greater (smaller) than the median for α during the crisis.

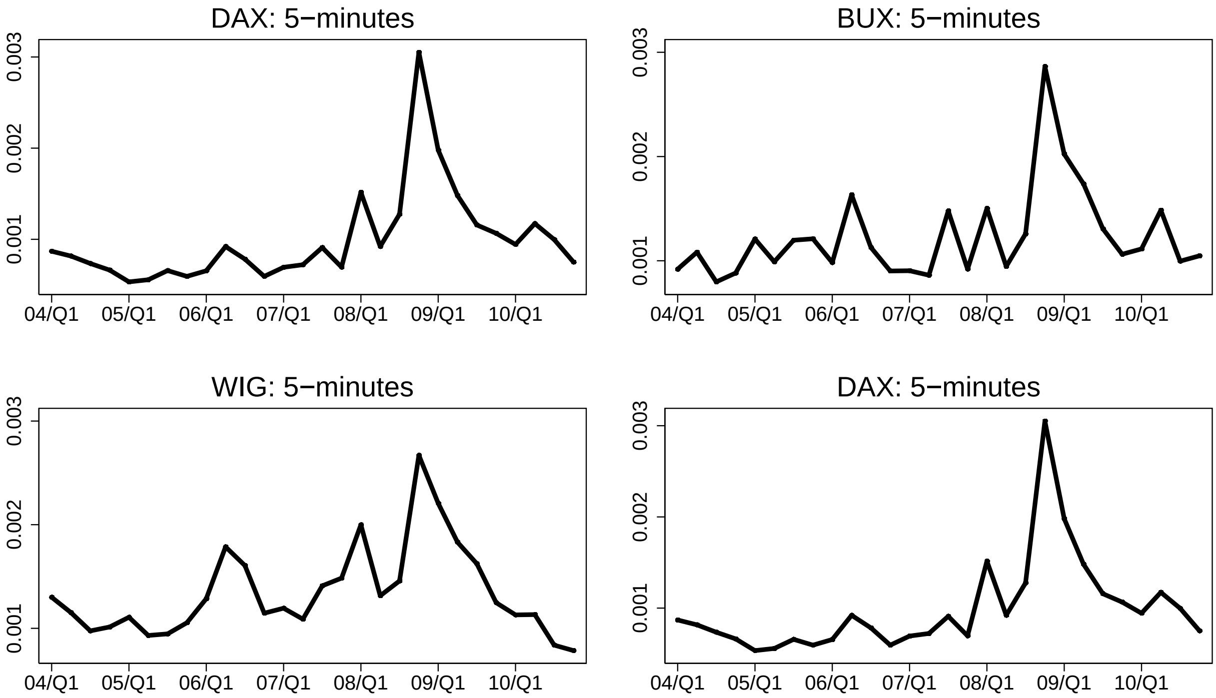
5.4. The Effect of Financial Crisis on Price Jumps

We can follow the strategy of pair-wise comparisons and ordering the indexes by their jumpiness or more precisely by their characteristic coefficient α also for periods before, during, and after the economic crisis. For that purpose we define the financial crisis as the period that started at 2009/Q1 and lasted until 2010/Q4, thus eight quarters in total. For robustness we also consider several different specifications, for example with the crisis starting at 2008/Q4 (nine quarters of financial crisis). The results were very similar, therefore we do not present them here but they are available upon request. The above ordering of the indexes and a comprehensive summary of all non-parametric comparisons are presented in Table 5.

We used the Wilcoxon test to compare the characteristic coefficients before and during the crisis for every stock market index, every frequency and both price jump indicators with both directions separately. We have also included the difference between the characteristic coefficient up and down, or $\Delta = \alpha^{Up} - \alpha^{Down}$.

Table 6 contains the Wilcoxon statistics for the test between the period before the crisis and the period during the crisis. The results suggest that there was no prevalent change in the price jump component of the price-generating process. The difference between the characteristic coefficients up and down further strengthens the findings and suggests, except in three cases, no change before or during the crisis. This result is in agreement with Novotný (2010) and suggests that there was either no change in the underlying price generating process at all or the entire price generating process was scaled

Figure 4: Standard deviation for returns by quarter.



Note: Plotted is the standard deviation by quarters for 5-minute returns.

up in such a way that the distribution of extreme price movements was similar. Using Figure 4, which depicts the standard deviation of returns for all four stock market indexes at a 5-minute frequency, we can conclude that the latter explanation is the case, i.e., the overall price generating process scaled up during the crisis, but the rate of price jumps remained untouched.

6. Conclusion

We performed an extensive analysis of price jumps using high-frequency data (5-, 10-, 15-, and 30-minute frequencies) for three emerging stock market indexes (PX, BUX, and WIG20) from the CEE Visegrad region. As a benchmark representing a geographically close and mature EU market we use the German DAX index. The time period of the data is from June 2003 to December 2010. For our analysis we employed two different indicators of price jumps: the price jump index and normalized returns. The analysis of returns revealed that the data substantially deviates from a Gaussian distribution and tends to support the presence of price jumps. We also analyze if we would observe larger negative extreme price movements compared to positive ones. However, the reverse is true and the intuitive asymmetry favoring negative price jumps does not hold, moreover, this result was robustly confirmed by both indicators.

Further, the Prague Stock Exchange differs with respect to the presence of price jumps when lower frequencies are used. Based on the theory, one would assume that the lower the frequency, the more price jumps will be observed. However, the PX index reveals almost the opposite behavior, so the behavior of the PX index significantly differs from the remaining three market indexes. One can speculate that this difference could be explained by the composition of the PX index: a small number of components, a relatively high number (and weight) of stocks with dual trading, prices determined in other exchanges, and some components not being traded with high-frequency. Simply, a relatively small number of trades with a few stocks could have a large impact on the entire PX index. These explanations, however, would need additional analysis and the market micro-structure perspective should be tested across the markets, which is beyond the scope of this study.

We have estimated the price jump properties quarter by quarter. This allows us to compare the estimated characteristic coefficients across stock market indexes and over time. We have thus employed quarterly estimates and the Wilcoxon test and show that there is no significant difference in the distributions of the characteristic coefficients moving up with respect to those moving down. Further, we have answered the question whether the price-generating process, or its price jump component, differs for all stock market indices. The results of the Kruskal-Wallis test used with quarterly estimates suggests a deviation among the indexes for high-frequency returns. A detailed pair-wise comparison using the Wilcoxon test revealed that it is the PX index that causes the disagreement and the results thus further support the presence of a PX Puzzle. Another pattern, consistent across all frequencies, both jump price indexes, and up and down moves shows that PX was clearly the most jumpy index while WIG had the smallest propensity to jump. This calls for further research, suggesting a link between market micro-structure and jump propensity. In particular, higher market volatility and also higher propensity to jump is explained by differences in the population of investors (Prague is dominated by foreign investors, while Warsaw is dominated by strong domestic institutional investors, namely pension funds), differences in the regulatory framework in Prague, where there are much weaker margin regulatory requirements, and much higher leverage possibilities in Prague.

Finally, we tested for the stability of the price jump component in time in particular during the recent financial crisis. The statistical tests suggest that the price jump component is stable before and during the financial crisis, although there are few cases when the processes were different. These disagreements occurred especially for high-frequency data.

Overall, we cast light on the issue of extreme price movements and their its distributions in CEE emerging markets. The quantitative understanding of price jumps can obviously help to decrease the risk connected with irregular but abrupt price changes and can be used to develop various financial models by computing associated risk measures. The empirical analysis presented in this study can also serve as a starting point for a larger study of the integration of financial markets, including the role of market micro-structure and the regulation of price jumps.

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Aït-Sahalia, Y. (2004). Disentangling diffusion from jumps. *Journal of Financial Economics*, 74(3):487–528.

Aït-Sahalia, Y., Cacho-Diaz, J., and Hurd, T. (2009). Portfolio choice with jumps: A closed-form solution. *The Annals of Applied Probability*, 19(2):556–584.

Aït-Sahalia, Y. and Jacod, J. (2009a). Estimating the degree of activity of jumps in high frequency data. *The Annals of Statistics*, 37(5A):2202–2244.

Aït-Sahalia, Y. and Jacod, J. (2009b). Testing for jumps in a discretely observed process. *The Annals of Statistics*, 37(1):184–222.

Bachelier, L., Davis, M., and Etheridge, A. (2006). *Louis Bachelier's theory of speculation: The origins of modern finance*, volume 13. Princeton Univ Press.

Barndorff-Nielsen, O., Graversen, S., Jacod, J., Podolskij, M., and Shephard, N. (2006). A central limit theorem for realised power and bipower variations of continuous semimartingales. In Kabanov, Y., Liptser, R., and Stoyanov, J., editors, *From Stochastic Calculus to Mathematical Finance*, pages 33–68. Springer Berlin Heidelberg.

Barndorff-Nielsen, O., Hansen, P., Lunde, A., and Shephard, N. (2008). Designing realized kernels to measure the ex post variation of equity prices in the presence of noise. *Econometrica*, 76(6):1481–1536.

Barndorff-Nielsen, O. and Shephard, N. (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of financial econometrics*, 2(1):1–37.

Barndorff-Nielsen, O. and Shephard, N. (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics*, 4(1):1–30.

Becketti, S. and Roberts, D. (1990). Will increased regulation of stock index futures reduce stock market volatility? *Economic Review*, pages 33–46.

Beirne, J., Caporale, G., Schulze-Ghattas, M., and Spagnolo, N. (2010). Global and regional spillovers in emerging stock markets: A multivariate garch-in-mean analysis. *Emerging Markets Review*, 11(3):250–260.

Broadie, M. and Jain, A. (2008). The effect of jumps and discrete sampling on volatility and variance swaps. *International Journal of Theoretical and Applied Finance*, 11(8):761–797.

Chorafas, D. (1994). *Chaos theory in the financial markets: applying fractals, fuzzy logic, genetic algorithms, swarm simulation & the Monte Carlo method to manage market chaos & volatility*. McGraw-Hill.

Cornell, B. and Sirri, E. (1992). The reaction of investors and stock prices to insider trading. *Journal of Finance*, pages 1031–1059.

Eryigit, M., Ēukur, S., and Eryigit, R. (2009). Tail distribution of index fluctuations in world markets. *Physica A: Statistical Mechanics and its Applications*, 388(9):1879–1886.

Fama, E. (1970). Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, 25(2):383–417.

Fengler, M., Härdle, W., and Mammen, E. (2007). A semiparametric factor model for implied volatility surface dynamics. *Journal of Financial Econometrics*, 5(2):189–218.

Fortune, P. (2001). Margin lending and stock market volatility. *New England Economic Review*, 2001(4):3–26.

Gatheral, J. (2006). *The volatility surface: a practitioner's guide*, volume 357. Wiley.

Giot, P., Laurent, S., and Petitjean, M. (2010). Trading activity, realized volatility and jumps. *Journal of Empirical Finance*, 17(1):168–175.

Gopikrishnan, P., Plerou, V., Amaral, L., Meyer, M., and Stanley, H. (1999). Scaling of the distribution of fluctuations of financial market indices. *Physical Review E*, 60(5):5305–5316.

Hanousek, J., Kočenda, E., and Novotný, J. (2011). The identification of price jumps. *CERGE-EI Working Paper*, 434.

Jayasuriya, S. (2011). Stock market correlations between china and its emerging market neighbors. *Emerging Markets Review*, 12(4):418–431.

Jiang, G. and Oomen, R. (2008). Testing for jumps when asset prices are observed with noise. *Journal of Econometrics*, 144(2):352–370.

Jiang, J., Li, W., Cai, X., and Wang, Q. (2009). Empirical study of recent chinese stock market. *Physica A: Statistical Mechanics and its Applications*, 388(9):1893–1907.

Johannes, M. (2004). The statistical and economic role of jumps in continuous-time interest rate models. *The Journal of Finance*, 59(1):227–260.

Joulin, A., Lefevre, A., Grunberg, D., and Bouchaud, J. (2008). Stock price jumps: News and volume play a minor role. *Wilmott Magazine*, Sep/Oct 2008:1–79.

Kennedy, D., Sivakumar, R., and Vetzal, K. (2006). The implications of ipo underpricing for the firm and insiders: Tests of asymmetric information theories. *Journal of Empirical Finance*, 13(1):49–78.

Kleinert, H. (2009). *Path integrals in quantum mechanics, statistics, polymer physics, and financial markets*. World Scientific Pub Co Inc.

Kruskal, W. and Wallis, W. (1952). Use of ranks in one-criterion variance analysis. *Journal of the American statistical Association*, pages 583–621.

Lee, S. and Mykland, P. (2008). Jumps in financial markets: A new nonparametric test and jump dynamics. *Review of Financial studies*, 21(6):2535–2563.

Li, X.-M. and Rose, L. (2009). The tail risk of emerging stock markets. *Emerging Markets Review*, 10(4):242–256.

Lin, L., Ren, R., and Sornette, D. (2009). A consistent model of ëxplosivef̈inancial bubbles with mean-reversing residuals. *Swiss Finance Institute Research Paper Series*.

Madhavan, A. (2000). Market microstructure: A survey. *Journal of Financial Markets*, 3(3):205–258.

Mandelbrot, B. and Van Ness, J. (1968). Fractional brownian motions, fractional noises and applications. *SIAM review*, 10(4):422–437.

Mann, H. and Whitney, D. (1947). On a test of whether one of two random variables is stochastically larger than the other. *The annals of mathematical statistics*, 18(1):50–60.

Mantegna, R. and Stanley, H. (2000). *An introduction to econophysics: correlations and complexity in finance*. Cambridge Univ Pr.

Merton, R. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of financial economics*, 3(1-2):125–144.

Nietert, B. (2001). Pricing relevance of stock price jumps: Capm and variance decomposition. In *EFMA 2001 Lugano Meetings*.

Novotný, J. (2010). Were stocks during the financial crisis more jumpy: A comparative study. *CERGE-EI Working Paper*, 416.

Pan, J. (2002). The jump-risk premia implicit in options: Evidence from an integrated time-series study. *Journal of Financial Economics*, 63(1):3–50.

Plerou, V., Gopikrishnan, P., Amaral, L., Meyer, M., and Stanley, H. (1999). Scaling of the distribution of price fluctuations of individual companies. *Physical Review E*, 60(6):6519.

Tiniç, S. (1995). Derivatives and stock market volatility: Is additional government regulation necessary? *Journal of Financial Services Research*, 9(3):351–362.

Vaglica, G., Lillo, F., Moro, E., and Mantegna, R. (2008). Scaling laws of strategic behavior and size heterogeneity in agent dynamics. *Physical Review E*, 77(3):036110.

Wilcoxon, F. (1945). Individual comparisons by ranking methods. *Biometrics Bulletin*, 1(6):80–83.