Empirical Analysis of Price Jumps on the PSE and Visegrad Indexes

Jan Novotny

jnovotny@cerge-ei.cz

Abstract

In this paper, I study various characteristics of price jumps using high frequency data. I employ 5, 10, 15 and 30 minute market data on the main indexes from the Prague, Warsaw, Budapest and Frankfurt Stock Exchanges for the period spanning from June 2003 to the end of 2008. I consequently employ five different indicators for price jumps, namely returns, price jump index, normalized returns, price jump index for levels and extreme returns. The usage of more than one indicator gives the results robustness. I focus on the tail part of the distributions to confirm the power-law behavior and study its asymmetry for jumps up and down. I perform the same analysis on a monthly and quarterly basis to check for any change due to financial turmoil. I then study the distribution of jumps using various indicators over trading and also check for a change in behavior due to the financial crisis. My analysis confirms power-law behavior of the jumps with a distribution close to the inverse-cubic law. The asymmetry of the jumps does not follow intuitive asymmetry preferring the jumps down. The distribution of jumps over a trading day depends on the frequency and corresponding time window. All four indexes reveal a high activity around the opening of the US markets. The characteristics of price jumps were not significantly influenced by the financial turmoil.

JEL Classification Codes: G15, P59

1 Introduction

The proper understanding of the market dynamics is of very great interest from both academic as well as a practical point of view. The distribution of the stock market price fluctuations using high frequency data has been extensively analyzed before, mainly for mature markets. Extreme price movements or price jumps play a crucial role since they indicate that something important has happened on the market. The source of price jumps can originate in completely different events ranging from scheduled macroeconomic news announcements to weather conditions to non-public events like insider trading activity. The different source of price jumps is exactly the reason why it is difficult to ex ante predict all the characteristics of price jumps. We can rather estimate them empirically and, having in hand their properties, we can try to understand them. In this work, I make the first step towards understanding the price jumps by estimating the broad range of the price jumps' properties. Besides the standard characteristics of price jumps as a high-frequency process I also study properties like the stability the price jumps over time, the asymmetry of price jumps for movements up and down and the distribution of price jumps over a trading day.

The extreme price jumps, or, in other words, the tail properties of price jump distributions, have been studied for example in [1, 2, 3, 4, 5, 6]. The general empirical understanding of tail distributions suggests that price jumps follow a distribution close to an inverse cubic one, i.e., it does not behave either as a pure Gaussian, or as a Levy-like distribution with infinite variance. Such stylized fact is just a small part of the picture describing the full knowledge about price jumps.

The goal of this paper is to answer five questions on the properties of price jumps for the stock market indexes in the Visegrad region. In this analysis I employ three stock market indexes from three Visegrad countries[1], namely the Czech Republic (PX) [7], Hungary (BUX) [8], and Poland (WIG20) [9]. I have omitted the index from Slovakia [10] since its stock market is of the very low volume. To complete the analysis, I have also included the index from Frankfurt [11] (DAX), Germany, since this stock market represents the geographically closest mature market[2].

Price jumps can be simply defined as a (logarithmic) difference between two consecutive prices. The difference depends on the frequency of data. However, taking the returns on their own as a time series of independent changes misses the important criteria. The criteria is the fact that we naturally judge a given price movement to be a price jump if it is bigger compared to the previous price changes. Therefore, it is more convenient to define an indicator for price jumps. Such an indicator will be able to capture the real price jumps as they are perceived by market participants. These indicators have to take into account the current situation on the market, namely the current trend, usually estimated by moving averages or the market volatility, represented by the standard deviations over some time window. In this analysis I will use four different estimators to model price jumps. The first two are identical with the two definitions briefly introduced above, the price jump index and normalized returns. Further, I define the price jump index directly for levels, which mimics how the traders view the markets, and, finally, extreme jumps as those returns which exceed a given threshold. Having in hand the class of suitable indicators for price jumps, I can give the answer to five questions that are described below and which study the price jumps' characteristics from different point of view.

The first and very important question related to the extreme price jumps focuses on its underlying high frequency distribution. I take the time series of price jumps as a whole and ask for their properties globally to get the basic knowledge how much are extreme jumps likely and how the four indexes differ, or what they have in common. The basic high-frequency properties were extensively studied in the literature. For example, Eryigit *et al.* in [3] studied price jumps for a broad range of stock market indexes. The authors used normalized returns, defined as the difference between the return and its long-term moving average, normalized by standard deviation. They have tested a broad range of possibilities for tails of the price jump distributions, where they mainly focused on

[1]One of the important reasons for this choice is the availability of the data for the Visegrad markets and the Frankfurt Stock Exchange.

[2]One can easily point out that it would be more reasonable to include the London Stock Exchange [12], which is rather commodity oriented and thus more related to, say, the Prague Stock Exchange, which highly depends on the energy producing industry, e.g., CEZ; however, I am interested in the Visegrad geopolitical region and thus use Frankfurt as geographically the closest and most mature neighborhood market.

the power-law distribution and tested for characteristic exponents. They also focused on the relation between a characteristic exponent of the tail distribution and the degree of the maturity of a market. The tail distribution of emerging markets, namely for the Chinese markets, was also studied in [4]. It was shown there that power-law behavior is valid only for long term distributions, i.e., for moving averages with very long time window, while for short term distributions the tail behavior behaves more in an exponential manner. The tail part of the price jump distributions was also studied by Joulin *et al.* [5], where the authors used the price jump index and 1 minute data to conclude that the tail distribution of individual stocks follows an inverse cubic distribution.

The second question I answer in this work is whether there exists asymmetry between jumps up and down. The standard textbook approach assumes price movements follow a symmetric distribution which further produces symmetric price movements [13, 14, 15, 16]. However, it is a well known fact that the price movements are not symmetric, they rather tend to behave in an asymmetric way. The intuition behind that lies in the very negative perception of movements down, which are rather connected with events like market panic, which is nothing else but very fast irrational behavior following the crowd. On the other hand, the movements up cause the more slow reactions of market players. This is further confirmed by the fact that conventional VaR approaches use only the negative side of the distribution to evaluate possible extreme downward movements, while the movements up are rather used for building expected returns estimates. In theory, the asymmetry between movements up and down is reflected by the fact that the corresponding volatility of the process does not follow rules guided by symmetric distributions.

Advanced techniques that takes into the account also the asymmetric response, for example, extends the family of the standard ARCH/GARCH models [14] by, e. g., EGARCH [18], or NGARCH [19], for more complete references, see [20]. The key feature of these extensions lies in the fact that σ_t^2 responds asymmetrically on residuals with negative and positive signs. Another possible way to incorporate the asymmetric movements of stock indexes is the use of price ranges as a measure of price volatility, while the upper and lower bounds for price movements follows different dynamics; see for example [21, 22], where Conditional Autoregressive Range model is employed to estimate the asymmetric price movements. Generally, the most extreme events tend to be downward that are connected with crashes and market panic. For example, Plerou *et al.* [1] used normalized returns for stock prices of individual companies and estimated the tail behavior for positive and negative normalized returns separately. They have found that the extreme negative normalized returns tend to happen more often compared to the extreme positive cases. In the context of my analysis, I employ the price jump index and normalized returns to estimate the tail behavior for movements up and movements down separately. Despite the fact that both previous definitions of price jumps are not identical, they should produce equivalent results.

The third question studied in this work asks whether the price jumps have different properties over a trading day. To answer it, I use price the jump index and extreme returns and study their distribution over a trading day. A complementary sub-question related to the study of the properties during the trading day asks how likely we are going to observe a given number of price jumps per day. The answers help to shed some light on the market mechanisms during a day. In order to interpret results properly, we have to realize that the three Visegrad markets are regional players, which are not supposed to influence the mature markets, but rather to be influenced by them. Since the Visegrad markets operate in the time when the US and Asian markets are mainly closed, we can expect high activity immediately after the Visegrad markets start to operate, since they accommodate information from the previous day. The Budapest Stock Exchange starts to operate as the first one, thus we can expect that it will catch the results from the previous night and accommodate them as the first one, while the remaining two stock exchanges besides the effect itself can see how the effects were accommodated by the already opened Visegrad market(s). Any further similarity can also suggest the possible correlation between markets.

The fourth question I answer in this work is the effect of the different institutional micro-structure of the Visegrad Stock Exchanges on the price jump characteristics. The most striking difference in the micro-structure is in the case of the Warsaw Stock Exchange, where the Open Pension Funds (OPFs) represents the important role. The OPFs are one of the Pillars of the Polish Pension Fund. These OPFs are restricted by law to invest only five percent of their assets abroad [24], the rest has to be invested in domestic markets. Under normal circumstances, such an institutional restriction acts as a smearing mechanism, since when the prices in stock markets grow, OPFs are selling assets, which decreases the price of stocks, and vice versa for the case when stock prices falls. This is the reason why the Warsaw Stock Exchange is an important player in the region. However, in the time of financial turmoil, this mechanism can cause a feedback which can even broaden the impact of the global financial crisis, since the market is over-valued in the long term due to the permanent presence of investors creating a bubble, which can in extreme situations burst. The situation at the remaining two Visegrad Stock Exchanges is different, since there are no such institutional requirements for local players and therefore the institutional investors can behave more aggressively and therefore be more opportunistic. This institutional difference raises a question, whether there will be any difference in the language of price jumps, which can be explained exactly by this phenomena. The difference in the micro-structure of the markets can cause the difference among the three Visegrad markets; therefore, whenever the PX and BUX will have completely different properties compared to the WIG, I can seek to determine an economic intuition behind that based on such a difference in the micro-structure.

Finally, the last question answered in this work is related to the current financial turmoil. There is no doubt the current financial crisis is by its scale of the biggest in the modern history. The intuitive answer would be that the properties of the price jumps, as studied in this work, will be probably changed. To answer this question properly is, besides academic curiosity, also important from the institutional point of view, as it teaches us the experience from the crisis in 1987. Some experts believe the crisis was caused by the mass usage of the Gaussian distributions in their numerical models, since the Gaussian distributions are not able to explain extreme price jumps and models using them will misinterpret reality [23]. No matter whether we believe this story or not, the study of the change of price jumps in the long-run horizon and especially during the current financial crisis can help to explain what happened there. However, such a study would require data even in the beginning of 2009, since in the Central European countries, the crisis arrived later. Thus, I answer the question at least partially for the beginning of the financial turmoil by performing the analysis of the tail parts of distributions for the price jump index and normalized returns month by month and quarter by quarter and ask whether and how the various aspects of the price jumps studied in this work vary in the long-run horizon.

2 Data

I use data with 5 minute frequency for main indexes from the Prague Stock Exchange (the PX index), the Budapest Stock Exchange (the BUX index), the Warsaw Stock Exchange (the WIG20 index) and from the Frankfurt Stock Exchange (the DAX index) spanning from June 2003 to December 2008. Data were originally with 1 minute frequency, however, due to many missing values, I have obtained from these data four lower frequencies, namely 5, 10, 15 and 30 minute. Whenever there were missing values at a given minute, I have interpolated this datum from the neighborhood values. For 5 minute frequency, I have used ± 1 minute, for 10 and 15 minute frequency, I have used ± 2 minutes, and, finally, for 30 minute frequency, I have used ± 3 minutes. When there were no values, I treat it as missing value. Since the main purpose of this research is to study scaling of the market with respect to the phenomena present due to market dynamics, I have cut-off the very beginning and the very end of the trading day. Be more specific: i) PX: opening hours from 9:15 to 16:00 and trading period from 9:30 to 15:30; ii) DAX: opening hours from 9:00 to 17:30 and trading period from 9:30 to 17:00; iii) BUX: opening hours from 9:00 to 16:30 and trading period from 9:30 to 16:00; and, iv) WIG: opening hours from 9:00 to 16:20[3] and trading period from 9:30 to 16:00. In the following, I also distinguish between two types of times, the calendar time and the trading time, where the latter one runs over the trading periods only, i.e., the last minute at the end of the trading period is followed by the first minute of the next trading period.

The close prices of the four indices for 5 minute frequency are in the Figure 19 in the Appendix. There can be found also Figures for three lower frequencies. All the Figures clearly shows that all

[3] My data set is further restricted since up to the September 2005, data for WIG index starts at 10:00.

four indexes follow similar patterns, namely the sudden drop due to to the financial turmoil in the third quarter of 2008 is evident.

The Prague Stock Exchange (PSE) [7] is the stock exchange in the Czech Republic. The main index on the PSE is the PX index, which was created on March 20 2006 and continuously tied up on the previous PX 50 index. The index is based on the most prominent stock titles on the PSE. The Budapest Stock Exchange (BSE) [8] is the stock exchange in Hungary. The official index of the BSE is the BUX index, which is based on the basket of up to 25 different stock titles. The Warsaw Stock Exchange (WSE) [9] is stock exchange in Poland. The main index on the WSE is the WIG20 index, which is composed of 20 the biggest and most liquid companies.[4] Finally, I have completed the set of three indexes by the index from the Frankfurt Stock Exchange (FSE) [11], since the FSE is geographically close to three Visegrad stock exchanges and represents proxy for a mature market. The main index of the FSE is the DAX index, which measures the performance of 30 the largest German companies in terms of order book volume and market capitalization. It is based on the prices of the electronic trading system Xetra.

3 Returns

There are several possibilities how to technically define a jump. As a first step, I perform price jumps analysis on the log-returns itself. Returns are not the best proxy for estimation of price jumps since they do not relate the current realization of return to the previous situation on the market. Despite this fact, basic knowledge obtained from study of returns give the first insight on the behavior of jumps, since the usual indicators for price jumps are defined with the help of returns.

3.1 Statistics of Returns

Throughout this work I use returns at time t defined in a standard way as

$$r(t) = \log(R(t)/R(t-1)) ,$$

where $R(t)$ is the price level of a given index. Be more precise, the return itself should carry an index for frequency. However, I have omitted it throughout the paper, since it will be clear what frequency was used. The returns for all four indexes with 5 minute frequency are depicted in the Figure 1. Time frame used for this Figure is trading time, which captures just the information used in this study. The Figure clearly shows two important facts. First, there is no significant difference visible just by an eye between extreme movements upward and downward. This can suggest that if there will be any difference in the tail behavior between the two directions it should not be too significant. Second, the end of the period is characterized by higher volatility, which suggests that something happened on the market. This is nothing else, but the financial turmoil we have already discussed in the Introduction.

To show the difference between trading and calendar times, the Figures in the Appendix contain a comparison of returns for calendar time, trading time and the returns for time, which does not belong to trading time. There are Figures for all four indexes.

The standard statistics for the data used in this work can be found in the Table 1. At first sight, we can see data are not Gaussian-like. To test this claim formally, I apply the Jarque-Bera statistics [25]. The Jarque-Bera statistics is defined as

$$JB = \frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) , \tag{1}$$

with S being the skewness, K stands for the kurtosis and N is the number of observations. The test is asymptotically equal to χ^2_2 test with two degrees of freedom. The test specifies:

$$H_0 : S = 0 \wedge K = 3 \, (\Leftrightarrow \sim N(.,.))$$

[4]I use the symbol WIG throughout this paper for the index composed of the twenty the most capitalized companies traded on Warsaw Stock Exchange.

Figure 1: Returns for all four main indexes used in this study: the PX index (Prague Stock Exchange), the BUX index (Budapest Stock Exchange), the WIG index (Warsaw Stock Exchange) and the DAX index (Frankfurt Stock Exchange). Frequency of the data is 5 minute. Time period spans from 06/2003 to 12/2008. Time used is trading time, where a part of the period at the beginning and at the end of the trading day were cut-off (see text for more details).

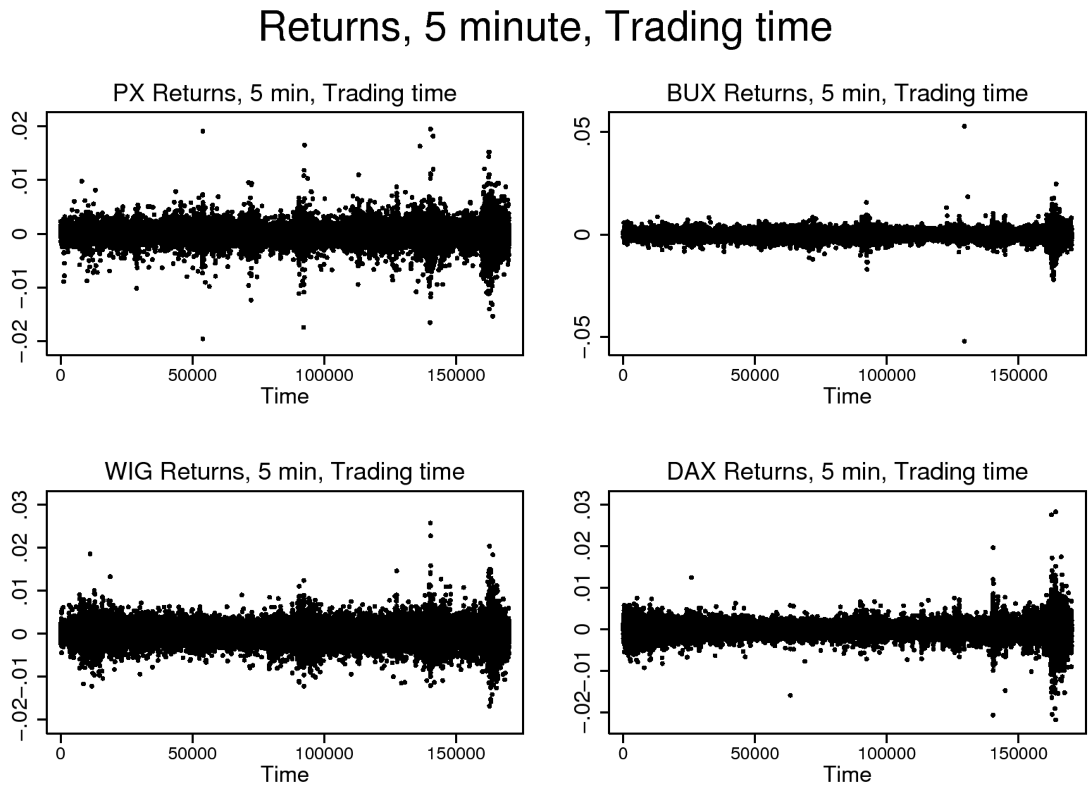
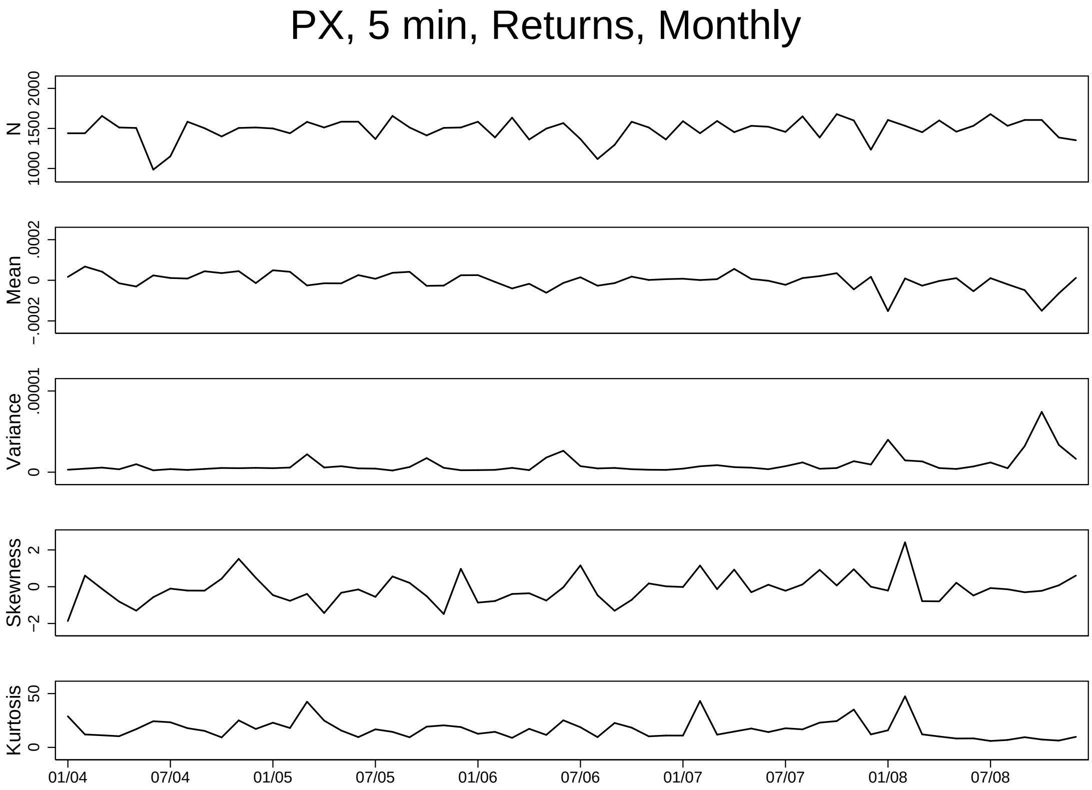


Table 1: The Table summarizes the standard statistics of log-returns $r(t)$ for all four main indexes used in this study: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange). All four frequencies were used: 5 minute, 10 minute, 15 minute and 30 minute. The Table shows for every combination of an index and frequency (f) the number of observations (N), mean of returns (μ), standard deviation (σ), skewness (S), kurtosis (K) and Jarque-Bera statistics (JB-stat.).

Index	f	N	μ	σ	S	K	JB-stat.
PX	5	99 802	-9.13e-07	.00096	-0.2914	33.13	3.77e6
PX	10	49 807	-1.34e-06	.00147	-0.6364	28.77	1.38e6
PX	15	33 333	1.86e-07	.00191	-0.6301	29.28	9.61e5
PX	30	16 594	-1.01e-06	.00279	-0.7119	21.08	2.27e5
BUX	5	10 890	-1.55e-05	.00123	-0.2485	78.42	2.58e6
BUX	10	55 138	-2.70e-05	.00181	-0.5953	48.33	4.72e6
BUX	15	37 222	-3.83e-05	.00227	-1.2000	44.58	2.69e6
BUX	30	19 298	-7.57e-05	.00338	-0.6181	22.91	3.20e5
WIG	5	104 075	-4.69e-06	.00145	0.1046	12.61	4.00e5
WIG	10	52 130	-8.51e-06	.00204	0.0152	11.62	1.61e5
WIG	15	34 746	-9.11e-06	.00246	0.0662	11.06	9.41e4
WIG	30	17 443	-2.66e-05	.00349	0.2007	13.01	7.30e4
DAX	5	127 661	-2.14e-06	.00106	-0.1076	34.06	5.13e6
DAX	10	64 521	-5.07e-06	.00151	-0.0778	31.70	2.21e6
DAX	15	43 480	-6.69e-06	.00184	-0.0997	24.19	8.13e5
DAX	30	22 429	-1.42e-06	.00286	3.5361	186.92	3.16e7

Figure 2: Basic statistical properties of the PX index (Prague Stock Exchange). Plotted are: number of observations (N), mean, variance, skewness and kurtosis for returns using 5 minute data. Statistical variables are calculated month by month, where the period for this calculation ranges from 01/2004 to 12/2008.



$$H_A : \not\sim N(.,.) .$$

The Jarque-Bera statistics is also summarized in the Table 1. In all the cases we can clearly see that statistics is high, which suggests that all time series used in this analysis are not Gaussian-like.

3.1.1 Returns during Financial Turmoil

In the following sub-section I focus on the study of returns' properties over time, i.e., how the statistical properties vary when they are calculated month by month. In the Figure 2, I present such an overview for the PX index using 5 minute returns. Namely, I plot number of observations, mean, variance, skewness and kurtosis of the sample as they are calculated month by month. The same plots for three remaining indexes BUX, WIG and DAX are in the Appendix.

The immediate observation one can make from the Figure is the increase of variance in the middle of 2008, when the financial turmoil appeared. The variance was rising just for few months, after this period markets accommodated the announcement of worse times and became less volatile. The financial crisis caused also the drop in levels of the stock prices, which can be seen from the low values of means for all four indexes. However, a first sign of bad news can be seen in the first month of 2008, when we observe a significant drop in mean and increase in variance for all four indexes. The extreme movements were swiftly corrected in the following few months.

3.2 Tail Behavior of Returns

In this paper I focus merely on the extreme events. The distribution of extreme returns can be depicted graphically as is in the Figure 3. The Figure shows the normalized probability distribution function of positive returns to be higher than a given threshold s, or, for negative returns the normalized probability distribution function of returns to be lower than a threshold value $-s$. The

Figure contains all four indexes at all four frequencies. The insight is that higher the frequency, the higher the probability for larger returns to occur.

Further, I test the tail behavior of returns, or, the probability distribution functions for the extreme returns, for a presence of the power-law behavior. In the case of positive returns, the power-law behavior reads:

$$P(z > s) \propto s^{-\alpha}\,, \tag{2}$$

where α is a characteristic scale parameter of the distribution, which generally depends on the frequency[5]. The eq. (3) says that probability of the return to be higher than a given threshold s is proportional to $s^{-\alpha}$. This relation tells us the higher is the characteristic coefficient α, the less likely we observe extreme returns. For negative returns, the power-law behavior can be analogously defined as:

$$P(z < -s) \propto -|s|^{-\alpha} \Leftrightarrow P(|z| > s) \propto s^{-\alpha}\,. \tag{3}$$

A simple way to reveal such a behavior is to plot the tail part of distributions in a log-log way, i.e., perform natural logarithms of eqs. (2,3). Being a power-law behavior valid, the log-log transformation linearizes the relation

$$\ln(P(z > s)) \propto -\alpha s\,. \tag{4}$$

The linearized version of the Figure 3 is presented in the Figure 4, where I plot the log-log quantity from the Figure 3. The both sides of linearized returns from this Figure tend to exhibit qualitatively a power-law behavior. The next step is to estimate the characteristic parameter α.

I employ the following algorithm to estimate the characteristic coefficient α for every frequency and index: Using OLS, I estimate α for various tail intervals of the log-log transformed probability distributions. I take the result which has the highest R^2. Such an algorithm is simple and in agreement with visual observation of the linear region of tails. The algorithm is simple thus I have performed an eye check of the results to avoid any excesses. The alternative approach would be to use e.g. MLE or Principal Component Analysis [27]. The results of the scale behavior analysis are in the following Table 2. In general, the characteristic coefficient is smaller

Validity of the fit can be also judged according to the value of the standard error. If the error is too big compared to other estimates, it question the fit itself, since the behavior captured by the fit is less linear. To illustrate the case, see the WIG at 5 minute frequency. The positive side of returns has relatively low standard error (compared to other values in the Table 2), which is in agreement with a visual inspection of the right hand side of the Figure 4. On the other hand, the negative side of the same variable has a large standard deviation, which is again visually confirmed on the left hand side of the Figure 4, where the behavior does not reveal a linear-like behavior.

4 Price Jump Index

The simple and intuitive indicator to study a price jumps and its distribution is to compare an absolute value of returns to the average value of the same quantity [5]. As an average value, I use the moving average of the length T, where T is counted in time steps and includes a current value,

$$< |r(t)| >_T = \frac{1}{N_T} \sum_{i=0}^{N_T} r(t-i)\,, \tag{5}$$

where N_T is the number of observations in the time window[6].

Hence, the price jump index $j_T(t)$ at the moment t using the time window T is defined as

[5] An alternative way to define a Probability distribution can be found in [26], where the authors use bins, i.e., they divide the r-axis into equidistant intervals Δr and count the number of r's in every bin. Then they normalize the number of hits per bin by the length of the bin and by the total number of data points.

[6] Since my data could contain a missing entries, the number of observations may not be necessary the same for every time window. If no missing value is present than $N_T = T - 1$.

Figure 3: The normalized probability distribution function of the negative and positive sides of returns to be higher/lower than a given threshold s. The Figure contains for all four indexes: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange), and all four frequencies: 5, 10, 15, and 30 minute. The symbols for frequencies are used as follows: (5 minute: $+$,10 minute: x, 15minute: $\triangle$, and 30 minute: $\diamond$).

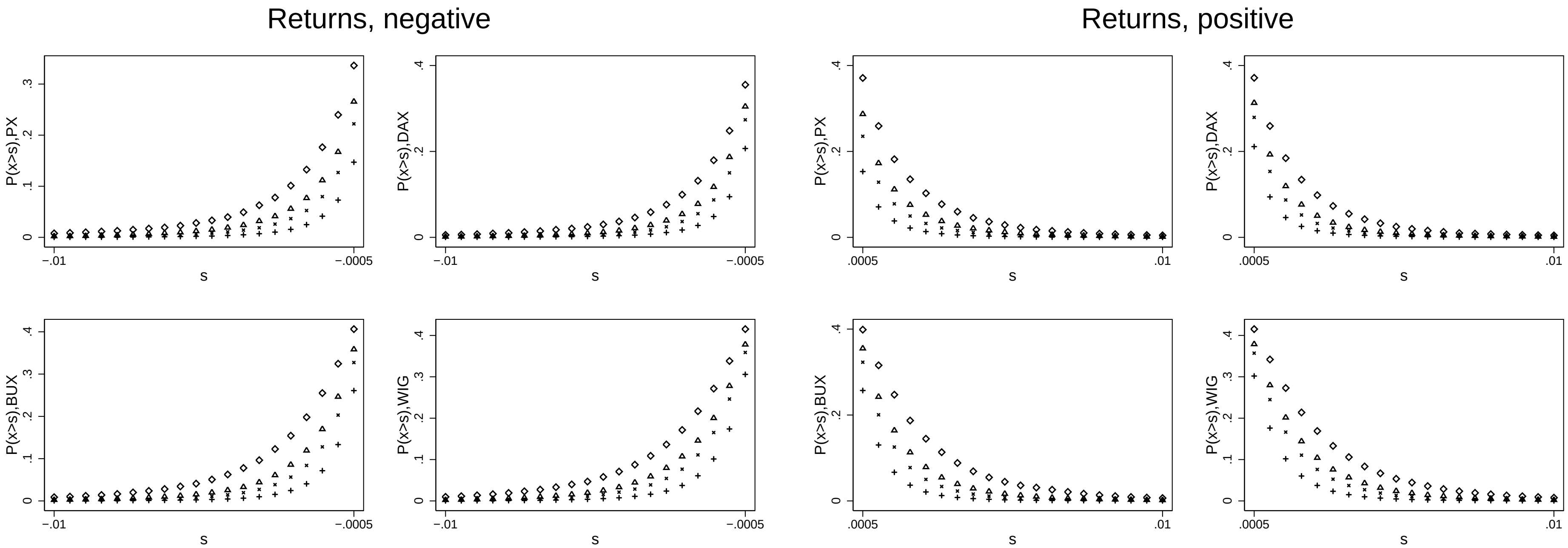


Figure 4: Negative and positive side of returns after the log-log transformation for all four indexes: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange), using all four frequencies: 5, 10, 15, and 30 minute. The symbols for frequencies are used as follows: (5 minute: +,10 minute: x, 15minute: △, and 30 minute: ◇).

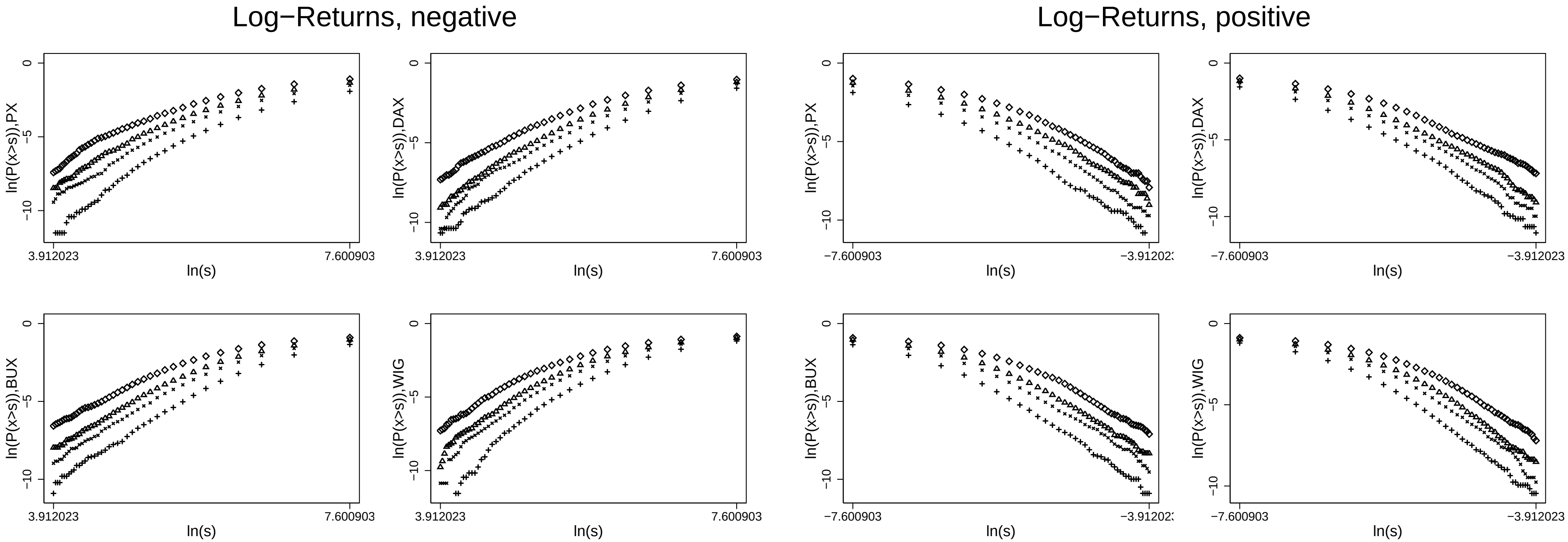


Table 2: The estimated characteristic coefficient α for returns for all four indexes: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange), using all four frequencies: 5, 10, 15, and 30 minute. Positive and negative returns are tested separately (the sign + stands for positive returns and − for negative returns). Value in brackets is the corresponding standard deviation. The values were estimated by OLS where the fit with the best R^2 were chosen.

$\alpha(\sigma_\alpha)$	5 min	10 min	15 min	30 min
PX, +	3.151 (.038)	3.411 (.044)	3.030 (.041)	3.084 (.047)
PX, -	4.159 (.093)	3.208 (.046)	3.266 (.055)	3.010 (.114)
BUX, +	3.351 (.037)	3.017 (.037)	3.124 (.039)	2.931 (.036)
BUX, -	3.177 (.027)	3.157 (.030)	3.086 (.023)	2.610 (.023)
WIG, +	3.716 (.046)	3.592 (.090)	3.787 (.038)	3.062 (.024)
WIG, -	6.050 (.204)	4.011 (.083)	3.746 (.042)	3.475 (.055)
DAX, +	3.600 (.055)	3.818 (.101)	2.884 (.071)	2.339 (.014)
DAX, -	3.476 (.065)	3.058 (.092)	3.541 (.057)	2.692 (.053)

$$j_T(t) = \frac{|r(t)|}{< |r(t)| >_T} \,. \tag{6}$$

The distribution of the price jump index $j_T(t)$ should follow a tail behavior [5] $\propto s^{-\alpha_T^{(f)}}$, i.e.,

$$P(j_T > s) \sim s^{-\alpha_T^{(f)}} \,, \tag{7}$$

where $\alpha_T^{(f)}$ depends both on the frequency of the data f and the length of the time window T. In the following, I will omit the frequency index, since it will not be necessary to keep it explicitly. On the other hand, the index for the time window T will be kept explicitly, since for every frequency I usually perform a robustness check over the length of the time window T. Joulin *et al.* in [5] advocate that the characteristic parameter α tends to be around 4 for a large set of US stocks[7], which corresponds to the inverse cubic distribution [1, 2]. On the other hand, the value of $\alpha \leq 3$ indicates the Levy-like behavior with an infinite volatility [28].

I estimate the characteristic coefficient α_T corresponding to the distribution of the price jump index for three Visegrad region main stock indexes: the PX index (Prague Stock Exchange), the BUX index (Budapest Stock Exchange), the WIG index (Warsaw Stock Exchange), and for one mature stock index: the DAX index (Frankfurt Stock Exchange). I employ four frequencies: 5, 10, 15, and 30 minute.

First, I will discuss the scale behavior depending on the length of the moving average. I explicitly take into account moving average with $T = 12$, 24, 100, 1000, 2000 and 5000 time steps, i.e., the length in minutes depends also on the used frequency. The time steps $T = 12$ and 24 are taken in the calendar time to focus on the current effects and does not take into account history. The time widows $T = 100$, 1000, 2000 and 5000 are used in the trading time to mainly focus on the long-term averages. The length of the time window defines a filtering property of the price jump index, i.e., it tunes up the frequency of processes which will be captured by the price jump index. Generally, the shorter the moving average, the higher frequency is mimicked and, therefore, the more noisy processes are captured. On the other hand, the very long time window means the price jump index will be sensitive even to very slow processes.

The Figure 5a depicts the tail behavior of the probability distribution from (7) for the main Prague Stock Exchange index PX, using all four frequencies: 5, 10, 15 and 30 minute, and all six time windows: $T = 12$, 24, 100, 1000, 2000 and 5000. The Figure clearly shows that, longer the time window T, the higher is the probability of the occurrence of extreme events. This result is not surprising, since when one take a short time window for moving average, one actually realizes a filter, which filters out current phase of a wave while it takes just a process with higher frequency.

[7] They use 1 minute frequency and $T = 120$, i.e., two hours moving average.

This is also in agreement with [1], where the exponential behavior of tails was explicitly tested. It is also supported by theory and empirical evidence presented in [28].

Analogously to returns, I transform eq. (7) by performing a natural logarithm on both sides to linearize it. The example of the log-log transformed distribution of the extreme value of the price jump index for the PX index is in the Figure 5b. One can simply see there a linear behavior of tails for all the long-term windows as well as clear non-linearity for the short-term ones. I have repeated the previous Figure for two remaining Visegrad indexes (BUX and WIG) and for the DAX index, which can be found in the Appendix.

Then, I estimate numerically the values of the characteristic exponent α_T for the PX index as it depends on: *i)* frequency: 5, 10, 15 and 30 minute; and *ii)* the length of the time window: $T = 1000$, 1000, 2000 and 5000. I have not used the two shortest time windows since the tails are clearly non-linear and therefore any estimation would be *a priori* false. The algorithm to estimate α_T is the same as in the case of returns: Using the OLS, I have estimated α for various tail intervals of linearized distributions. I have taken the result with the highest R^2.

The detailed summary of all results can be found in the Appendix. The values of α_T for the longest time window $T = 5000$ using all four indexes and all four frequencies is summarized in the Table 3. The Table gives interesting insight on the behavior of markets.

First, by comparing the characteristic coefficients for the highest frequency (5 minute), the PX index has significantly the lowest value of all the indexes. This result suggests the presence of extreme price jumps there. On the other hand, the WIG index has suppressed the presence of extreme events. The lower frequencies also reveals another important pattern that further distinguish the behavior of the PX index compared to other three indexes. In the case of the PX index, the characteristic coefficient increases as frequency decreases. This implies the less extreme events is present for lower frequencies. On the other hand, the remaining three indexes reveals completely different behavior, the characteristic coefficient is decreasing with decreasing frequency. Such a behavior tends to be supported by analogous results with shorter time window, see the Appendix. The behavior of the three indexes is similar to the behavior of returns. where the tail behavior is for high frequency exponentially suppressed. The exponential suppression can be viewed as a behavior with a very high characteristic coefficient. However, the PX index behaves in opposite way, which gives to rise a “PX puzzle”.

The explanation for the “PX puzzle” can lie in the fact that a lot of small emissions is traded on the PSE. The price of small emissions, or, the units of small volume, can be easily influenced in a very short time. The explanation can be seen in the often used parallel between volume of the traded assets and the mass in dynamics. The heavier the object is the more effort has to be spent to make it move[8]. Consequently, the fast movements, when viewed from the longer perspective, are averaged out and the movements are not so jumpy.

The micro-structure of the WIG index, as was described in the introduction, can also explain why there are so few extreme events compared to other three markets. The presence of the OPFs acts as a smoothing factor since they have to buy when price goes down and vice versa, therefore the price level is rather smooth.

Finally, the DAX index reveals for the slowest frequency the Levy-like behavior, which, in theory, implies infinite volatility.

4.1 Up/Down Asymmetry

The previous study did not distinguished between upward and downward movements. Both directions were considered together and the characteristic coefficient α_T was estimated for both directions together. However, it is reasonable to think the upward and downward movements separately. Hence, I have defined the price jump index in the same way as above; however, I have divided the price jump index into two groups according to the direction of movements they correspond. The zero

[8]Another parallel is to make the bell ring. Having a small bell, it is easy to make it ring fast. However, the big heavy bell needs more effort to make it ring even slowly. Further, the movement of fast ringing bell averages out when observed from a “frozen” perspective, analogous to taking photos with a long exposition, and thus seems to be more static than the slower big one.

Figure 5: LHS: a) Tail part of the price jump index distribution for the PX index, using all four frequencies and six different time windows T. The two short-term windows have more suppressed occurrence of the extreme events compared to the four long-term windows. RHS: b) The same distribution after performing a natural logarithms to both sides of eq. (7). Symbols used:$T = 12$: $\bullet$, $T = 24$: $\diamond$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$, and $T = 5000$: $\times$.

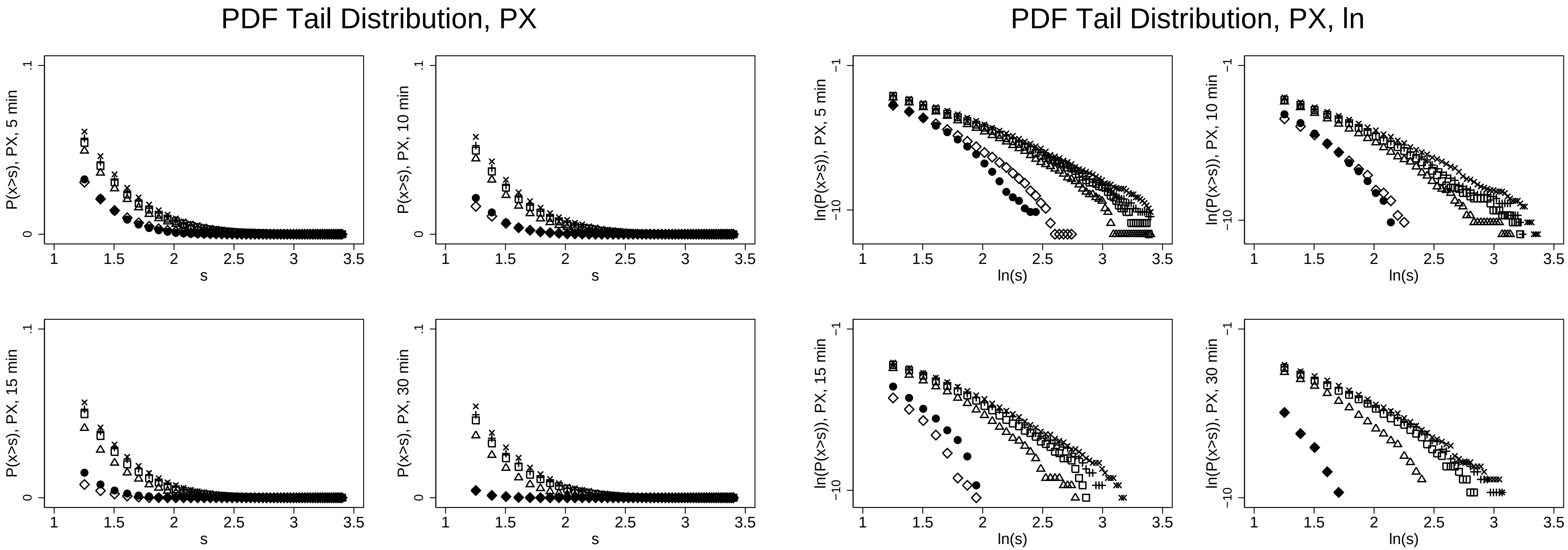


Table 3: The estimated α_T for all four indexes PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange), and all four frequencies: 5, 10, 15 and 30 minute, and for the time window $T = 5000$. The value in the bracket is the standard deviation. The higher the standard deviation is, the worse the estimation of the characteristic coefficient was found.

$\alpha_T(\sigma_\alpha)$	5 min	10 min	15 min	30 min
PX	3.554 (.033)	3.313 (.031)	3.713 (.040)	4.167 (.085)
BUX	3.809 (.024)	3.403 (.027)	3.493 (.029)	3.465 (.059)
WIG	4.949 (.083)	4.825 (.096)	4.517 (.099)	3.927 (.063)
DAX	4.098 (.046)	3.773 (.030)	3.542 (.049)	2.973 (.032)

Table 4: Estimated characteristic coefficient $\alpha_T^{\pm}$ for all four indexes: PX, BUX, WIG and DAX , using all frequencies: 5 minute, 10 minute, 15 minute and 30 minute, and the time window $T = 5000$. The characteristic coefficient is calculated separately for the upward movements (+) and the downward movements ($-$). The value in the bracket is the standard deviation.

$\alpha_T^{\pm}(\sigma_\alpha)$	5 min	10 min	15 min	30 min
PX, +	3.654 (.047)	3.721 (.058)	3.475 (.040)	3.516 (.077)
PX, -	3.435 (.041)	3.065 (.031)	4.028 (.053)	4.008 (.082)
DAX, +	3.836 (.037)	3.704 (.049)	3.310 (.044)	2.720 (.044)
DAX, -	4.277 (.057)	3.792 (.043)	3.703 (.068)	3.233 (.037)
BUX, +	3.887 (.027)	3.388 (.033)	3.712 (.062)	3.712 (.078)
BUX, -	3.769 (.033)	3.365 (.046)	3.352 (.045)	3.359 (.063)
WIG, +	4.208 (.075)	4.103 (.096)	3.979 (.067)	3.277 (.081)
WIG, -	6.205 (.293)	4.811 (.157)	4.769 (.159)	4.076 (.127)

movements were not taken into account. To study the upward/downward asymmetry, or up/down asymmetry, I have considered the price jump index for the longest time window $T = 5000$, which enables me to compare results with existing literature or with other results obtained in the next sections. The Table 4 contains the characteristic coefficients α_T for all four indexes: PX, BUX, WIG and DAX, and all four frequencies: 5, 10, 15 and 30 minute for the upward and downward movements estimated separately.

The very first result which can be clearly seen from this Table is the contradiction to papers [3, 4], where results from both papers says that $\alpha^{(+)} > \alpha^{(-)}$, i.e., the extreme jumps are more likely to be downward. My results do not confirm such a strict conclusion. Having in hand my results, I cannot generally conclude that one of the directions does significantly support more extreme events than the other one for all frequencies and all indexes.

4.2 Financial Turmoil

The current financial turmoil has its projections in all aspects of the real economy and financial world. Thus, an immediate question arises: Does the current financial turmoil have any significant effect on the behavior of price jumps? Or, more precisely, having the data set and the price jump indicator, can I observe any significant change of the characteristic coefficients during the financial turmoil with respect to the preceding periods? The answer to this question(s) is the goal of the following sub-section. The first problem to solve is the fact that my data set ranges to the end of 2008, while the main effect of the crisis arrived into the Europe[9] after the end of the year when various economic indicators were revealed. Therefore, I would be able to capture the initial phase of the financial crisis and not the time when the impact hit the region with full strength.

Generally, to stress that one period of time behaves differently, it is necessary to compare two or more comparable periods, i.e., to use two periods of time where the main difference does not come

[9] For example in the Czech Republic, many politicians claimed at the end of 2008 that crisis is the problem of the Western European countries but not the Czech Republic.

from seasonal effects. I restrict myself to the period from the beginning of 2005 to the end of 2008 and then divide the data set by months or by quarters and repeat the price jump analysis, as defined above, for such sub-periods.

Before I proceed with the analysis itself, I check the distribution of volatility on the market. I employ the standard deviation as an indicator for volatility on the market, which is defined as:

$$\sigma_T^2(t) = \frac{1}{N_T - 1} \sum_{i=0}^{N_T} (r(t-i) - < |r(t)| >_T)^2 \ . \tag{8}$$

The distribution of the volatility calculated on quarterly basis can be found in the Figure 6[10], where I have used data with 5 and 30 minute frequency and the shortest time window $T = 12$. The shortest time window was chosen since it is the most sensitive to any change in the underlying conditions, recall the discussion on the filtering properties and how they depend on the various lengths of the moving average window. The Figure clearly shows the increased volatility during the end of 2008, which confirms changed condition on the markets.

Further, I estimate the characteristic coefficient α_T for every month/quarter using the same data as were shown in the previous Figure, for all four indexes: PX, BUX, WIG and DAX, for 5 minute frequency and for time window $T = 12$ and 5000. The results for quarterly data are shown in the Figure 7. Figure for monthly data can be found in the Appendix. The Figure depicts estimated characteristic coefficient α_T using the OLS algorithm. Besides the coefficient itself, there is depicted also the confidence interval at 95% for each sub-interval.

The Figures do not reveal any significant pattern confirming a hypothesis that the financial turmoil results in the most volatile market and therefore the most extreme events are more probable. The connection between the high volatility and the lower characteristic coefficient supporting more extreme events is not therefore confirmed.

Another reason why any pattern was not statistically confirmed lies in the fact that any analysis focusing on extreme events requires a huge amount of data, since a very small part of them is relevant. Dividing my data set into the smaller sub-sets reduces significantly the available statistics, just recall the fact that dividing data sets by months, the available statistics decreases more than 50 times!

4.3 Price Jump Index Monthly & Quarterly

Since the intuition for the presence of any strong pattern due to the current financial turmoil is strong, I inspect the tail part of the distribution graphically. On the Figure 8 I plot the 3-D figure of the log-log distribution for extreme events for the PX index, using data with 5 and 30 minute frequency and time windows $T = 12$ and 5000. Time period spans from 2005 to 2008 and the distributions are plotted on a quarterly basis, i.e., 1 stands for the first quarter of 2005 and 16 for the last quarter of 2008. The Figure is an analogy to the Figure 5 (interpretation of $\ln(s)$ and the value on the vertical z-axis). Figures for the remaining three indexes and/or using a monthly basis are in the Appendix.

These Figures does not visually confirm any significant change in the last two quarters compared to the rest of the sample. Neither change in the behavior is present in the similar Figures for other indexes or for the distributions drawn on a monthly basis, see the Appendix. The message from the Figures one clearly reads is the fact that the short time window produces more uniform behavior over time, see the Figure 5 for 5 minute frequency and $T = 12$, while the long time window reveals higher degree of diversity among the quarters, see again the Figure 5 for 5 minute frequency and $T = 5000$. Having in hand these Figures, it is not surprising that my OLS algorithm does not produce significantly different estimation of the characteristic coefficients for the period of the financial turmoil.

[10] The same Figure on a monthly basis follows in the Appendix.

Figure 6: LHS: a) Volatility study of all four indexes: PX, BUX, WIG and DAX, using 5 minute data and the shortest time window $T = 12$. The Figure shows the distribution of the volatility for each quarter. RHS: b) The same volatility distribution but with 30 minute data.

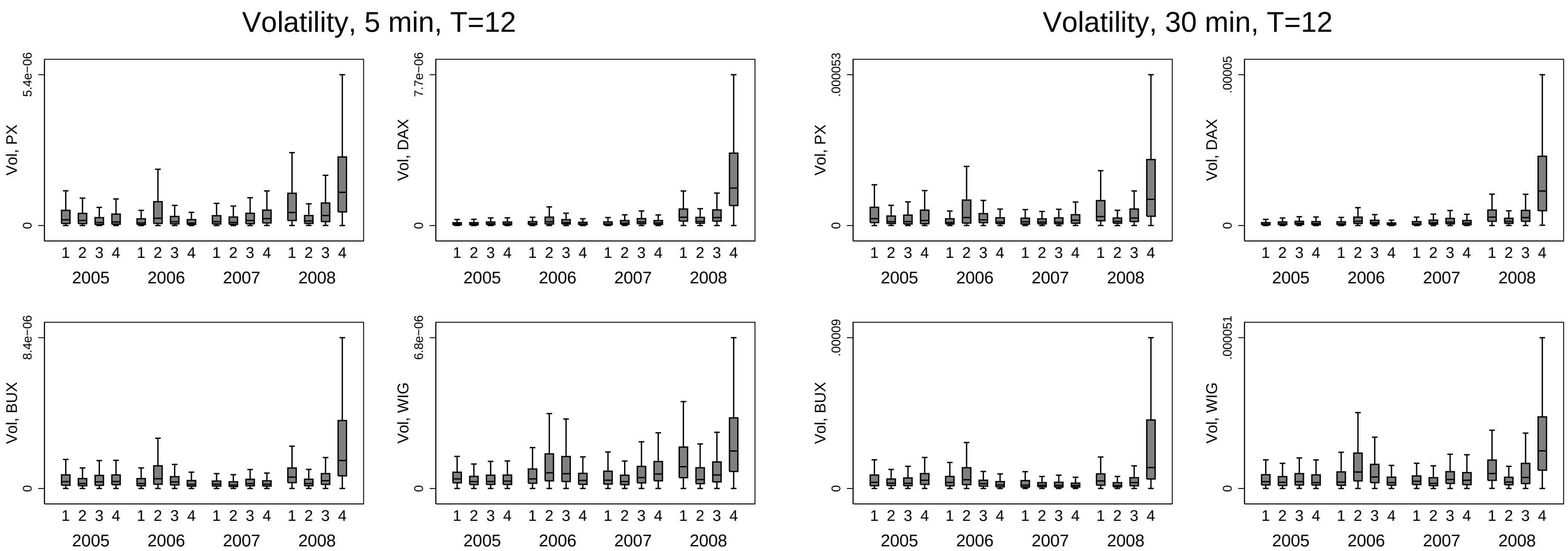
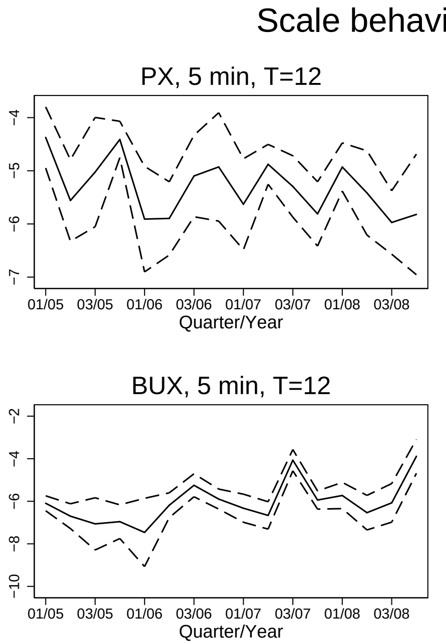


Figure 7: Estimated characteristic parameter α_T for all four indexes: PX, BUX, WIG and DAX, estimated on quarterly basis for period spanning from 2005 to 2008. LHS: The characteristic parameter for $T = 12$. RHS: The characteristic coefficient for $T = 5000$. Dashed line stands in both sub-figures represent the 95% CI.



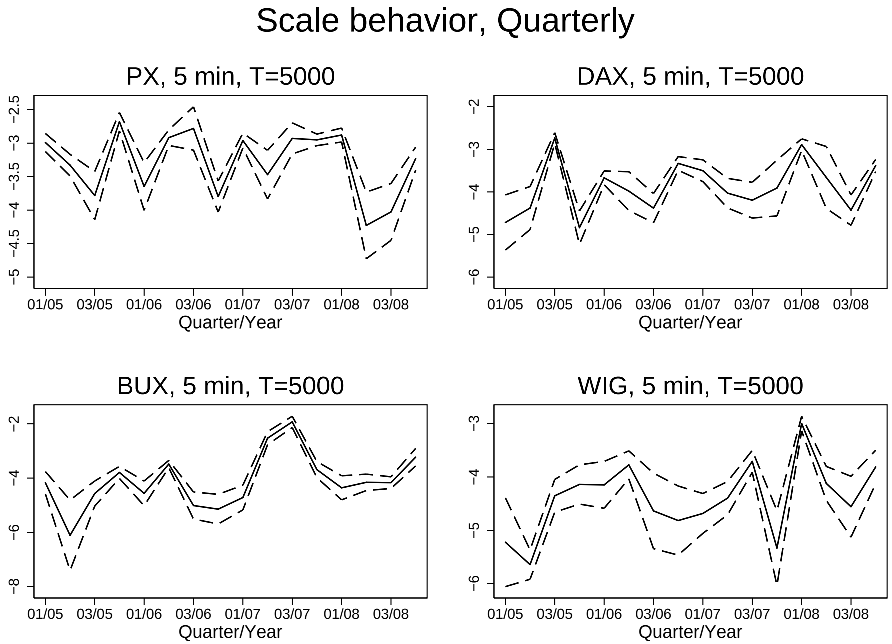
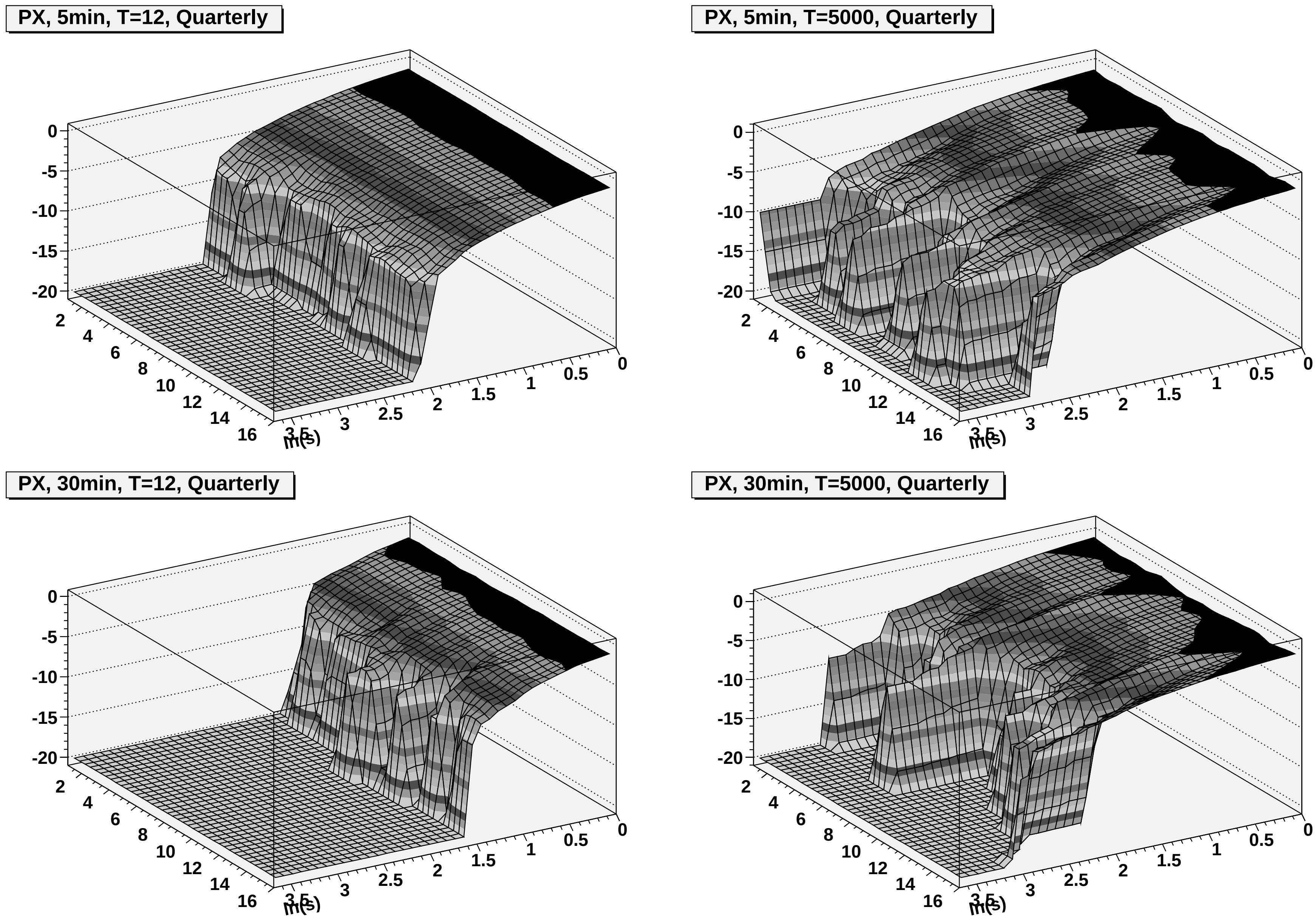


Figure 8: The tail part of the log-log distribution of extreme events for the PX index. Data spans from 2005 to 2008 and are taken on quarterly basis, where 1 stands for the first quarter of 2005 and 16 for the last quarter of 2008. The ln(s) is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.



4.4 Daily Distribution of the Price Jump Index

Another interesting aspect connected to the price jump index lies in the distribution of the extreme value of the price jump index over a trading day. Such an analysis can shed the light on some market mechanisms, especially whether there are some periods of the trading day, where the price vary a lot – the market is quite hot. This information is interesting from the practical point of view, since, for example, if the market raises during the calm period, it is less likely to observe an extreme change in the trend[11].

I have several possibilities to perform the study of the distribution of the extreme values of the price jump index over a trading day. I can study the distribution of times of four the highest values of the price jump index every day and draw a histogram of the times over some long period. However, all the days are not the same, the market activity can vary across days, and, what is the most important thing, this variation may not be necessarily random, since some days in every month are certainly more active than others[12]. Thus, I calculate the threshold as a given centile of the price jump index for each month and then study how the price jumps, whose price jump index lies above the threshold, are distributed over a trading day.

Namely, I calculate 95%, 97% and 99% centiles for all four indexes using frequencies 5 and 30 minute and two time windows $T = 12$ and 5000. The frequencies and the lengths of the time window are chosen to cover the behavior of data over the entire data set. I have calculated centiles both for the period spanning from 2004 to 2008 and divided it into the sub-sets by months.

The Figure 9 contains 95%, 97% and 99% centiles of the price jump index of the main the PX index for frequencies 5 and 30 minute and for time windows $T = 12$ and 5000. The Figure shows both the centiles for the entire period spanning from 2004 to 2008 (straight lines) and the centiles calculated on monthly basis. Figures for three remaining indexes and Figures summarizing all four indexes using a given frequency and time window follow in the Appendix.

The distribution of extreme price jumps over a trading day, where extreme jumps are those higher than a 95% centile calculated for each month separately is depicted in the Figure 10. There are all four combinations of 5 and 30 minute frequencies and time windows $T = 12$ and 5000. The Figures for three remaining indexes as well as Figures for the same analysis using the 97% and 99% centiles can be found in the Appendix. I have also repeated this analysis using centiles calculated for the entire period. These Figures are also summarized in the Appendix.

The Figure for the PX index suggests that in the case of the short time window, the most extreme changes occur around 2 pm, i.e., in the time when US markets wake up. Similar conclusion follows for the remaining three markets as well. The situation for the long time window is different. In this case there is also apparent a small "peak" around the starting activity of US markets, however, it is not a dominant one. The dominant period of extreme jumps prevails to be at the beginning of a trading day. The situation for two remaining Visegrad indexes is very similar to the PX index case[13]. However, the DAX index behaves differently. There is still dominant the opening of the US markets, the effect is much higher than in the case of the Visegrad indexes.

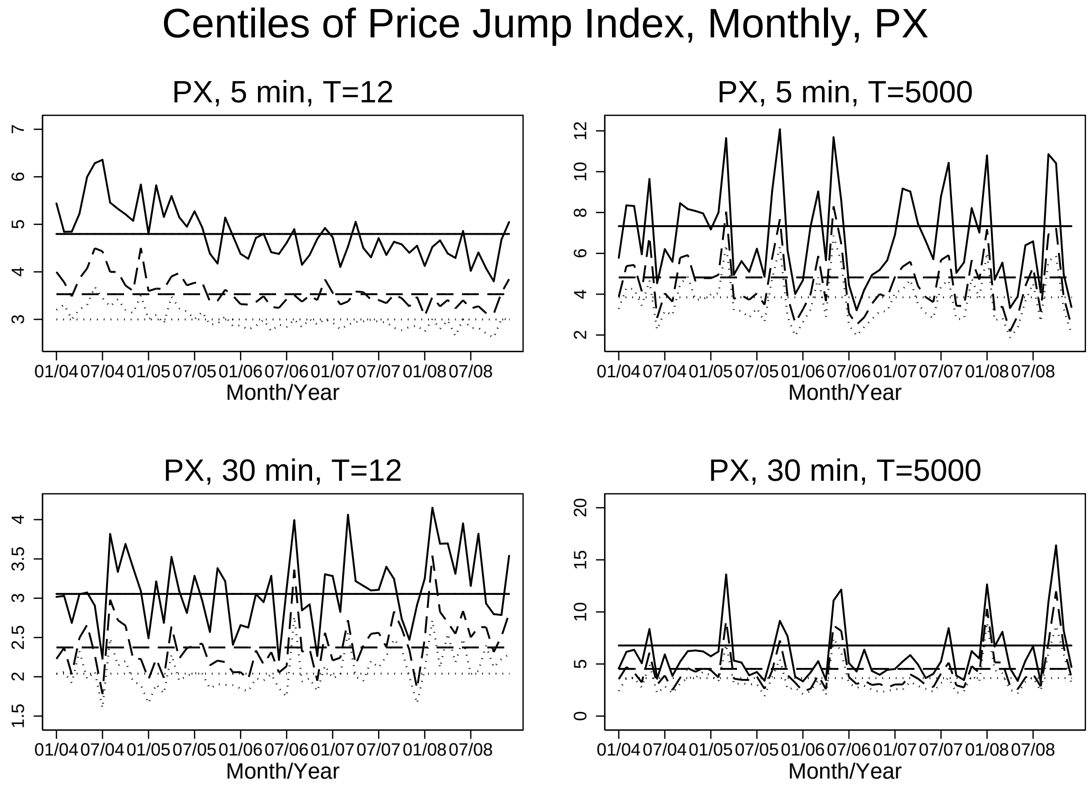
At this point, I can also ask a question, whether there is any significant change in the distribution of extreme jumps over a trading day during the period of the current financial turmoil. To see any such pattern, I plot the same distribution as I did in the previous Figure 10, but now month by month in a three dimensional manner. Namely, the Figure 11 shows the temporal distribution of extreme price jumps for the PX index using 5 minute frequency and time windows $T = 12$ and $T = 5000$. The extreme events are characterized in the same way as above, i.e., as those higher than 95% centile, where centiles are calculated every month separately. The time period covers years from 2004 to 2008, where 1 in both Figures represents 01/2004 and 60 represents 12/2008. The Figure does not reveal any significant change in the market behavior during the beginning of the financial turmoil.

[11] This has to be also adjusted to the fact that we are talking about price jump index, not about the returns!

[12] For example futures on various market products use to be sold in one particular day in each month.

[13] In the case of the WIG index, there is a double peak at the beginning of the trading day, which is caused by the fact that part of my data are chopped at 10 a.m.

Figure 9: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for the PX index using 5 and 30 minute frequency, and $T = 12$ and $T = 5000$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.



4.5 Price Jump "Condensation"

In the previous sub-section, I have presented the distribution of extreme price jumps over a trading day. Furthermore, we can also ask how many extreme price jumps we can usually observe during an average day. To answer this question I calculate how many price jumps higher than a given threshold occur in one day. As a threshold, I use the 95% centile calculated every month. In the Figure 12 I have plotted a distribution of the number of extreme jumps per day using the PX index with 5 and 30 minute frequency and time windows $T = 12$ and 5000. The time period spans from 2004 to 2008. Figures for remaining three indexes are in the Appendix.

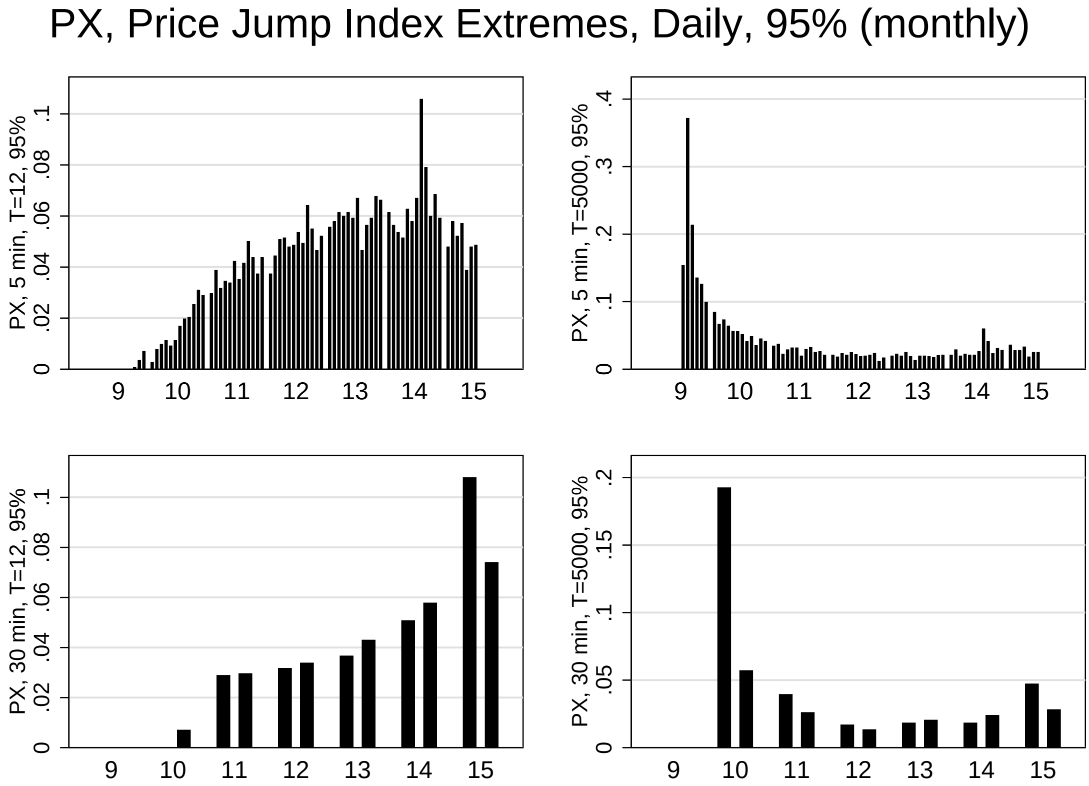
The distribution of the number of price jump per day varies significantly according to frequency. In the case of high frequency data, i.e., for 5 minute frequency, the part of the distribution for non-zero number of extreme price jumps has a local maximum no matter what time window T we use. In the case of $T = 12$, the most probable number of extreme price jumps per day is 3. In the case of $T = 5000$, the most probable number of price jumps is 0. If we take the days with non-zero number of extreme price jumps only, the most probable number is 2. On the other hand, in the case of lower frequencies, i.e., in this case for 30 minute frequency, this distribution is monotonically decreasing function and the most probable number of extreme price jumps per day is 0.

5 Normalized returns

Normalized returns [1, 4] represent a second possible definition of the price fluctuations. Normalized returns are defined as

$$r^n_T(t) = \frac{r(t) - < r(t) >_T}{\sigma_T(t)}, \tag{9}$$

Figure 10: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis.



where $< r(t) >_T$ is the same as above but with true values of returns and $\sigma_T(t)$ is the standard deviation as was defined in (8). Such a definition normalizes price movements by the market volatility and, therefore, it enables us to compare normalized returns for various different time series.

In this analysis, I have used the longest time window $T = 5000$ since it is convenient in literature [1, 4]. In the Figure 13 I plot the log-log distribution for normalized returns of the PX index using all four frequencies 5, 10, 15 and 30 minute. Despite the good motivation for the long time window, I have performed a robustness check for the length of the time window, namely, I have plotted this distribution for $T = 12$, 24, 100, 1000, 2000 and 5000. This Figure shows both the negative normalized returns (on the left hand side part of the Figure) and the positive normalized returns (on the right hand side of the Figure). Figures for the remaining three indexes BUX, WIG and DAX are in the Appendix.

Generally, we can say that the longer the time window the more linear-like shape the distribution has. It is also apparent that there is a problem with statistics, since the tails for a majority of distributions are distorted. This could even poison the estimates and conclusions based on them. Such a problem was not present in the case of the price jump index.

I have estimated the characteristic coefficient α_T for normalized returns using the same OLS algorithm as in the previous cases. I have used the longest time window only, i.e., $T = 5000$. Results are in the Table 5.

The story from this analysis is the same is in the case of the price jump index. We cannot simply conclude that negative extreme cases are more probable than the positive ones. Further, the standard deviations for nearly all the estimates are higher than in the case of the price jump index. This is probably caused by the distorted tails, as was mentioned above.

Figure 11: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 minute frequency, and $T = 12$. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The period spans from 2004 to 2008, where 1 represents 01/2004 and 60 represents 12/2008.

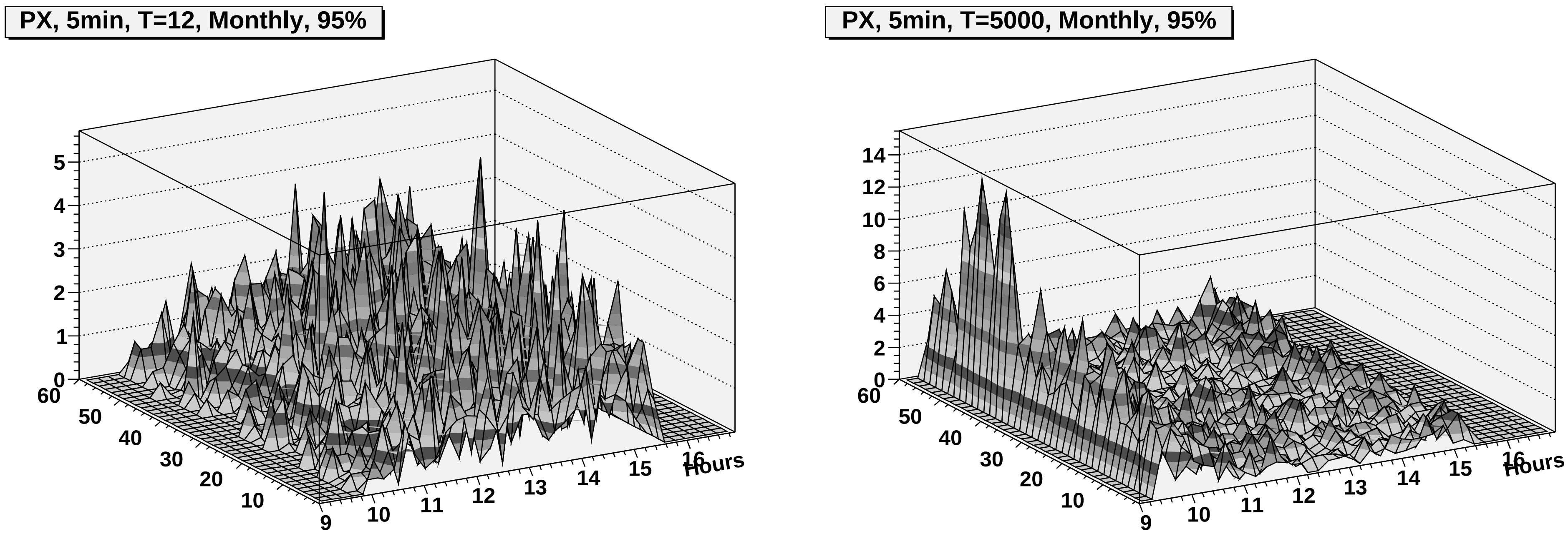


Figure 12: The distribution of the number of extreme price jump index per day for the PX index using 5 and 30 minute frequency, and $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The time period spans from 2004 to 2008.

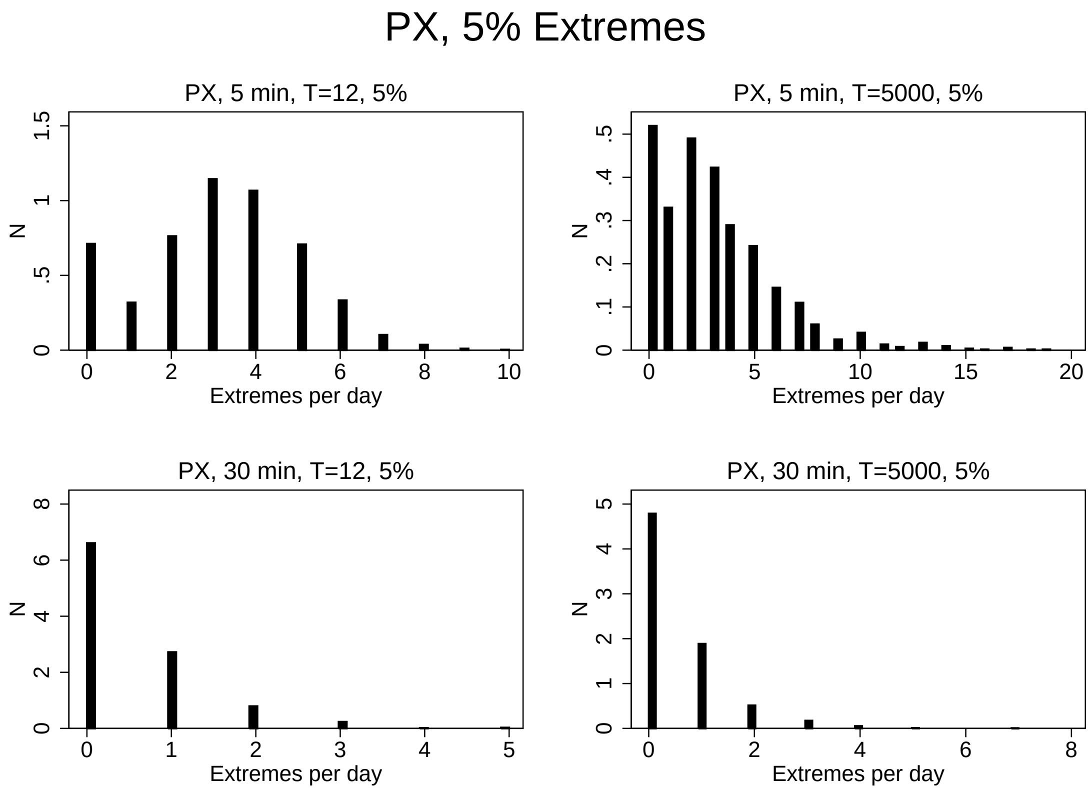
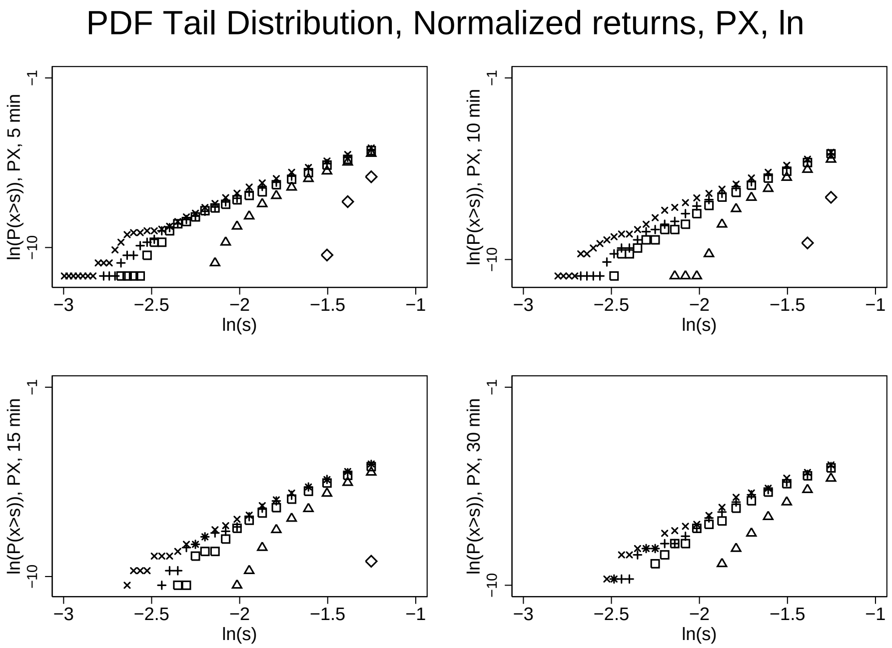


Table 5: The estimated characteristic coefficient α_T for normalized returns for all four indexes: PX, BUX, WIG and DAX. The length of the time window is $T = 5000$. Value in brackets means standard deviation. Estimated both negative $(-)$ and positive $(-)$ sides of normalized returns. Parameters were estimated using the standard OLS algorithm.

$\alpha(\sigma_\alpha)$	5 minute	10 minute	15 minute	30 minute
PX, +	3.659 (.066)	3.607 (.080)	3.651 (.038)	3.397 (.093)
PX, -	3.277 (.070)	3.186 (.027)	3.932 (.105)	3.654 (.141)
BUX, +	3.972 (.044)	3.636 (.060)	3.827 (.103)	4.144 (.138)
BUX, -	4.060 (.058)	3.756 (.037)	3.689 (.072)	3.874 (.133)
WIG, +	3.992 (.092)	4.316 (.098)	4.140 (.103)	3.472 (.079)
WIG, -	5.153 (.110)	4.872 (.215)	5.636 (.234)	4.652 (.246)
DAX, +	4.288 (.067)	4.025 (.036)	3.275 (.061)	2.699 (.115)
DAX, -	4.064 (.095)	3.815 (.061)	3.830 (.088)	3.388 (.082)

Figure 13: Log-log distribution of the tail part of normalized returns for the PX index using 5, 10, 15, and 30 minute frequencies. On the left hand side, the negative side of the distribution is depicted, while on the right hand side, positive normalized returns are shown. I have used following symbols: $T = 12$: $\bullet$, $T = 24$: $\diamond$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$ and $T = 5000$: $\times$.



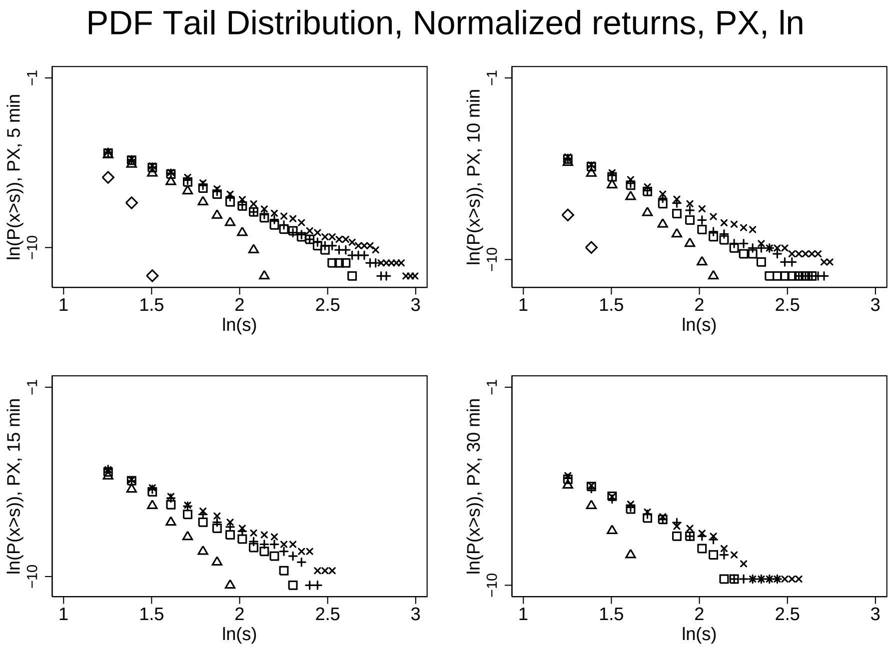


Table 6: Comparison of the up/down asymmetry as it was estimated using the price jump index and normalized returns. There are compared for all four indexes PX, BUX, WIG and DAX and all four frequencies: 5, 10, 15, and 30 minute. The length of the time window is $T = 5000$. In each entry I compare the characteristic coefficient α_T for positive (+) and negative (−) sides of the price jump index and normalized returns, respectively. For illustration, lets focus on the upper left entry, i.e., on the case of the PX index at 5 minute frequency and $T = 5000$. The entry tells us that in the case of the price jump index: the α_T^+ is bigger than α_T^-, and in the case of normalized returns: α_T^+ is also bigger than α_T^-.

PJI/NR	5	10	15	30
PX	$+>-,+>-$	$+>-,+>-$	$+<-,+<-$	$+>-,+<-$
BUX	$+>-,+<-$	$+<-,+<-$	$+>-,+>-$	$+>-,+>-$
WIG	$+<-,+<-$	$+<-,+<-$	$+>-,+<-$	$+<-,+<-$
DAX	$+>-,+>-$	$+>-,+>-$	$+<-,+<-$	$+<-,+<-$

5.1 Comparison to Price Jump Index

Having two different indicators for price jumps, we can compare the results they imply. Direct comparison of the values of the estimated characteristic coefficients α_T could misleading, since, in general, we are comparing different quantities which do not need to be appropriately normalized to each other. Rather, I just focus on the up/down asymmetry they imply and compare it.

In the Table 6 I compare the relation between the characteristic coefficients $\alpha_T^{(+)}$ and $\alpha_T^{(-)}$, i.e., for positive and negative sides of price movements, using first the price jump index, the first inequality in the entry, and normalized returns, the second inequality in the entry.

The inequalities agree in all but three cases. The first two disagreements, namely for the PX index at 30 minute frequency and the BUX index at 5 minute frequency differ, but the differences in the absolute values of the characteristic coefficients are very small, see, e.g., the BUX index case at 5 minute frequency in the Table 6. The third disagreement is for the WIG index at 15 minute frequency, which could come from the fact that the standard deviations for the WIG index are generally high and thus the OLS estimation algorithm is for them less reliable.

6 Jumps on the absolute levels

In the preceding two sections, I have defined price jumps in terms of returns, since working with returns is practically and theoretically more convenient, just recall a well known difference between returns and levels, where levels use to be non-stationary, while the first differencing makes the series stationary. On the other hand, human traders[14] perceive at the very first moment levels of a given index or commodity and then translates the movements of levels into the price changes either by using various indicators, candle graphs or any other tools.

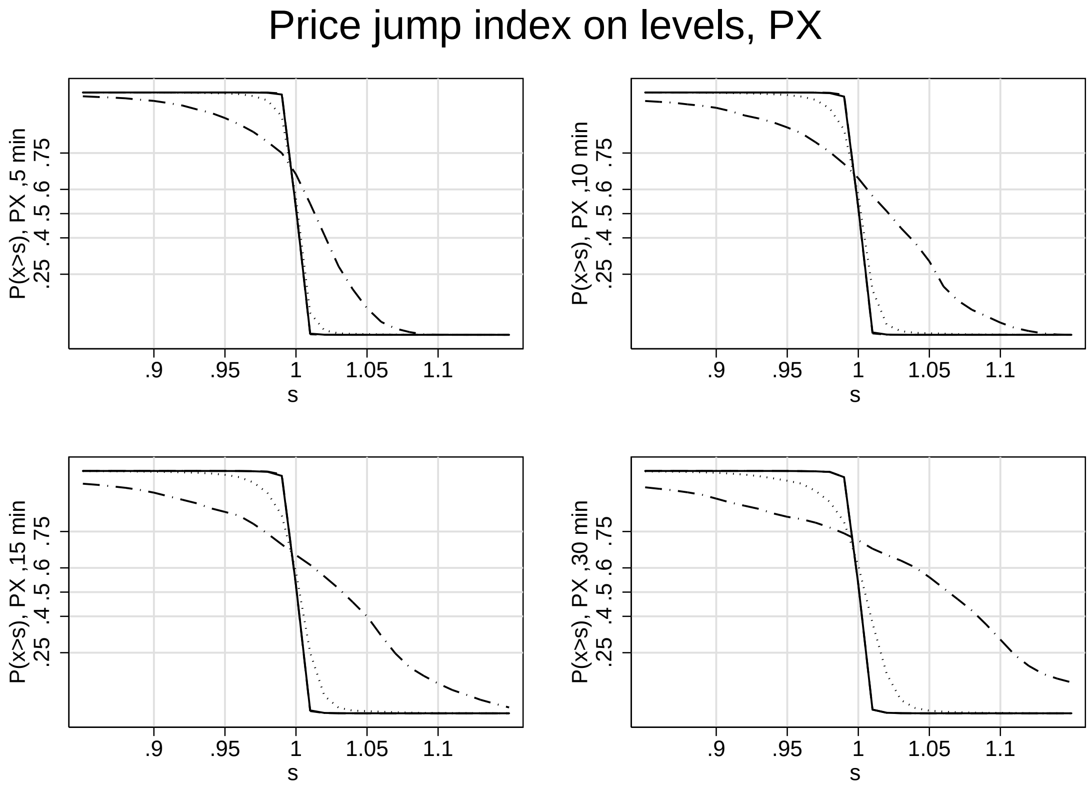
To proceed with this analysis I can choose one of the definitions for price jumps, where instead of returns I will use the levels. I choose the price jump index. I employ eq. (10) to define such a quantity, where instead of returns $r(t)$ I use absolute levels $R(t)$. I employ several lengths of the moving time window, namely $T = 12$ and 24, as a short-term representatives, and $T = 100$ and $T = 2000$ for long-term properties. As I have used before, $T = 12$ and 24 are defined in calendar time, while $T = 100$ and 2000 are defined in trading time. This quantity is motivated by practical purposes, thus the moving average starts at the previous time being and runs T time steps into history. The price jump index for levels is therefore defined as

$$j_T^L(t) = \frac{R(t)}{< R(t) >_T}. \tag{10}$$

Having defined and calculated this quantity, I analyze the distribution of the price jump index for levels. Since the fundamental features of this quantity are very different from the price jump

[14] I want to distinguish between human traders and quants, i. e., computer codes using various mathematical models to estimate future price movements.

Figure 14: The price jump index defined on levels for PX using all four frequencies: 5, 10, 15, and 30 minute. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).



index defined on returns, the analysis of the tail part of the distribution could be misleading. This can be further supported by the fact, that price jump index for returns considered just the values higher than one, while in this case, both values above and below one are plausible. Therefore, I have focused on the central part of the distribution. In the Figure 14 I plot the central part of the price jump index for levels for the PX index, using all four frequencies 5, 10, 15, and 30 minute, and all four time windows $T = 12$, 24, 100, and 2000. Figures for the remaining three indexes BUX, WIG and DAX as well as comparison across frequencies can be found in the Appendix.

The Figures shows that longer the time window, the more probable are movements down relative to movements up. In a perfectly symmetric case, all the lines should cross the point $(1, 0.5)$.

7 Extreme Returns over Day

Finally, I analyze the distribution of extreme returns over a trading day, a study similar to the one performed using the price jump index. To perform this analysis, I classify jumps in the following manner: I use returns and group them by months for a period spanning from 2004 to 2008. In each month, I pick up an $X\%$ of the most extreme events and perform a temporal analysis of their occurrence over a trading day. Extremes are calculated independently for both negative and positive movements, i.e., there is a one threshold value for negative returns and one threshold value for positive returns. The extreme negative returns are those lower than a first threshold while positive returns are those higher than a second threshold. Namely, I use three pairs of threshold values based on the following centiles calculated month by month: *i*) 1% and 99%, *ii*) 3% and 97%, and *iii*) 5% and 95%. In the following Figure 15 I plot the critical values for all four indexes PX, BUX, WIG and DAX, using 5 minute frequency.

The Figure supports the previous claim that during a financial turmoil, there is higher volatility in the market, as well as we can conclude a similar claim for the first months of 2008.

Figure 15: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures all four indexes: PX, BUX and WIG, and using 5 minute frequency. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

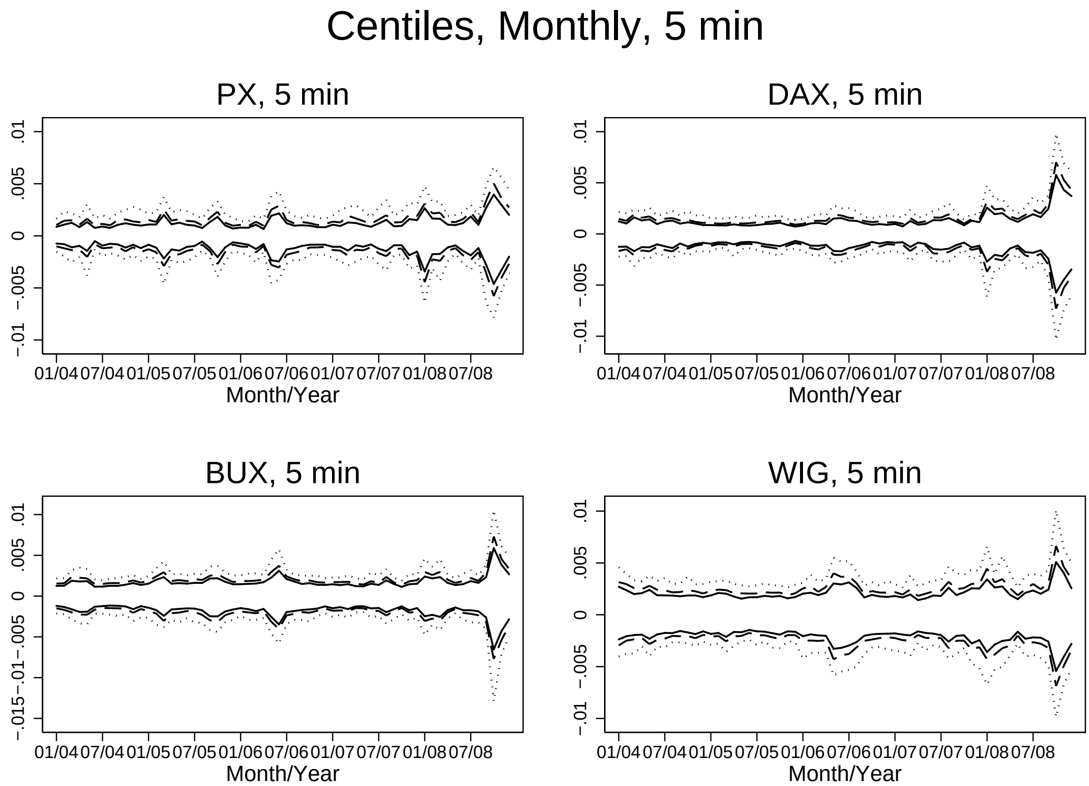
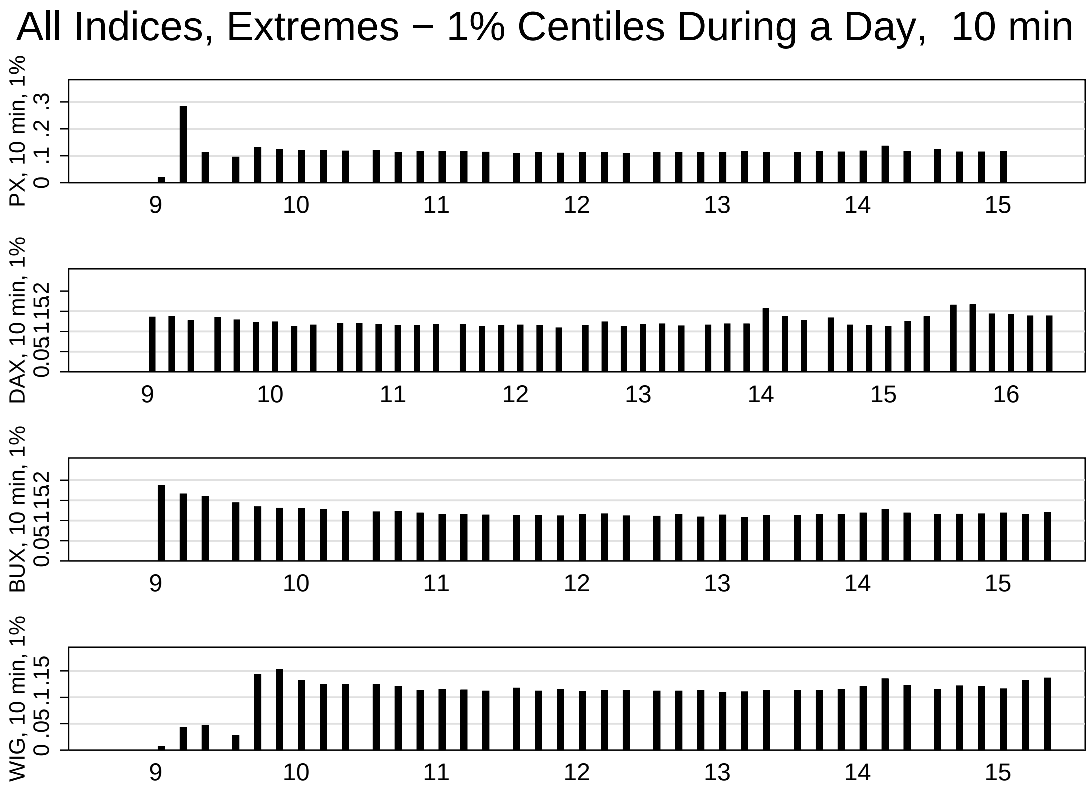


Figure 16: The distribution of extreme returns during a trading day for all four indexes PX, BUX, WIG and DAX, using 10 minute frequency and threshold values defined as 1% and 99% centiles.



Next, I have plotted the distribution of the extreme returns. The Figure 16 contains such a distribution over a trading day for all four indexes PX, BUX, WIG and DAX, using 10 minute frequency[15] and threshold defined as 5% and 95% centiles. Figures employing three other frequencies and other threshold values can be found in the Appendix.

First, the distribution for the WIG index is again corrupted by the fact that my data set contains part of the data starting at 10 a.m. The more relevant result stemming from this Figure is the fact that there is no extreme peak around the opening of the US markets. This could suggest that opening of the US markets rather suggests the trend reversal, which will be more visible in the price jump index than on the returns.

In the Figure 17 I have performed an analysis of the distribution of extreme returns, as they are defined above, for each month in the period from 2004 to 2008 separately. I use the PX index and 10 minute frequency with thresholds equal to 1% and 99% centiles, respectively, and plot this distribution month by month in a three-dimensional plot.

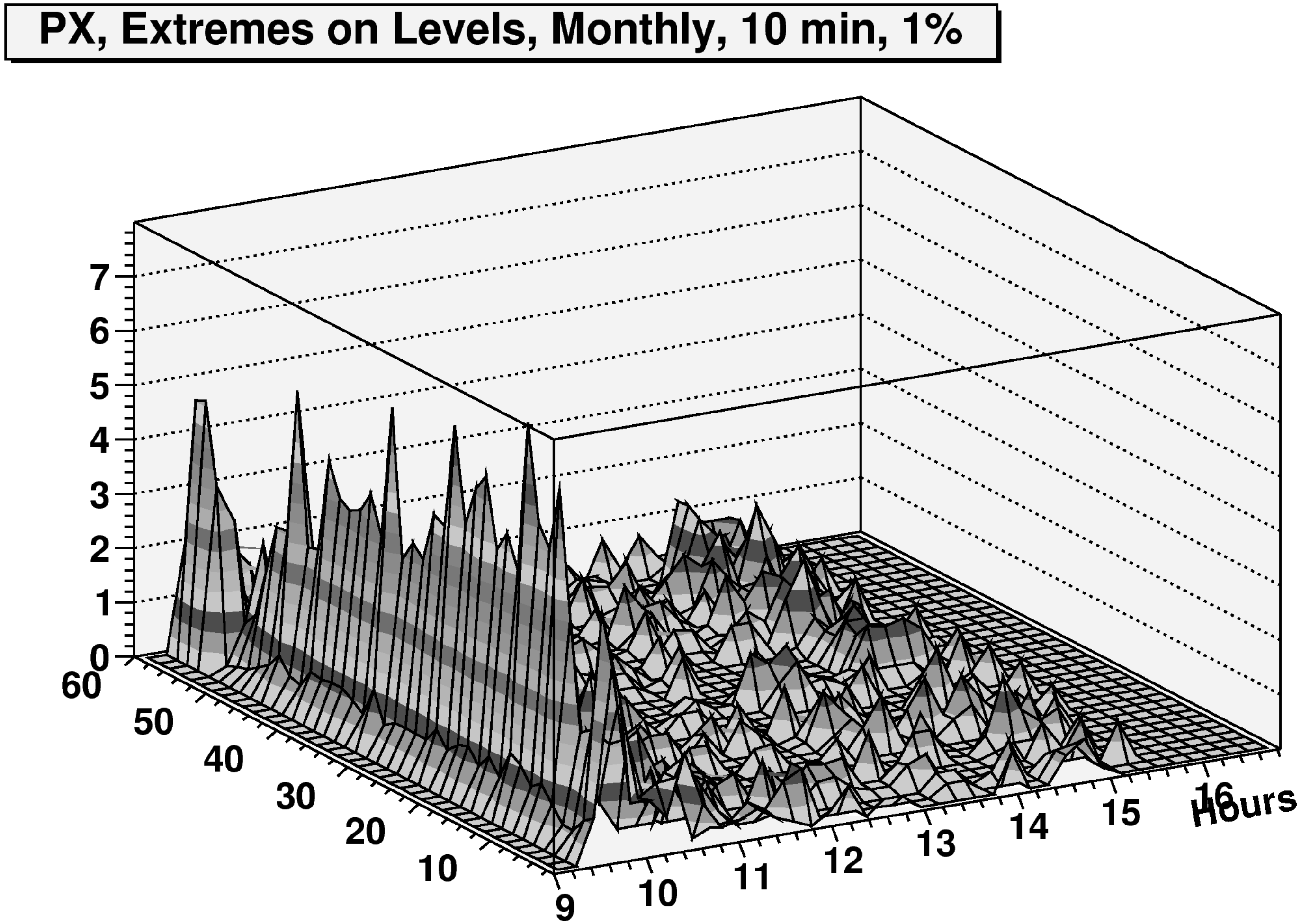
In this Figure we can see a light change in the behavior of extreme returns during a financial turmoil, since the most extreme returns tend to occur sooner. A slight change in the behavior of extreme returns can be seen also for other indexes, see the Appendix. This suggests an impatience on the market during a financial turmoil.

8 Conclusion

I have performed an extensive analysis of price jumps using high frequency data (5, 10, 15 and 30 minutes} for three Visegrad indexes (PX, BUX and WIG20) and for DAX, representing a regionally close mature market, for the time period ranging from June 2003 to December 2008. I have employed five different estimators for price jumps, four excluding the returns itself, namely the price jump index, normalized returns, price jump index defined for levels and extreme returns defined as those

[15] I present the lower frequency to illustrate the results in a more wide scope.

Figure 17: The distribution of extreme returns during a trading day for the PX index using 10 minute frequency and thresholds defined as 1% and 99% centiles calculated month by month. The distribution is plotted for every month from 01/2004 to 12/2008, where 1 stands for 01/2004 and 60 for 12/2008.



returns exceeding a given threshold. The analysis of returns confirmed the well accepted facts that the returns are not Gaussian and possesses the different tail behavior for different frequencies. After that, I have focused on the distributions for price jumps. To do that, I have used the price jump index and normalized returns to study the power-law behavior of the tail part of distributions. I have estimated the characteristic coefficient using a simple OLS algorithm which was scanning for the linear-like part of the tail distribution. Its value is in agreement with literature and is around the value corresponding to the inverse cubic law. However, both estimators for jumps do not give the same numbers, as was not expected since both are different and take into account different market characteristics. The price jump index is related to the current trend on the market while the normalized returns take into account also the market volatility. To strengthen my results I have employed various lengths of the time window, which was necessary to choose for the calculation of the market trend and volatility.

Comparing the characteristic coefficient as a function of frequencies, we can divide the indexes into two groups. The first group contains BUX, WIG and DAX indexes, for which the characteristic coefficients decreases with decreasing frequency. This means the lower the frequency the more likely the extreme events occur. This is in agreement with behavior of returns that behaves in the similar manner. On the other hand, the second group contains just PX index, for which exactly the opposite. Moreover, the PX index at high-frequency supports the occurrence of the extreme events much likely compared to other three indexes. The strange behavior of the PX index can be partly caused by the smaller emissions traded there since the lower the volumes are the more easy one can make the index oscillate. At lower frequencies, the oscillations are averaged out and thus not present. However, the full explanation of this micro-structure caused "PX puzzle" is still needed.

Further, I have tested the difference in the characteristic coefficients for price jumps up and down separately using the previous two estimators. My results do not agree with the literature which claims that the extreme movements down are more likely than the movements up for a broad range of frequencies and lengths of the time window. My results rather suggest that this relation depends on a given frequency. Both estimators give the same results which strengthen my analysis since in the previous literature, where the authors used either of the two estimators for price jumps. Another aspect of the price jumps extensively studied in this paper is the change in the price jump properties over time, and especially during a financial turmoil. This analysis is motivated by the fact that returns itself reveals an increased volatility in the second half of 2008 – the beginning of the financial turmoil. I have employed the price jump index and estimated the characteristic coefficient month by month and quarter by quarter. Such an analysis does not reveal any significant change in the characteristic coefficient due to the financial turmoil. Since the amount of the data for the analysis was on a monthly and quarterly basis significantly reduced, I have checked the results visually and nor the visual inspection of the data, see the three-dimensional plots, did not reveal any significant change.

As a next step, I have studied the distribution of jumps over a trading day. I have employed the price jump index and the two time windows, the short one and the long one, to ask when the price jumps occurs. The short time window reveals a high number of jumps during the opening time of the US markets while the long time window reveals the increased number of jumps at the beginning of the trading day. This observation does not change significantly over time and is not affected by the financial turmoil. The analysis also shows that for a short time window the distribution of the number of extreme price jumps per day reveals the local maximum, i.e., taking 5% of the most extreme price jumps we will more likely observe four extreme price jumps per day than two.

Further I have defined a price jump index for levels, since it is easily motivated by the way how the traders perceive the price movements. However, this definition does not enable us to study the distribution of extreme events. I have therefore used this estimator to show the asymmetry of the price movements towards the downward movements.

Finally, I have employed extreme returns, defined as those exceeding a given threshold and focused on their distribution during a trading day. The three emerging markets show quite a high activity in the beginning of the day while the DAX index has the biggest weight around the opening of the US markets. The same distribution, as plotted month by month, reveals a slight change during financial turmoil, where the markets tends immediately react in the very beginning of their operations, which suggests increased impatience on the markets during financial turmoil. The similar behavior over a trading day suggests the correlation between market; first, the intuitive dependence of emerging markets on the mature markets, however, it does not reveal more information about the correlation among the emerging markets, whose quantitative estimate is important since it can complete the picture about the economic and financial integration of the countries inside the Visegrad region.

In conclusion, my analysis presented a broad range of empirical facts concerning the extreme price jumps on the three emerging markets of the Visegrad region and the main index of the Frankfurt Stock Exchange. The focus is on the more than one of the characteristic aspects, as is usual in the literature, reveals us a complex picture. Such a picture helps us to formulate some stylized empirical facts concerning the price jumps. However, the detailed description of the markets raises other questions, namely those related to the degree of correlation of markets with each other. Besides the correlation across the markets, the connection of the price jumps and their source is still missing. The distribution of price jumps for Visegrad indexes over a trading day suggests the origin of price jumps could lie, at least partially, in the events on the mature market. However, the source of price jumps for the mature markets is still unclear. Last but not least, the open question on the behavior of the characteristic coefficient during the beginning of the financial turmoil calls for the extension of this analysis to include at least the first half of 2009 and repeat the test for stability of the price jumps' parameters. This analysis could, therefore, serve as a starting point for the next step in finding a bridge between price jumps and the real economic reasons behind them. Such a bridge is missing but needed. Hopefully, the knowledge of the complex quantitative description of downward (as well as upward) price jumps – one of the symptoms of market panic – could enable the open market public to rationalize their behavior during the warm periods of time and thus reduce the panic itself.

Acknowledgement

I thank Jan Hanousek for stimulating discussions and helpful suggestions. This work is supported by the GAČR grant (402/08/1376), which is gratefully acknowledged.

Appendix

In this Appendix I keep all the Figures and Tables which do not fit into the body of the main text. The paper with the full Appendix can be found in [29] or upon request. The Figures and Table are organized into sections which follow in the same order as in the body of the paper. The Figures and Tables are briefly introduced; however, they do not stand on its own since they serve as a complement to the main text.

Introduction

The Figure 18 shows graphically trading hours for all four indexes as they were in 2008.

Data

The Figure 19 depicts price level data for all four indexes using 5 minute frequency.

Returns

The Figures 20, 21 and 22 shows returns for all four indexes using frequencies: 10, 15 and 30 minute, and trading time.

Further, the Figures 23, 24, 25 and 26 shows returns index by index, where for every index all frequencies are shown.

The following series of Figures depicts comparison of returns when different times are used. Namely, for every index and every frequency there are three sub-figures that show returns when trading time, calendar time and cutted time (the difference between calendar and trading times) are used. The Figures 28-30 show the PX index, the Figures 31-34 show the BUX index, the Figures 35-38 show the WIG index[16], and finally the Figures 39-42 show the DAX index.

The Figures 43, 44 and 45 depict basic statistical properties of the remaining three indexes: BUX, WIG and DAX. The properties are calculated using 5 minute frequency.

[16] The strange behavior of this index is due to the fact that the first half of data started later, see the text.

Figure 18: Trading hours of all four stock exchanges valid for the year 2008. The Stock Exchanges used in this study are: the Prague Stock Exchange, the Budapest Stock Exchange, the Warsaw Stock Exchange and the Frankfurt Stock Exchange. The longest trading period is for the Frankfurt Stock Exchange. The Stock Exchanges of the Visegrad region start to operate in the following order: Budapest, Prague and Warsaw.

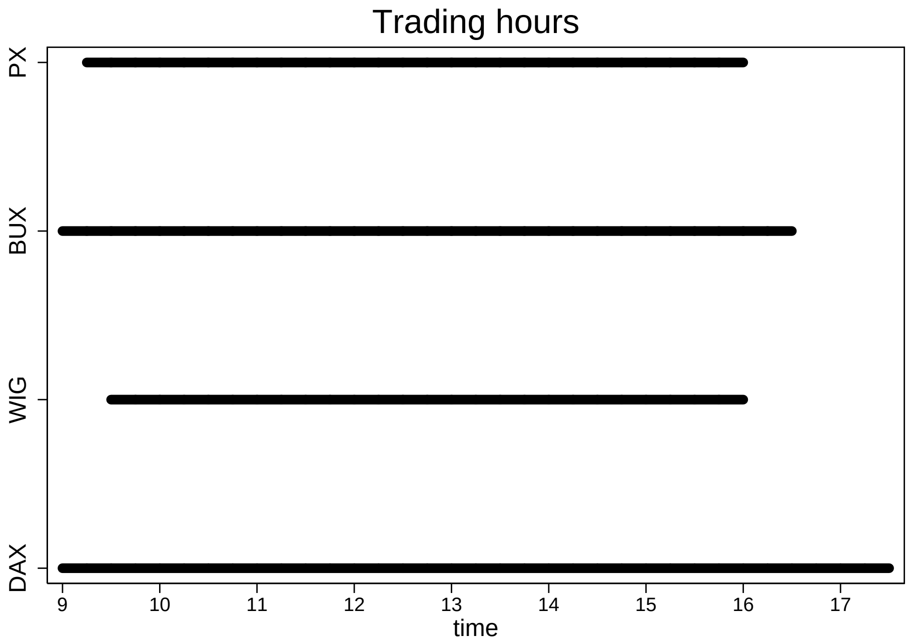


Figure 19: The price levels for all four main stock indexes from the Visegrad countries and Frankfurt (Germany): PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange). The working frequency is 5 minute and the time period spans from 06/2003 to 12/2008. We can see all four indexes follow similar patterns.

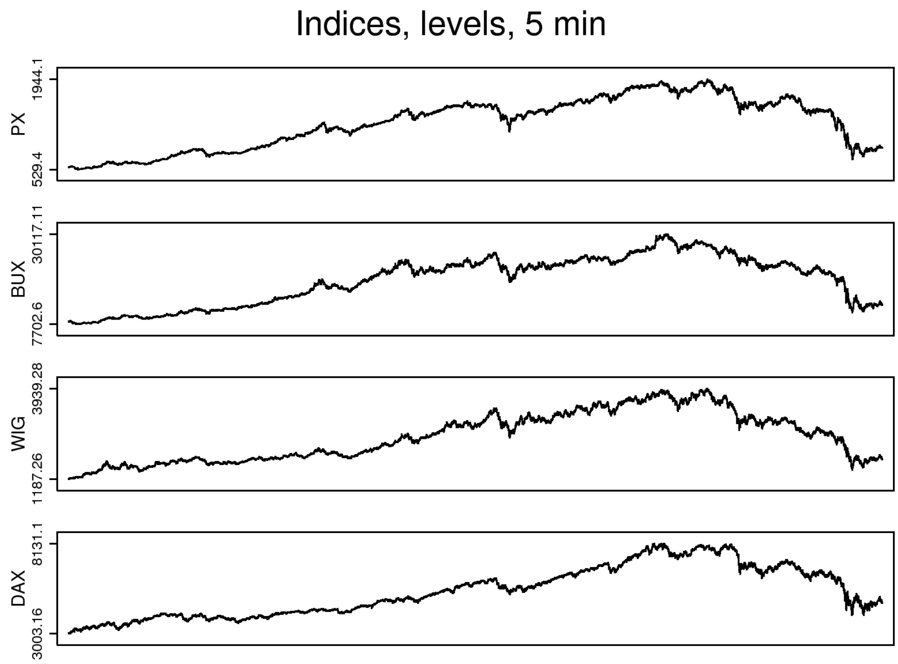


Figure 20: Returns for all four main indexes used in this study: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange). Frequency of the data is 10 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time.

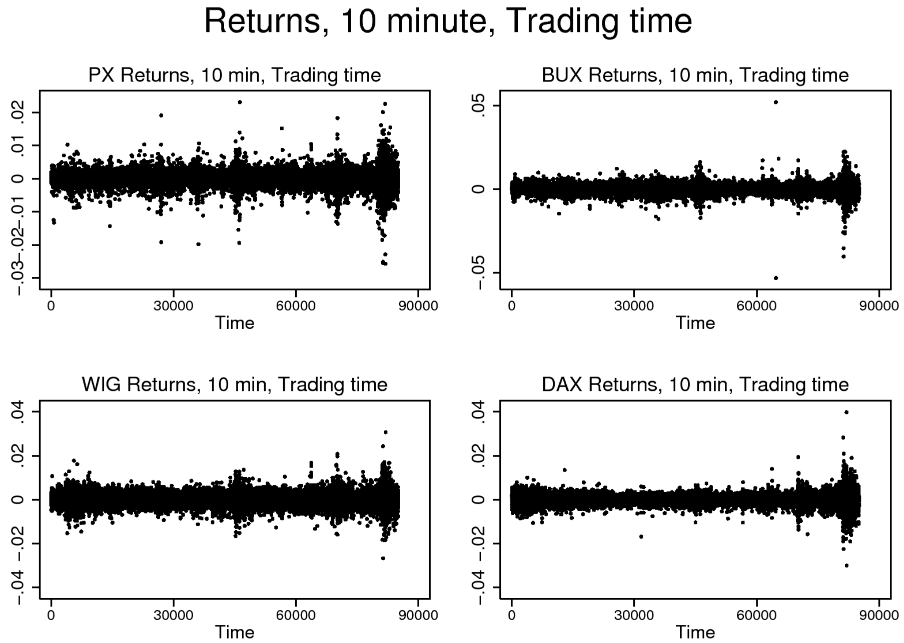


Figure 21: Returns for all four main indexes used in this study: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange). Frequency of the data is 15 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time.

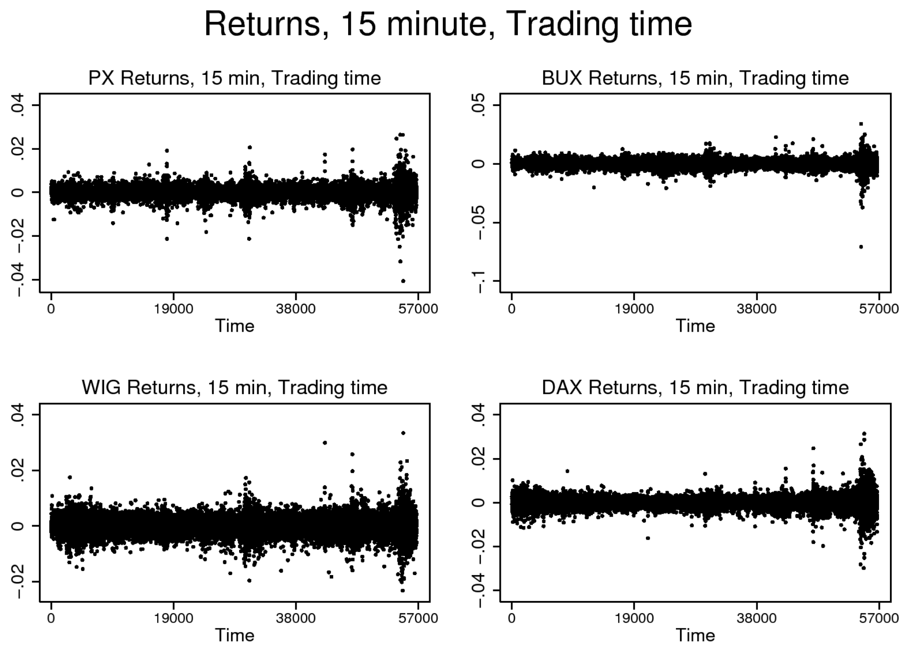


Figure 22: Returns for all four main indexes used in this study: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange) and DAX (Frankfurt Stock Exchange). Frequency of the data is 30 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time.

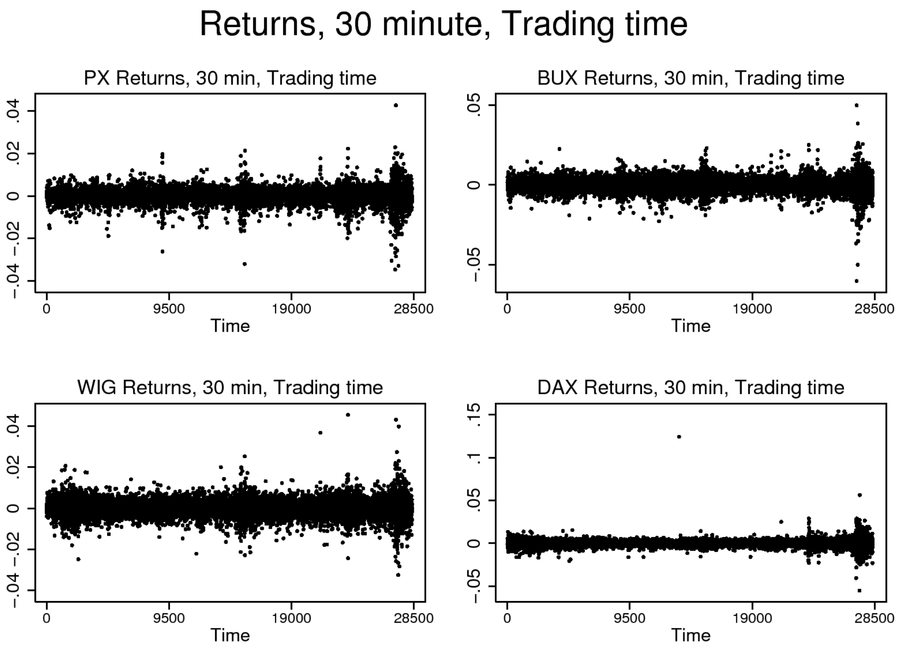


Figure 23: Returns for the PX (Prague Stock Exchange) index for all four frequencies: 5, 10, 15 and 30 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time. For the Prague Stock Exchange the opening hours are from 9:15 to 16:00 and trading period is defined from 9:30 to 15:30.

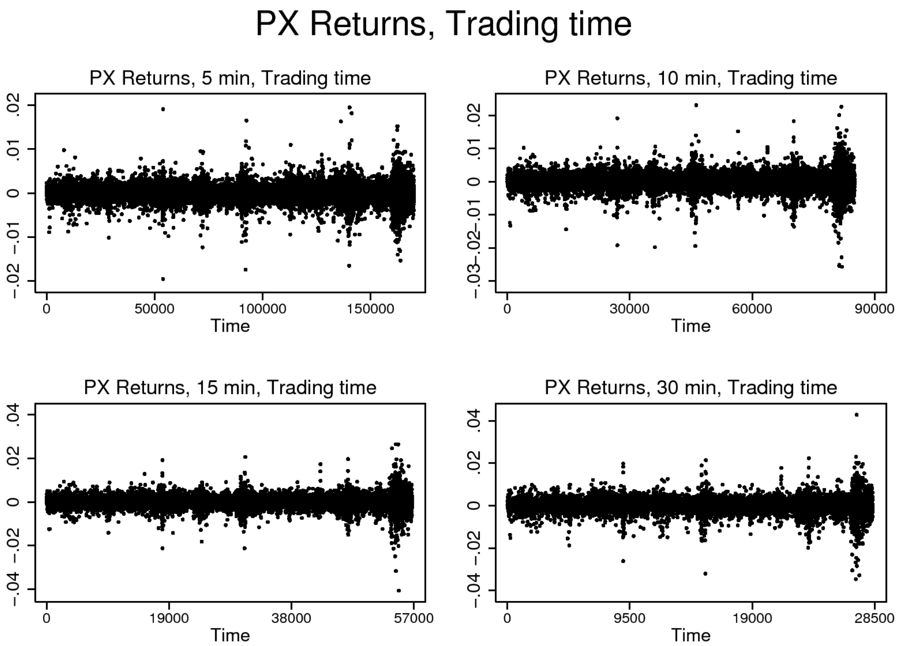


Figure 24: Returns for the BUX (Budapest Stock Exchange) index for all four frequencies: 5, 10, 15 and 30 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time. For the Budapest Stock Exchange the opening hours are from 9:00 to 16:30 and trading period is defined from 9:30 to 16:00.

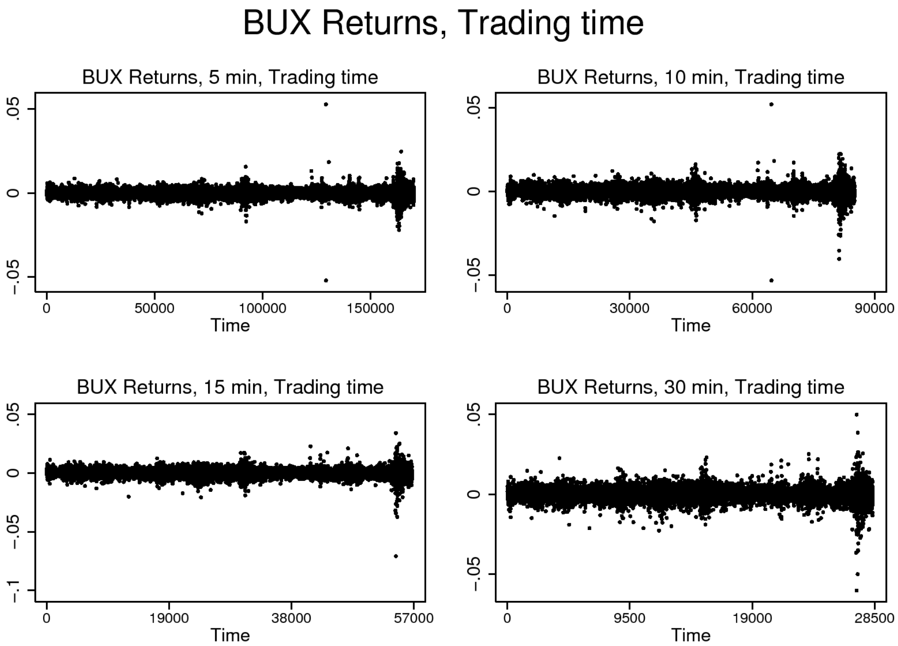


Figure 25: Returns for the WIG (Warsaw Stock Exchange) index for all four frequencies: 5, 10, 15 and 30 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time. For the Warsaw Stock Exchange the opening hours are from 9:00 to 16:20 and trading period is defined from 9:30 to 16:00.

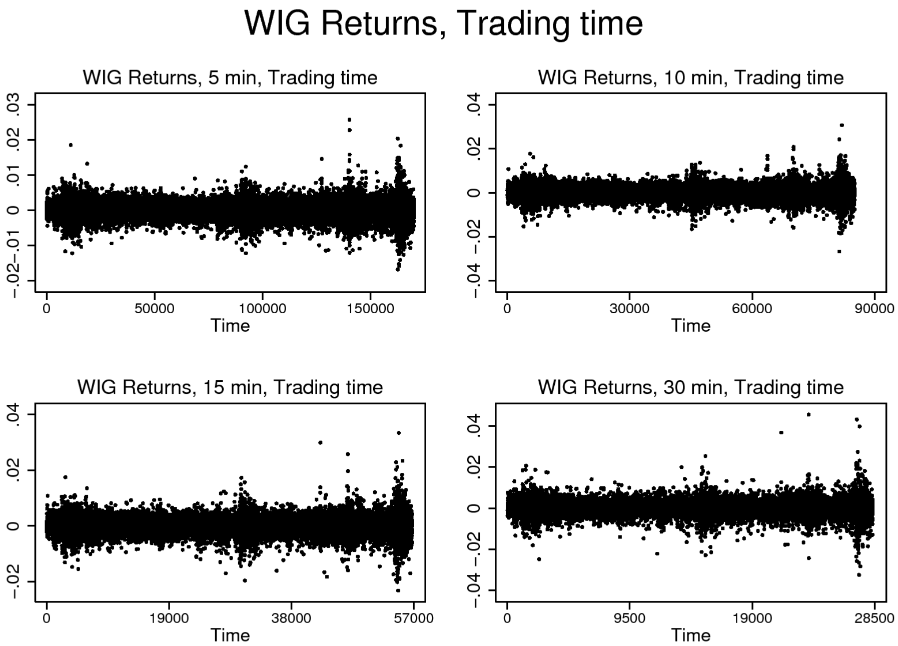


Figure 26: Returns for the DAX (Frankfurt Stock Exchange) index for all four frequencies: 5, 10, 15 and 30 minute. Time period spans from 06/2003 to 12/2008, further, the time used is trading time. For the Frankfurt Stock Exchange the opening hours are from 9:00 to 17:30 and trading period is defined from 9:30 to 17:00.

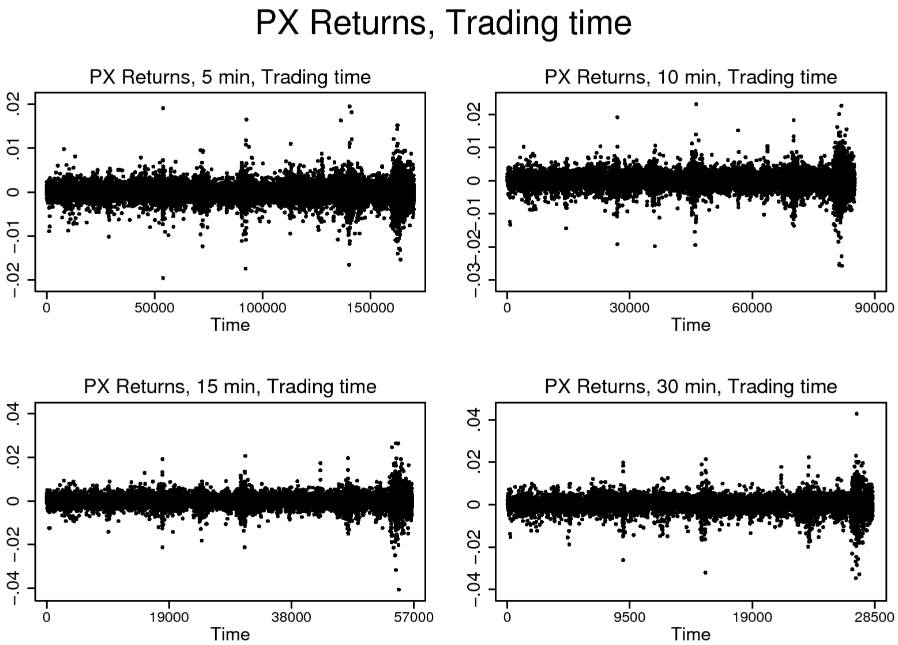


Figure 27: Returns for the PX (Prague Stock Exchange) index using 5 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

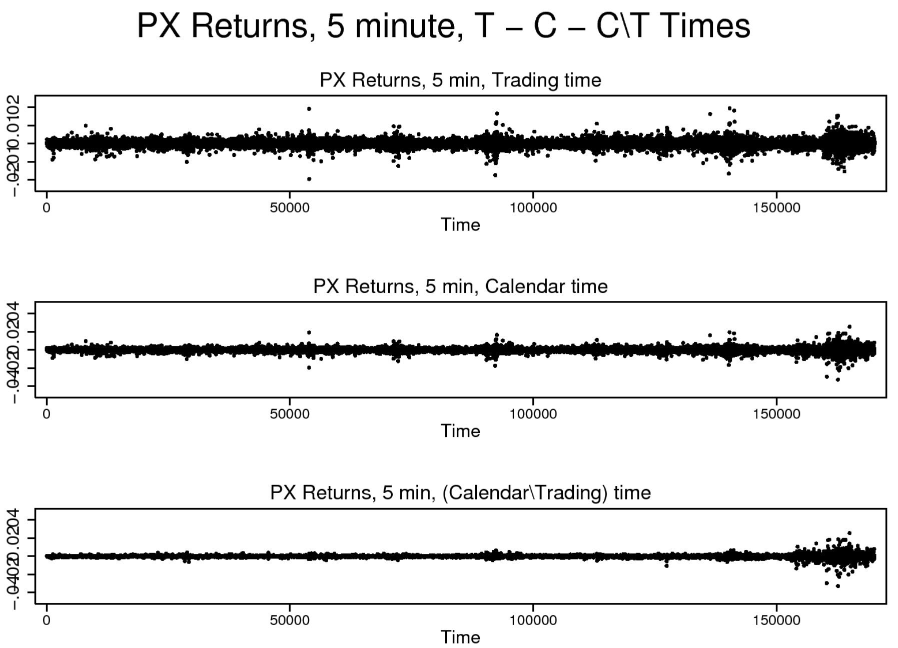


Figure 28: Returns for the PX (Prague Stock Exchange) index using 10 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

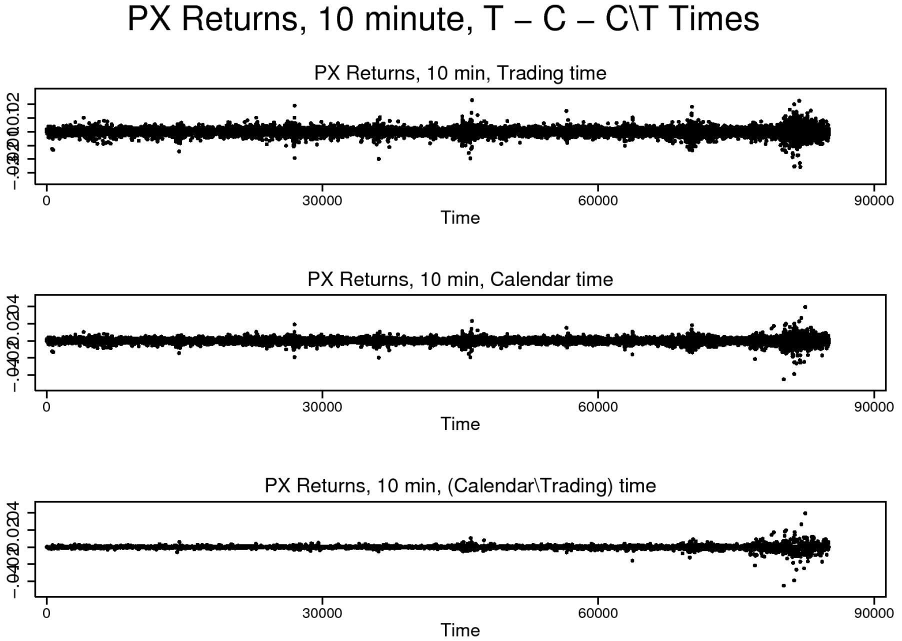


Figure 29: Returns for the PX (Prague Stock Exchange) index using 15 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

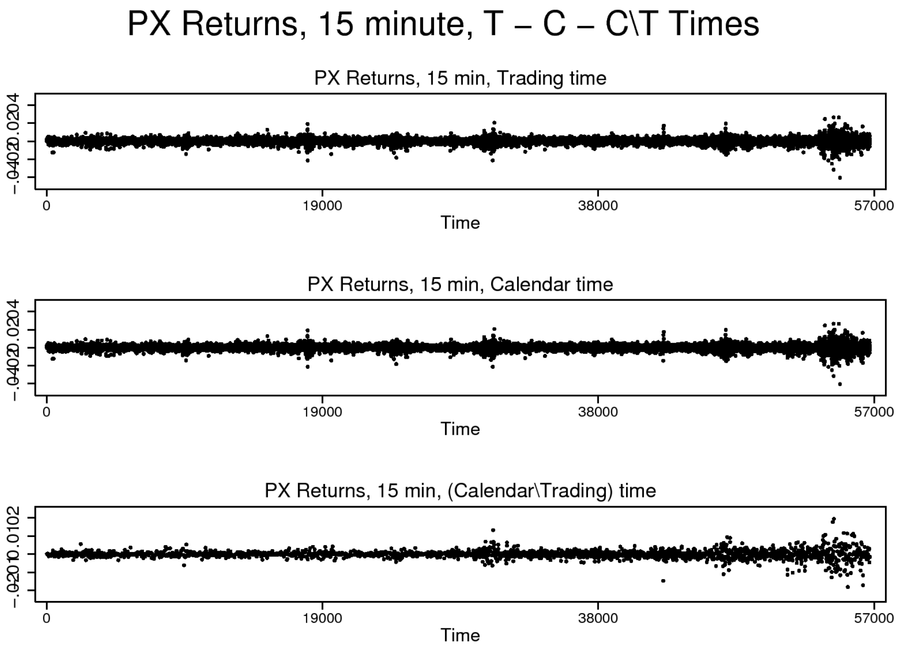


Figure 30: Returns for the PX (Prague Stock Exchange) index using 30 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

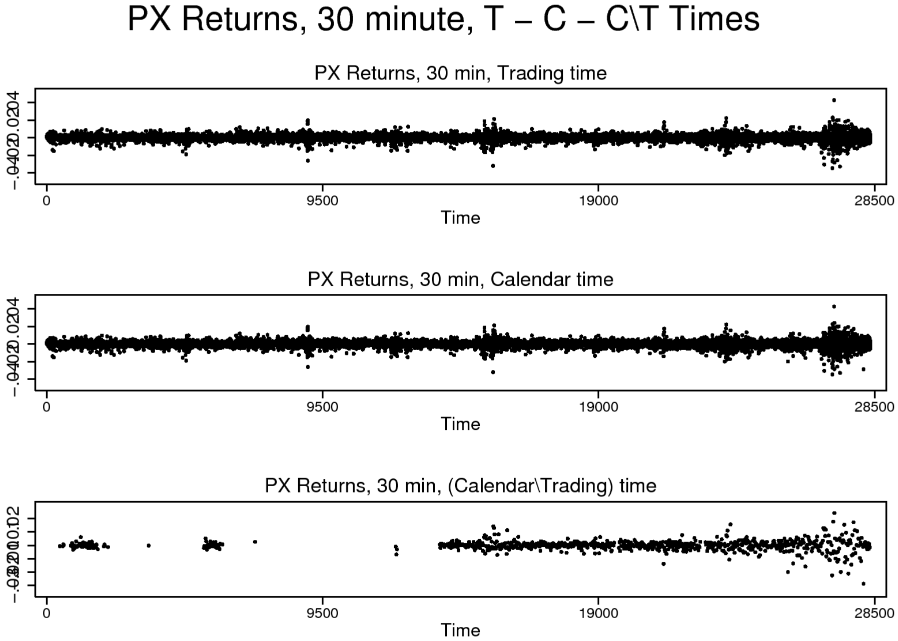


Figure 31: Returns for the BUX (Budapest Stock Exchange) index using 5 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

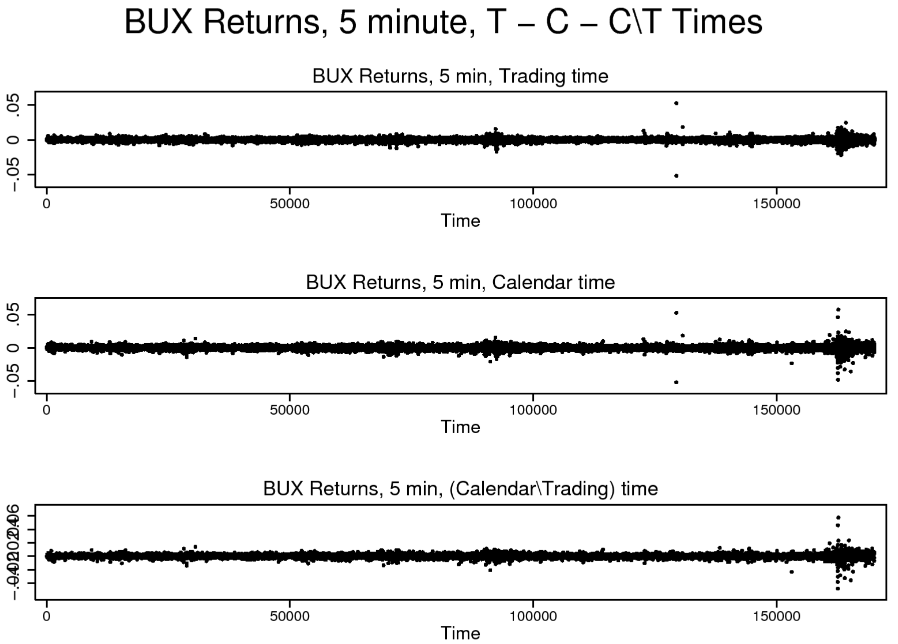


Figure 32: Returns for the BUX (Budapest Stock Exchange) index using 10 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

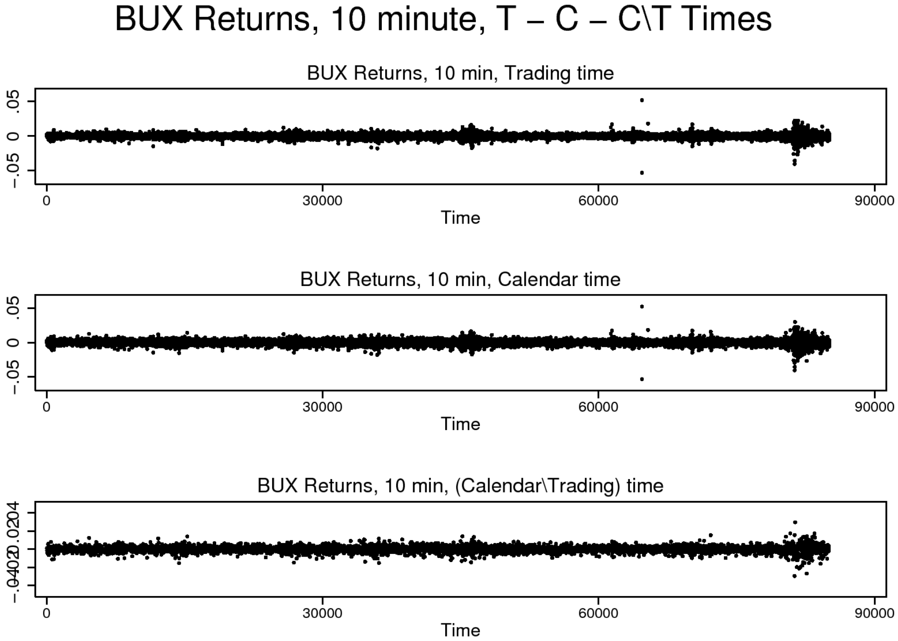


Figure 33: Returns for the BUX (Budapest Stock Exchange) index using 15 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

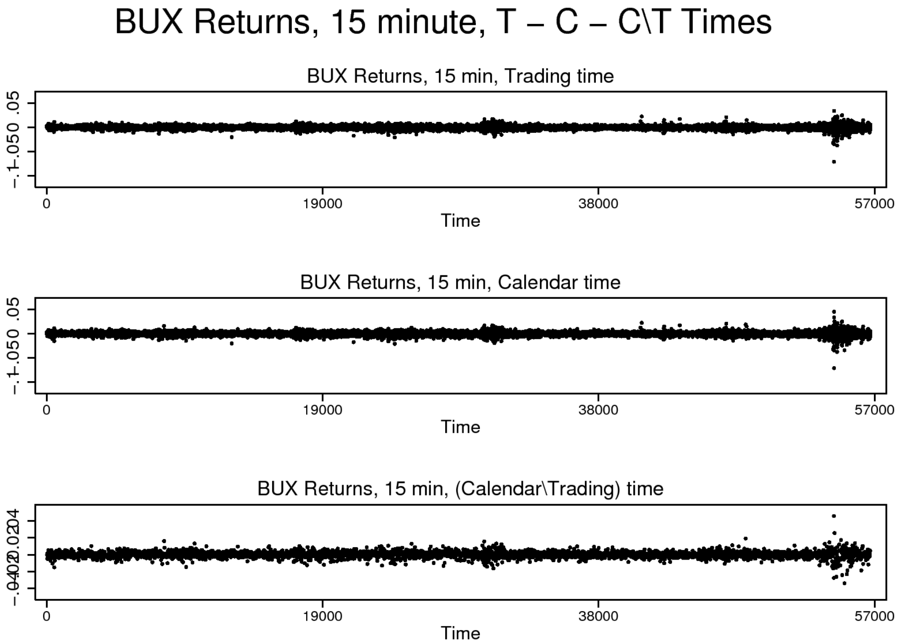


Figure 34: Returns for the BUX (Budapest Stock Exchange) index using 30 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

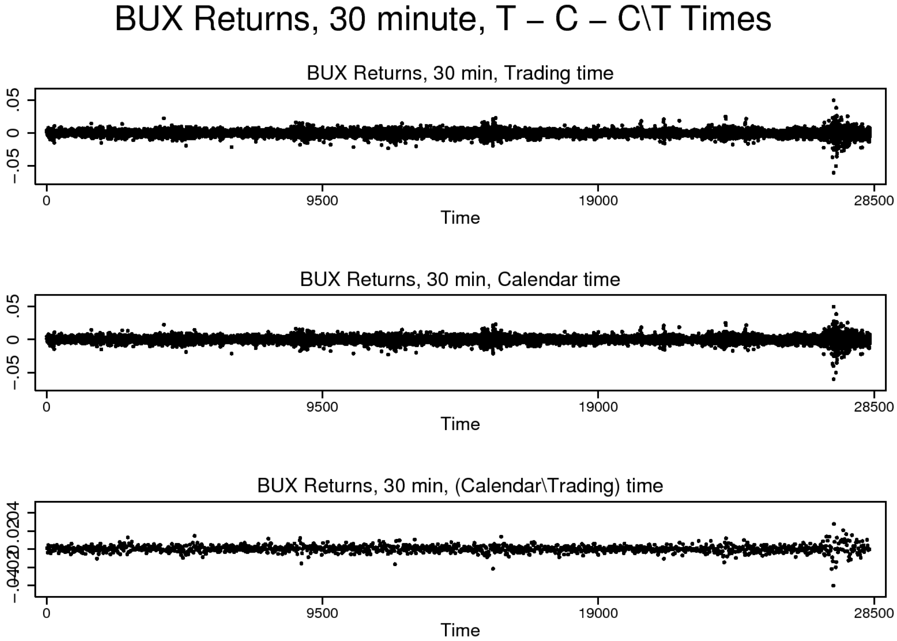


Figure 35: Returns for the WIG (Warsaw Stock Exchange) index using 5 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

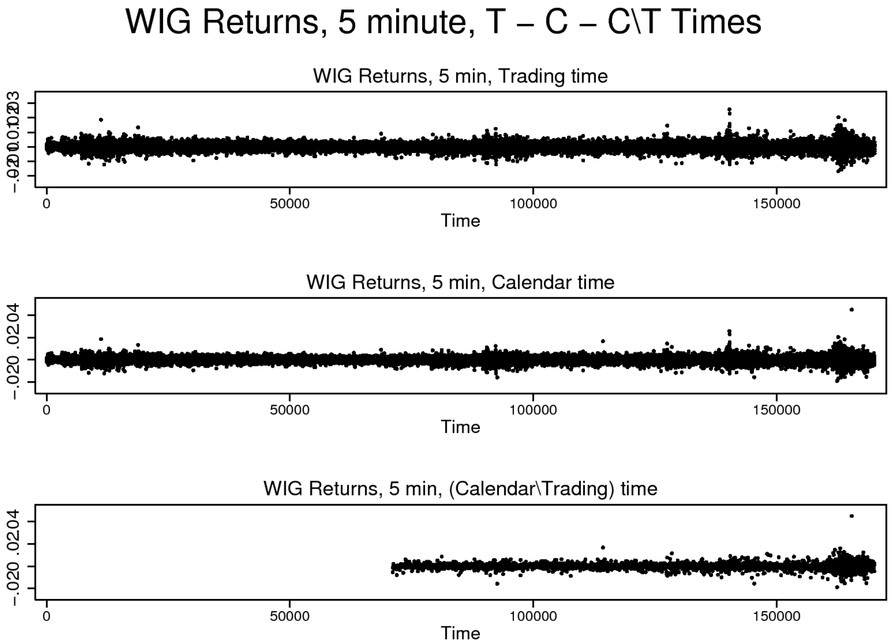


Figure 36: Returns for the WIG (Warsaw Stock Exchange) index using 10 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

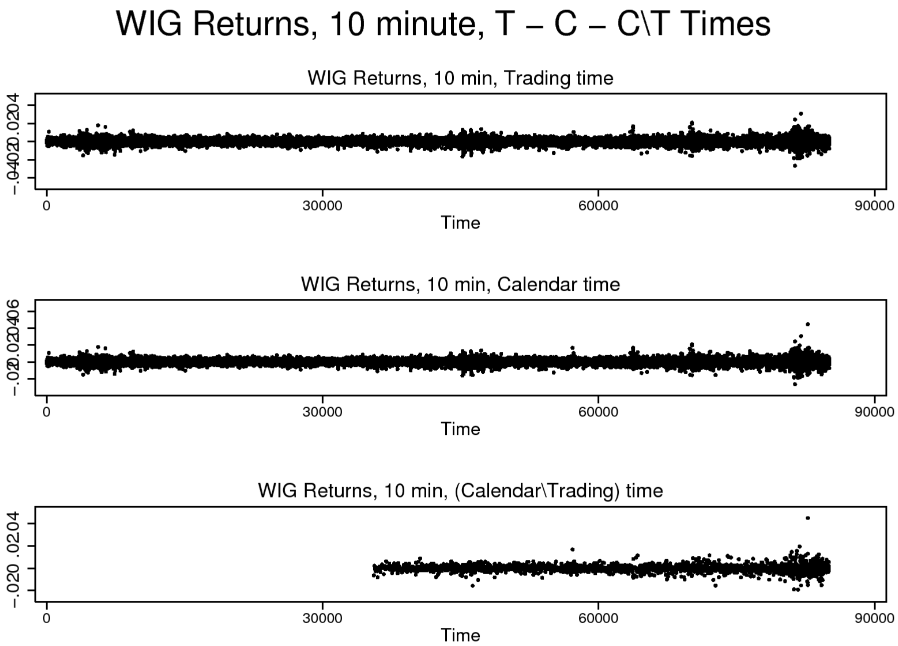


Figure 37: Returns for the WIG (Warsaw Stock Exchange) index using 15 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

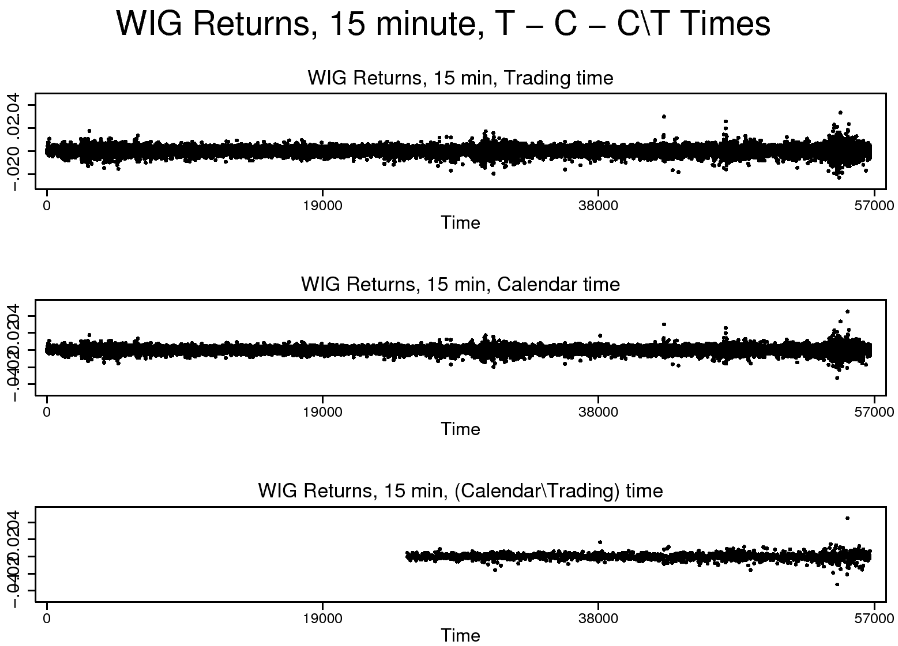


Figure 38: Returns for the WIG (Warsaw Stock Exchange) index using 30 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

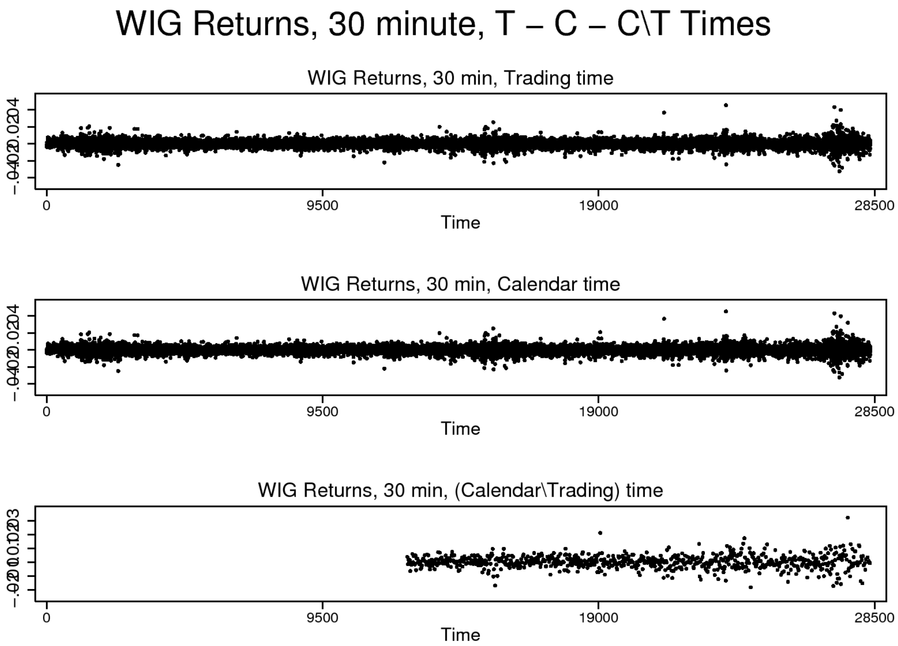


Figure 39: Returns for the DAX (Frankfurt Stock Exchange) index using 5 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

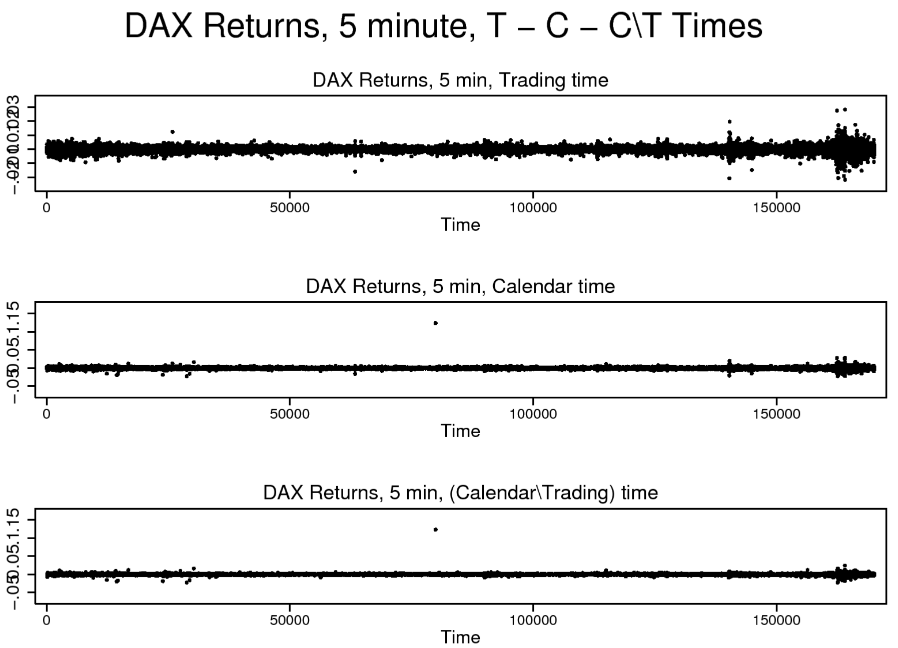


Figure 40: Returns for the DAX (Frankfurt Stock Exchange) index using 10 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

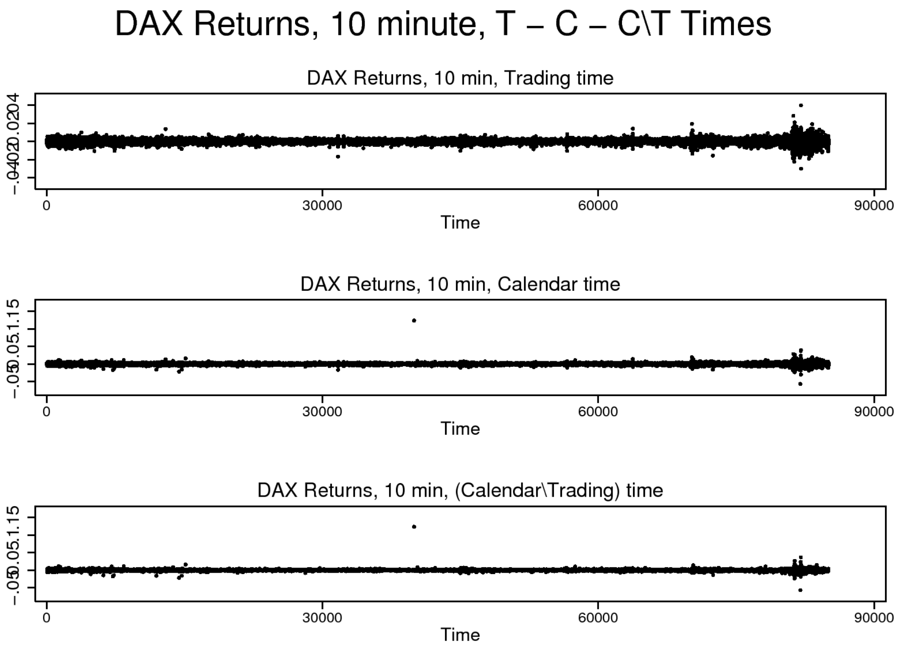


Figure 41: Returns for the DAX (Frankfurt Stock Exchange) index using 15 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

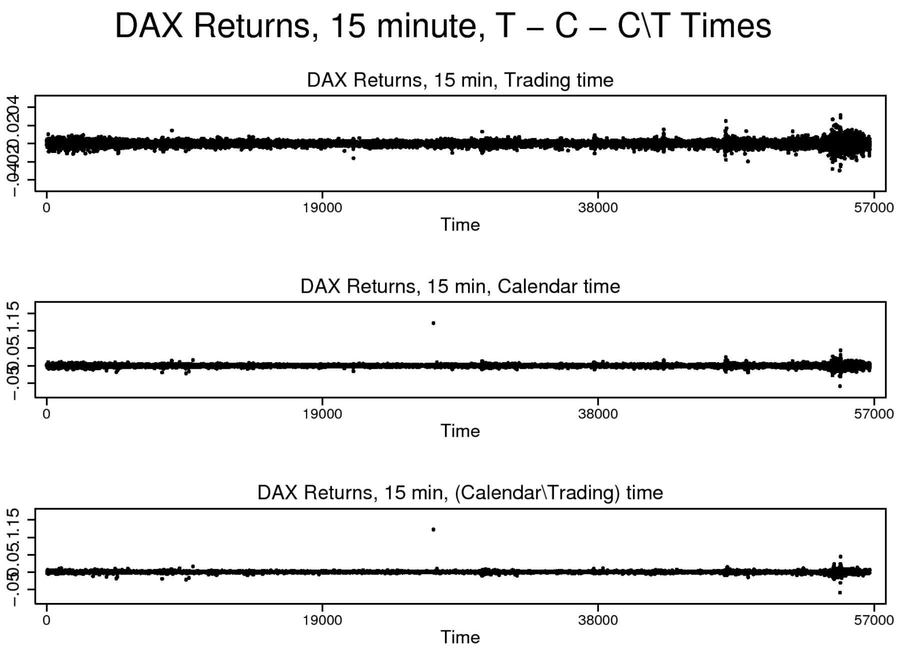


Figure 42: Returns for the DAX (Frankfurt Stock Exchange) index using 30 minute frequency. The Figure contains: trading time, calendar time and the difference between calendar and trading times (i.e., the data omitted from the trading time).

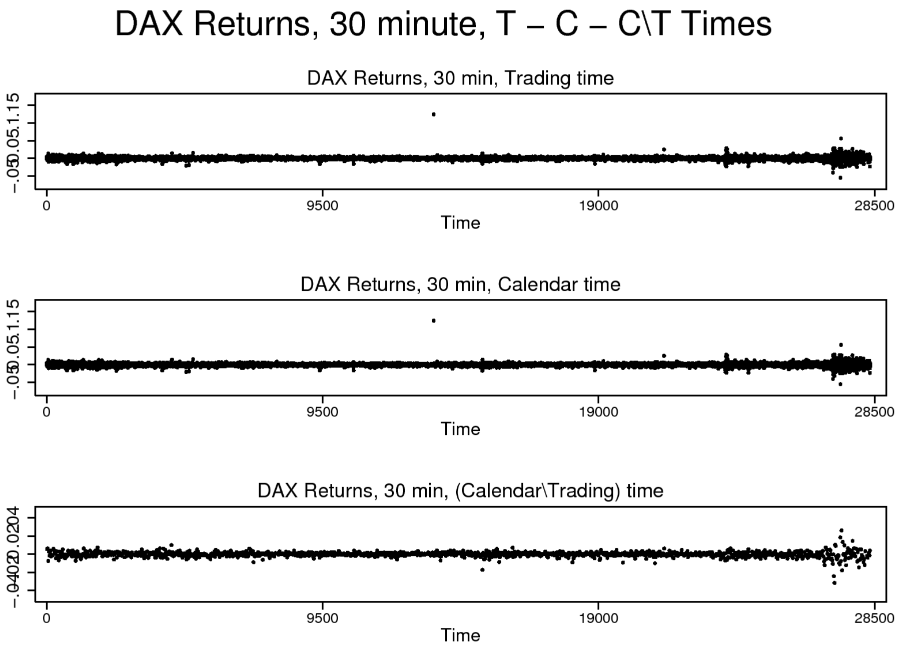


Figure 43: Basic statisitical properties of the BUX index (Budapest Stock Exchange). Plotted are: number of observations (N), mean, variance, skewwness and kurtosis for returns using 5 minute frequency. Statistical variables are calculated month by month, where the period for this calculation ranges from 01/2004 to 12/2008.

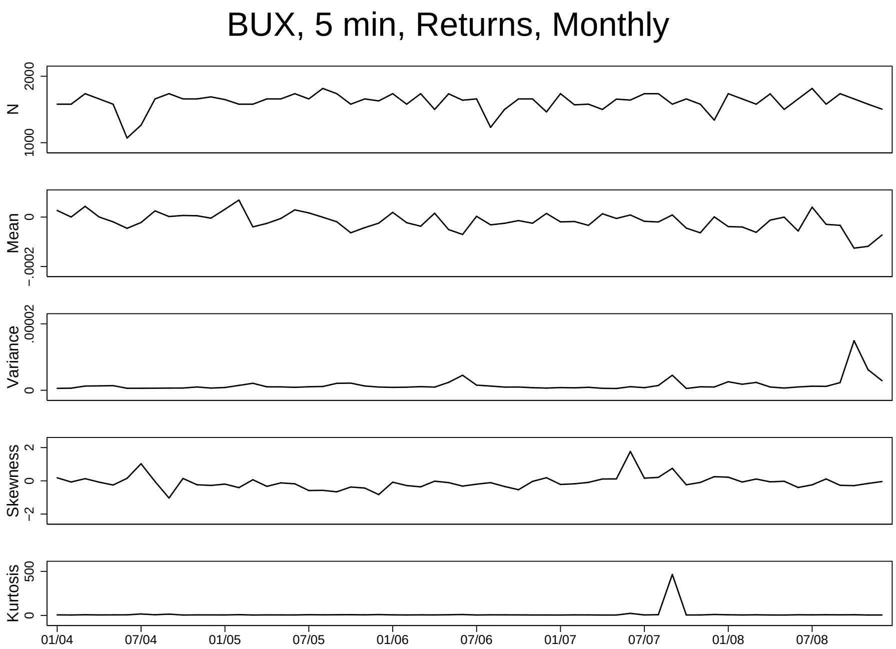


Figure 44: Basic statisitical properties of the WIG index (Budapest Stock Exchange). Plotted are: number of observations (N), mean, variance, skewwness and kurtosis for returns using 5 minute frequency. Statistical variables are calculated month by month, where the period for this calculation ranges from 01/2004 to 12/2008.

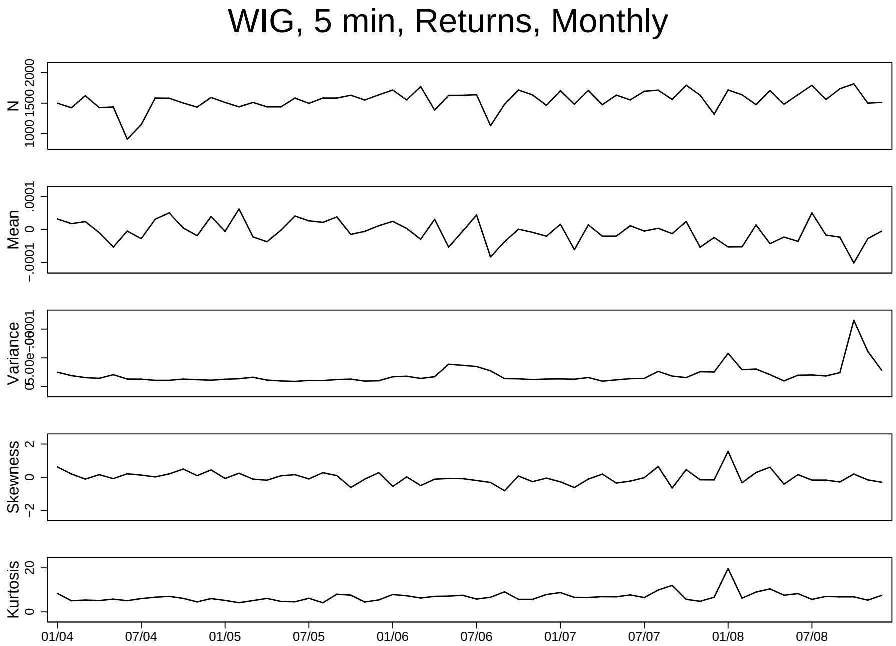
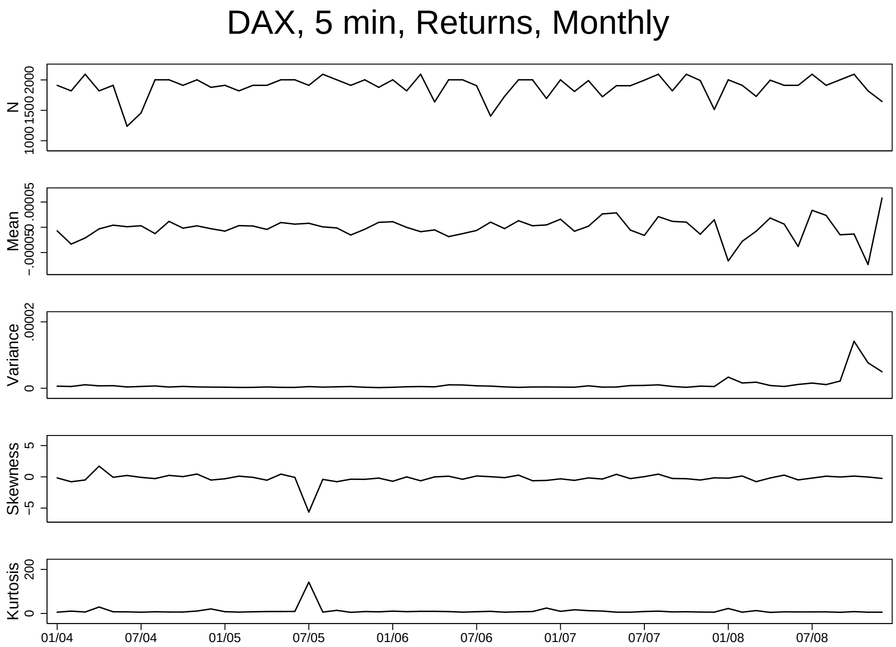


Figure 45: Basic statisitical properties of the DAX index (Budapest Stock Exchange). Plotted are: number of observations (N), mean, variance, skewwness and kurtosis for returns using 5 minute frequency. Statistical variables are calculated month by month, where the period for this calculation ranges from 01/2004 to 12/2008.



Price jump index

The Figures 46, 47 and 48 show the tail part of the distribution of the price jump index for three indexes BUX, WIG and DAX for all four frequencies and all six time windows. There is also depicted the same quatity after log-log transformation.

The Tables 7 – 10 contain estimated characteristic coefficient α_T for all four indexes using all four frequencies and four time windows $T = 100$, 1000, 2000 and 5000.

Financial Turmoil

The volatility study for monthly data is in the Figure 49. The volatility distribution for all four indexes using 5 and 30 minute frequency with the shortest time window $T = 12$.

Estimated characteristic coefficient α_T on mohtly basis for all four indexes is depicted in the Figure 50. There are also 95% CI boundaries.

Price Jump Index Monthly & Quarterly

The tail part of the log-log distributions of extreme events for three remaining indexes calculated on quarterly basis are in the Figures 51 – 53. The Figures 54 – 57 show the same quantity but using monthly basis. The Figures are for all four indexes.

Daily Distribution of the Price Jump Index

The Figures 58 – 60 show 99%, 97% and 95% centiles calculated for remaining three indexes using 5 and 30 minute frequency and $T = 12$ and 5000. Theare are centiles calculated for the entire period spanning from 01/2004 to 12/2008 as well as the centiles calculated month by month.

The Figures 61 – 64 present comparison of centiles among indexes for every frequency (5 and 30 minute) and every time window ($T = 12$ and 5000). Centiles are defined as above and are calculated for the entire period as wellas on monthly basis.

The Figures 61 – 64 present comparison of centiles among indexes for every frequency (5 and 30 minute) and every time window ($T = 12$ and 5000). Centiles are defined as above and are calculated for the entire period as wellas on monthly basis.

The Figures 65 – 67 show the distribution of the extreme values of the price jump index over a trading day for three remaining indexes using 5 and 30 minute frequency and $T = 12$ and 5000. Extreme events are those for which the price jump index is higher than a 95% centile, where centiles are calculated on motnhly basis.

The Figures 68 – 71 represent the same quantity as above, but with a changed threshold for extreme events which is now equal to 97% centile. The Figures are for all four indexes.

The Figures 72 – 75 represent the same quantity as above, but with a changed threshold for extreme events which is now equal to 99% centile. The Figures are for all four indexes.

The Figures 76 – 79 represent the same quantity as above, but with a changed threshold for extreme events which is now equal to 95% centile and is calculated for the entire period. The Figures are for all four indexes.

The Figures 80 – 83 represent the same quantity as above, but with a changed threshold for extreme events which is now equal to 97% centile and is calculated for the entire period. The Figures are for all four indexes.

The Figures 84 – 87 represent the same quantity as above, but with a changed threshold for extreme events which is now equal to 99% centile and is calculated for the entire period. The Figures are for all four indexes.

The Figures 88 – 90 represent the temporal distribution of extreme price jumps for three remaining indexes using 5 minute frequency and time windows $T = 12$ and $T = 5000$. The extreme events are characterized as those higher than 95% centile, where centiles are calculated every month separately. The time period covers years from 2004 to 2008, where 1 in both Figures represents 01/2004 and 60 represents 12/2008.

Price Jump "Condensation"

The Figures 91 – 93 represent the distribution of the number of extreme jumps per day using three remaining indexes with 5 and 30 minute frequency and time windows $T = 12$ and 5000. The time period spans from 2004 to 2008.

Table 7: The estimated α_T for all four indexes the PX index using all four frequencies: 5, 10, 15 and 30 minute, and four time windows $T = 100$, 1000, 2000 and 5000. The value in the bracket is the standard deviation.

$\alpha(\sigma_\alpha)$	$T = 100$	$T = 1000$	$T = 2000$	$T = 5000$
PX, 5 min	4.156 (.055)	3.539 (.038)	3.901 (.040)	3.554 (.033)
PX, 10 min	4.902 (.139)	4.145 (.060)	4.330 (.072)	3.313 (.031)
PX, 15 min	5.626 (.157)	4.134 (.059)	3.774 (.053)	3.713 (.040)
PX, 30 min	5.436 (.241)	5.180 (.159)	4.668 (.108)	4.167 (.085)

Table 8: The estimated α_T for all four indexes the BUX index using all four frequencies: 5, 10, 15 and 30 minute, and four time windows $T = 100$, 1000, 2000 and 5000. The value in the bracket is the standard deviation.

$\alpha(\sigma_\alpha)$	$T = 100$	$T = 1000$	$T = 2000$	$T = 5000$
BUX, 5 min	4.583 (.077)	4.662 (.059)	4.757 (.080)	3.809 (.024)
BUX, 10 min	4.344 (.074)	3.637 (.060)	3.757 (.038)	3.403 (.027)
BUX, 15 min	4.202 (.112)	3.771 (.065)	3.743 (.031)	3.493 (.029)
BUX, 30 min	5.148 (.162)	3.858 (.043)	3.683 (.077)	3.465 (.059)

Table 9: The estimated α_T for all four indexes the WIG index using all four frequencies: 5, 10, 15 and 30 minute, and four time windows $T = 100$, 1000, 2000 and 5000. The value in the bracket is the standard deviation.

$\alpha(\sigma_\alpha)$	$T = 100$	$T = 1000$	$T = 2000$	$T = 5000$
WIG, 5 min	5.635 (.212)	6.501 (.126)	5.562 (.101)	4.949 (.083)
WIG, 10 min	6.467 (.344)	5.971 (.270)	5.273 (.188)	4.825 (.096)
WIG, 15 min	5.204 (.182)	5.559 (.139)	5.381 (.131)	4.517 (.099)
WIG, 30 min	5.514 (.152)	4.106 (.123)	4.000 (.071)	3.927 (.063)

Table 10: The estimated α_T for all four indexes the DAX index using all four frequencies: 5, 10, 15 and 30 minute, and four time windows $T = 100$, 1000, 2000 and 5000. The value in the bracket is the standard deviation.

$\alpha(\sigma_\alpha)$	$T = 100$	$T = 1000$	$T = 2000$	$T = 5000$
DAX, 5 min	4.707 (.054)	4.411 (.038)	4.477 (.048)	4.098 (.046)
DAX, 10 min	5.609 (.119)	4.602 (.052)	4.074 (.046)	3.773 (.030)
DAX, 15 min	4.986 (.083)	3.939 (.075)	3.596 (.054)	3.542 (.049)
DAX, 30 min	4.140 (.064)	2.249 (.068)	3.000 (.055)	2.973 (.032)

Figure 46: LHS: a) Tail part of the price jump index distribution for the BUX index, using all four frequencies and six different time windows T. The two short-term windows have more suppressed occurrence of the extreme events compared to the four long-term windows. RHS: b) The same distribution after performing a natural logarithms to both sides of eq. (7). Symbols used:$T = 12$: $\bullet$, $T = 24$: $\diamond$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$, and $T = 5000$: $\times$.

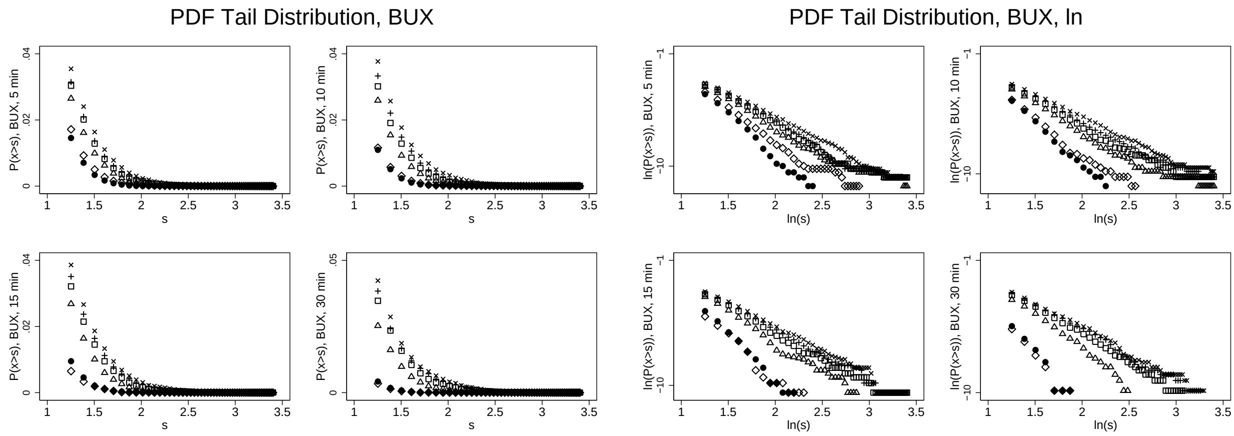


Figure 47: LHS: a) Tail part of the price jump index distribution for the WIG index, using all four frequencies and six different time windows T. The two short-term windows have more suppressed occurrence of the extreme events compared to the four long-term windows. RHS: b) The same distribution after performing a natural logarithms to both sides of eq. (7). Symbols used:$T = 12$: $\bullet$, $T = 24$: $\diamond$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$, and $T = 5000$: $\times$.

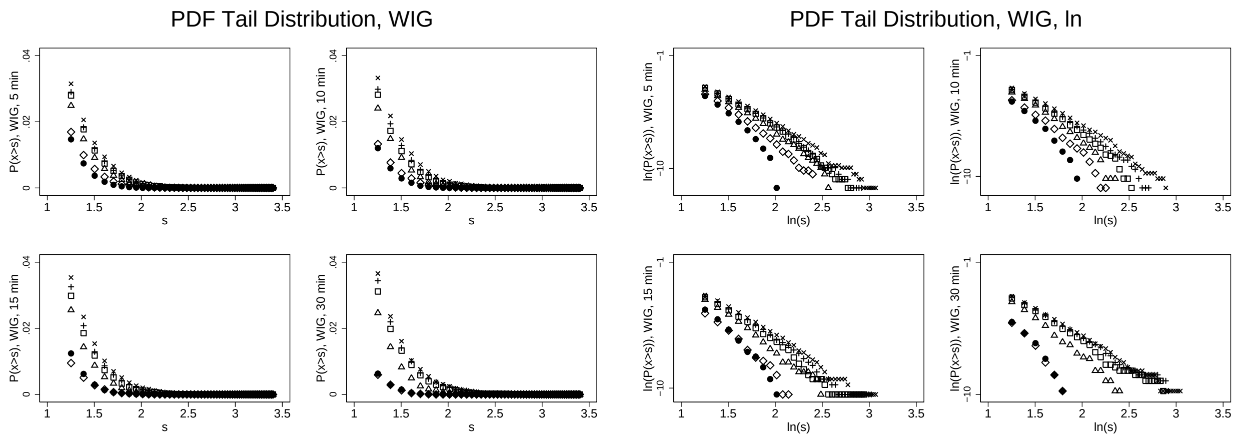


Figure 48: LHS: a) Tail part of the price jump index distribution for the PX index, using all four frequencies and six different time windows T. The two short-term windows have more suppressed occurrence of the extreme events compared to the four long-term windows. RHS: b) The same distribution after performing a natural logarithms to both sides of eq. (7). Symbols used:$T = 12$: $\bullet$, $T = 24$: $\diamond$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$, and $T = 5000$: $\times$.

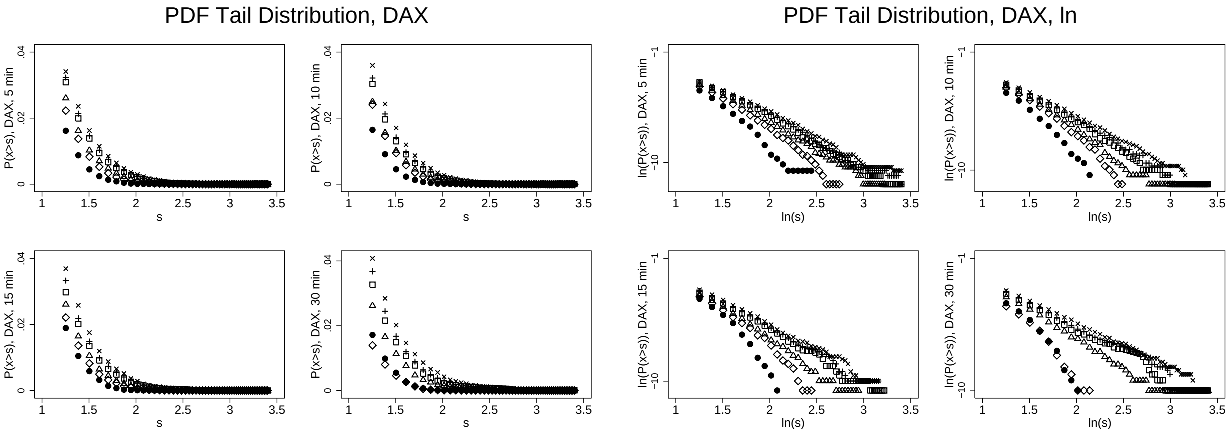


Figure 49: LHS: a) Volatility study of all four indexes: PX, BUX, WIG and DAX, using 5 minute data and the shortest time window $T = 12$. The Figure shows the distribution of the volatility for each month. RHS: b) The same volatility distribution but with 30 minute data.

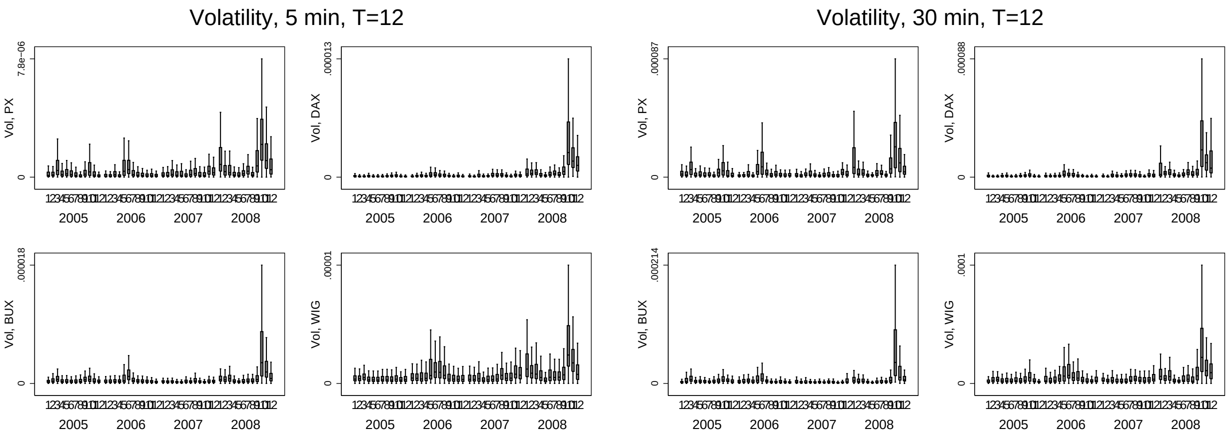


Figure 50: Estimated characteristic parameter α_T for all four indexes: PX, BUX, WIG and DAX, estimated on monthly basis for period spanning from 2005 to 2008. LHS: The characteristic parameter for $T = 12$. RHS: The characteristic coefficient for $T = 5000$. Dashed line stands in both sub-figures represent the 95% CI.

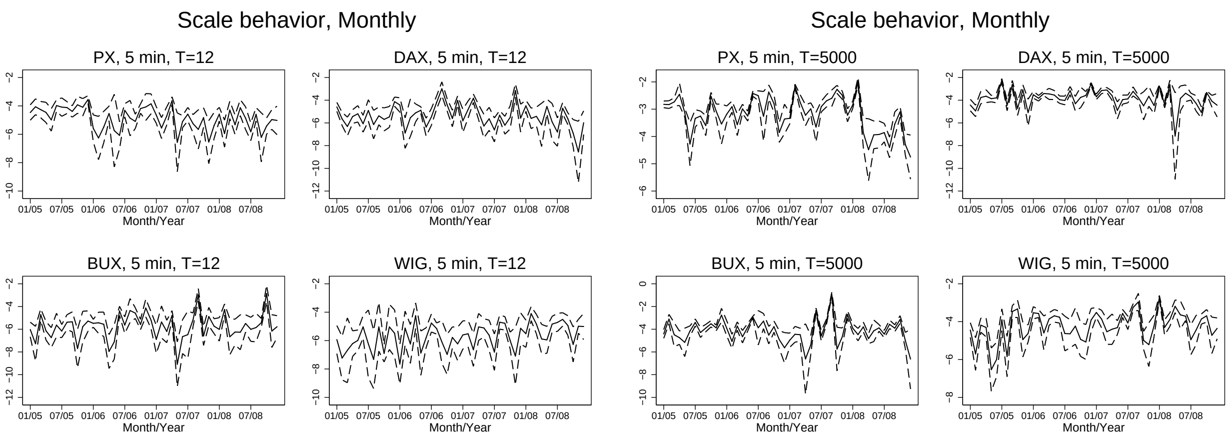


Figure 51: The tail part of the log-log distribution of extreme events for the BUX index. Data spans from 2005 to 2008 and are taken on quarterly basis, where 1 stands for the first quarter of 2005 and 16 for the last quarter of 2008. The ln(s) is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

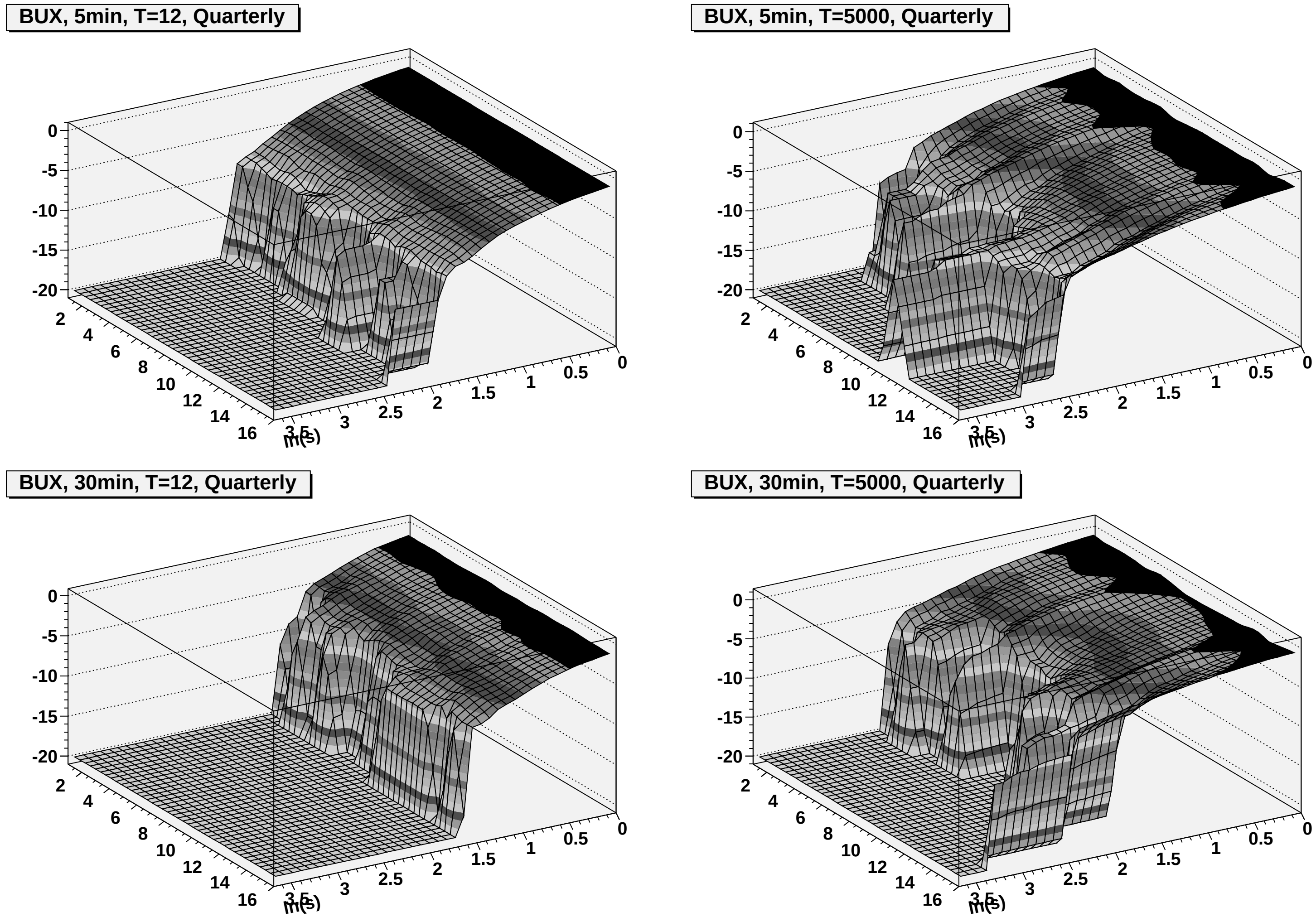


Figure 52: The tail part of the log-log distribution of extreme events for the WIG index. Data spans from 2005 to 2008 and are taken on quarterly basis, where 1 stands for the first quarter of 2005 and 16 for the last quarter of 2008. The ln(s) is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

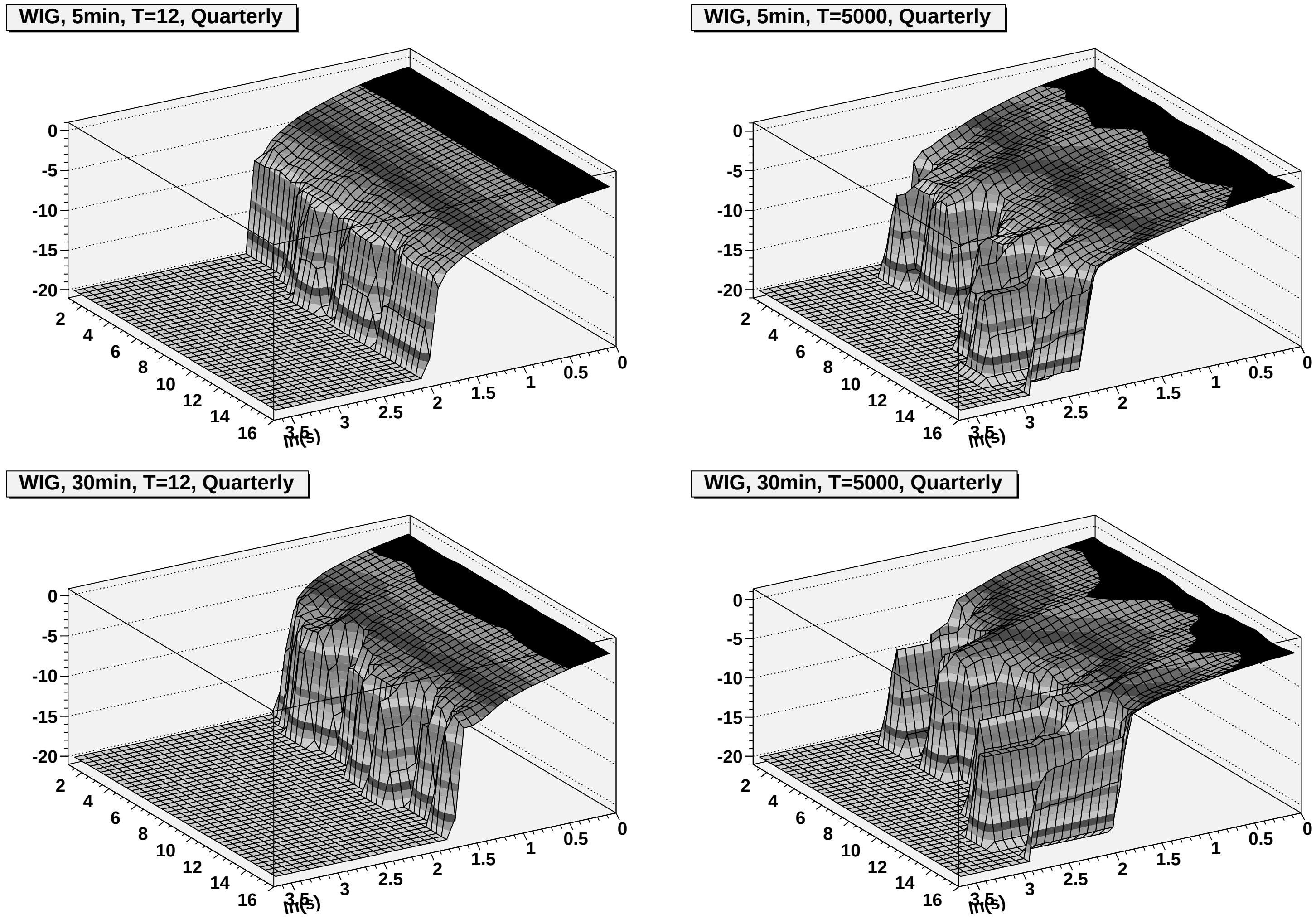


Figure 53: The tail part of the log-log distribution of extreme events for the DAX index. Data spans from 2005 to 2008 and are taken on quarterly basis, where 1 stands for the first quarter of 2005 and 16 for the last quarter of 2008. The ln(s) is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

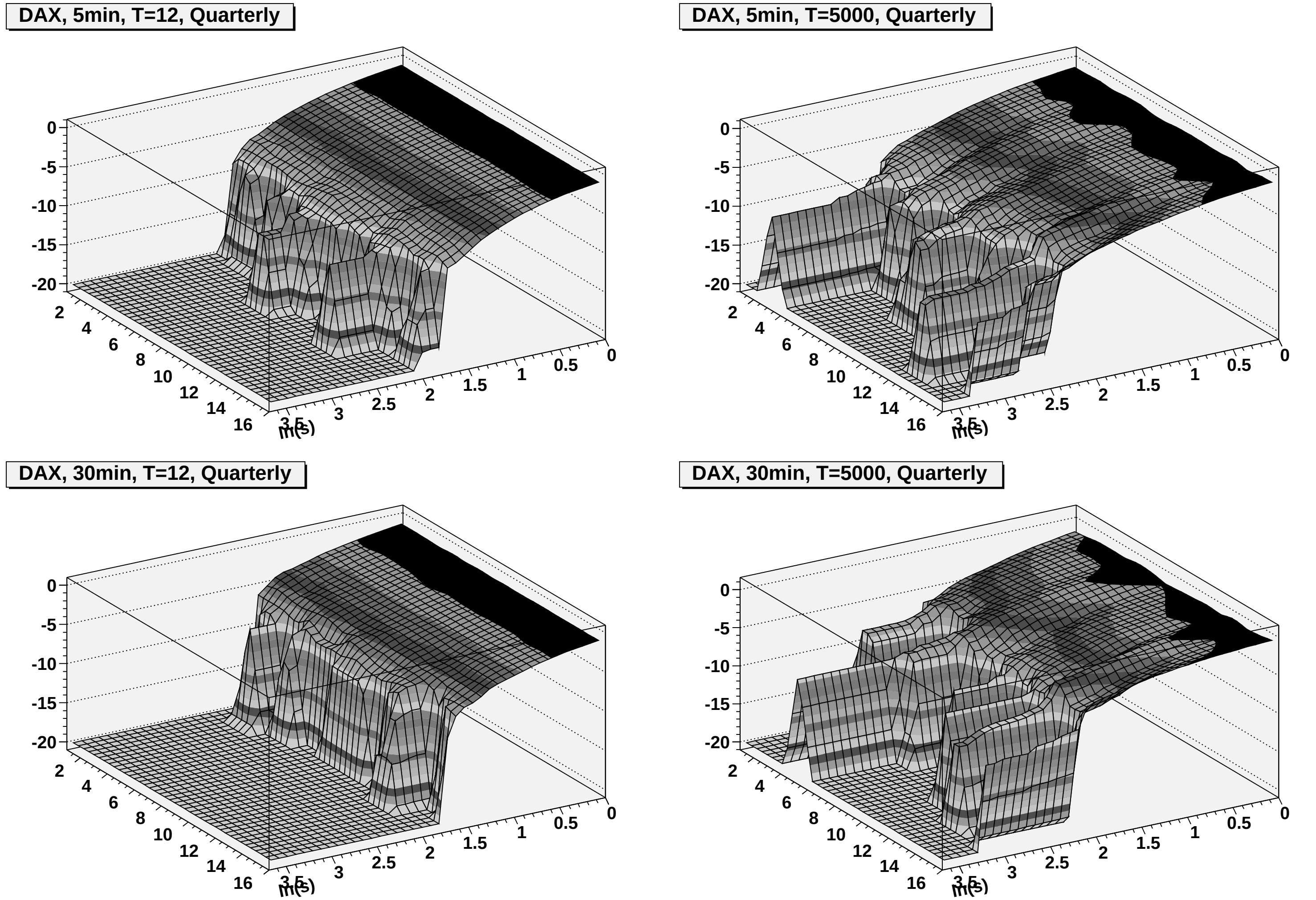


Figure 54: The tail part of the log-log distribution of extreme events for the PX index. Data spans from 2005 to 2008 and are taken on monthly basis, where 1 stands for 01/2005 and 48 for 12/2008. The $\ln(s)$ is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

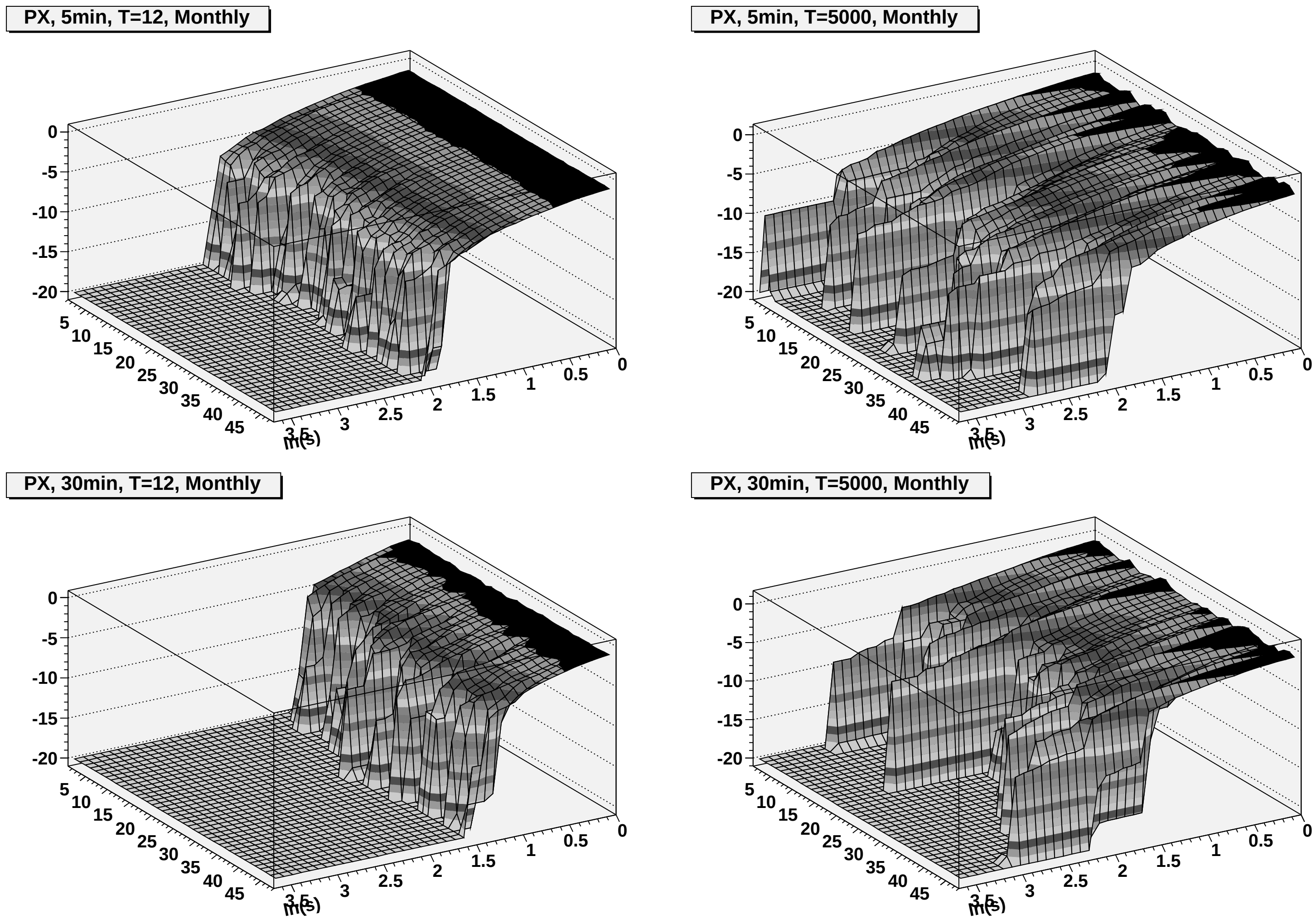


Figure 55: The tail part of the log-log distribution of extreme events for the BUX index. Data spans from 2005 to 2008 and are taken on monthly basis, where 1 stands for 01/2005 and 48 for 12/2008. The $\ln(s)$ is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

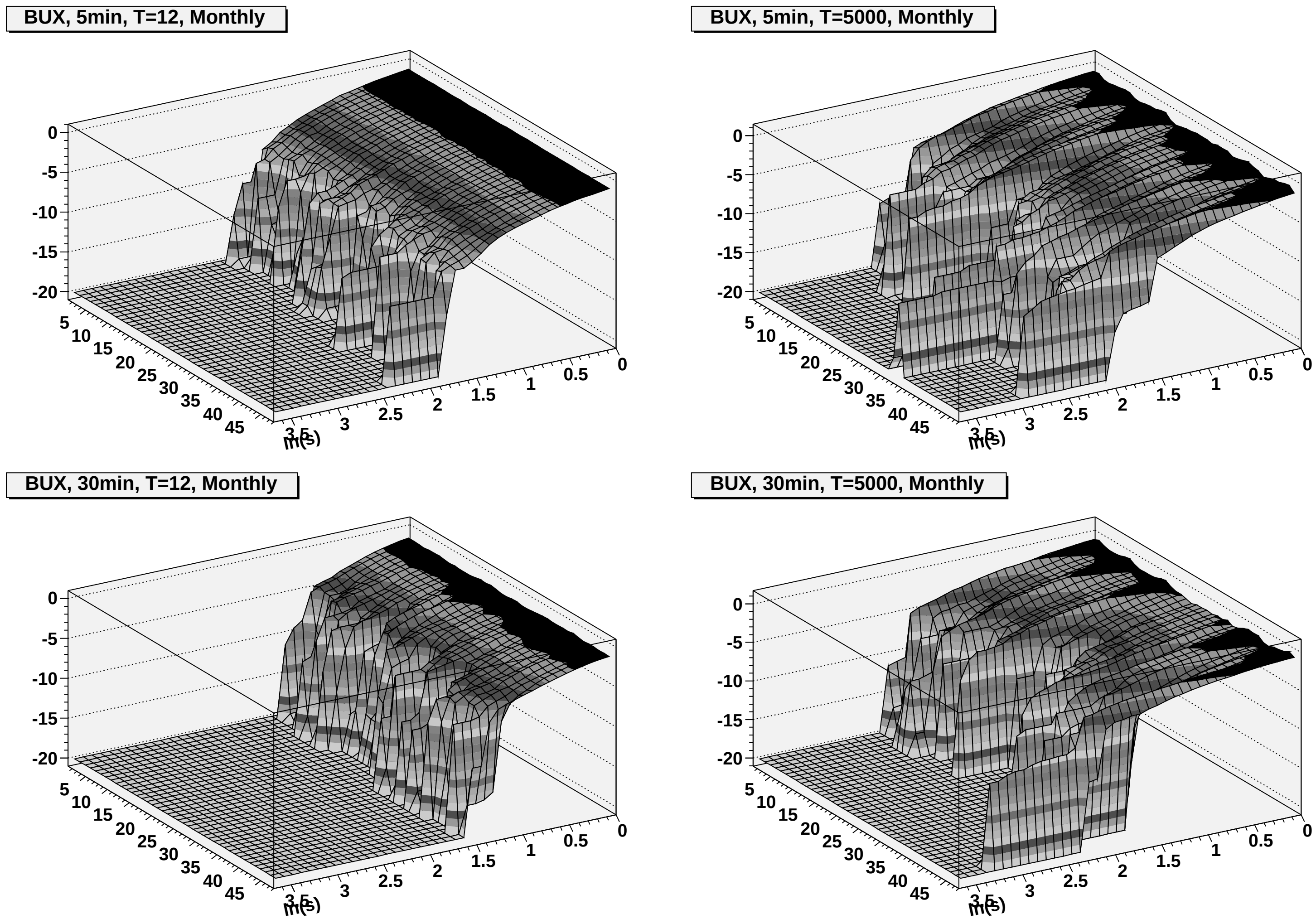


Figure 56: The tail part of the log-log distribution of extreme events for the WIG index. Data spans from 2005 to 2008 and are taken on monthly basis, where 1 stands for 01/2005 and 48 for 12/2008. The $\ln(s)$ is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

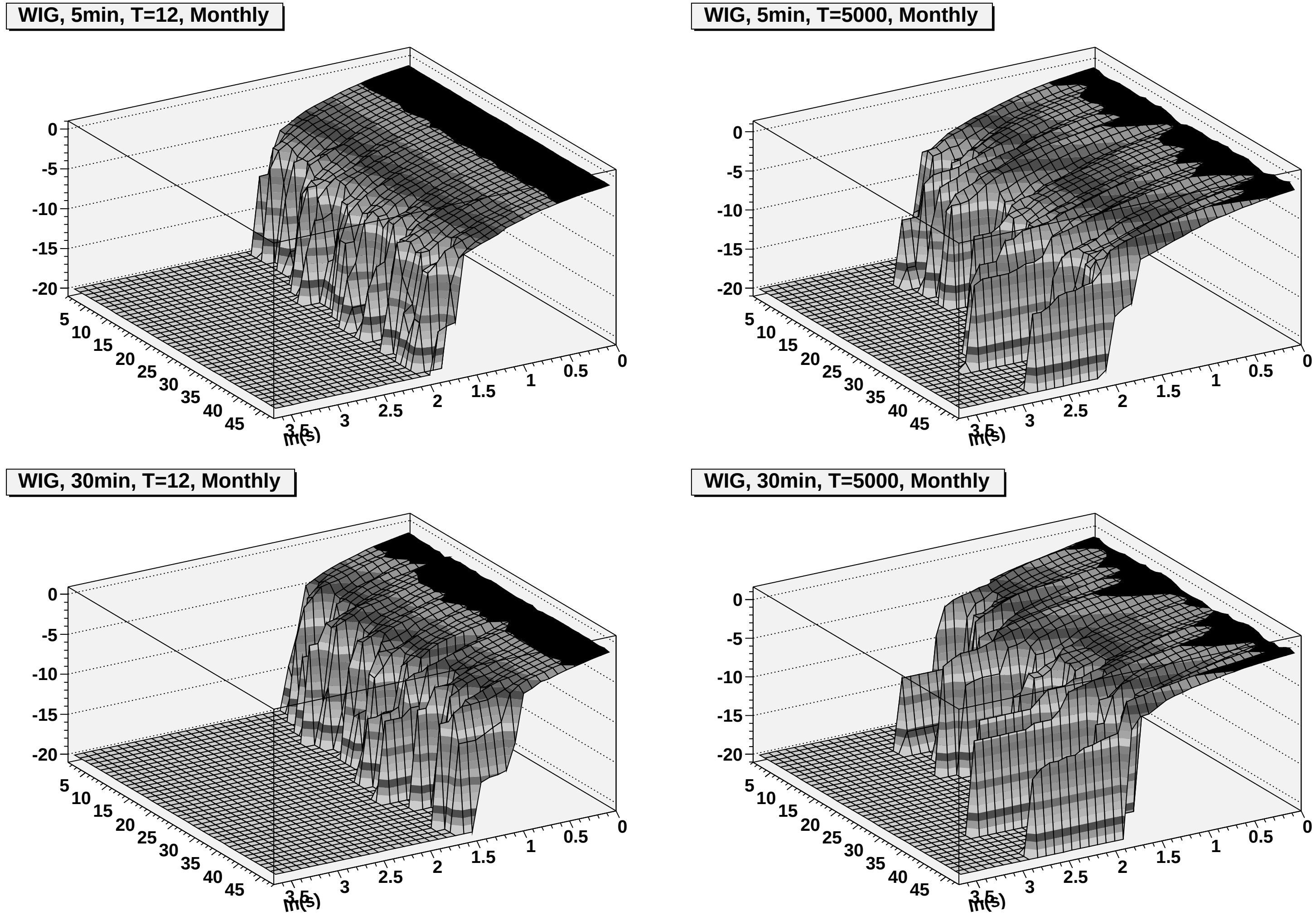


Figure 57: The tail part of the log-log distribution of extreme events for the DAX index. Data spans from 2005 to 2008 and are taken on monthly basis, where 1 stands for 01/2005 and 48 for 12/2008. The ln(s) is the same as defined in eq. (4). This Figure is a 3-D analogy to Figures like Figure 5.

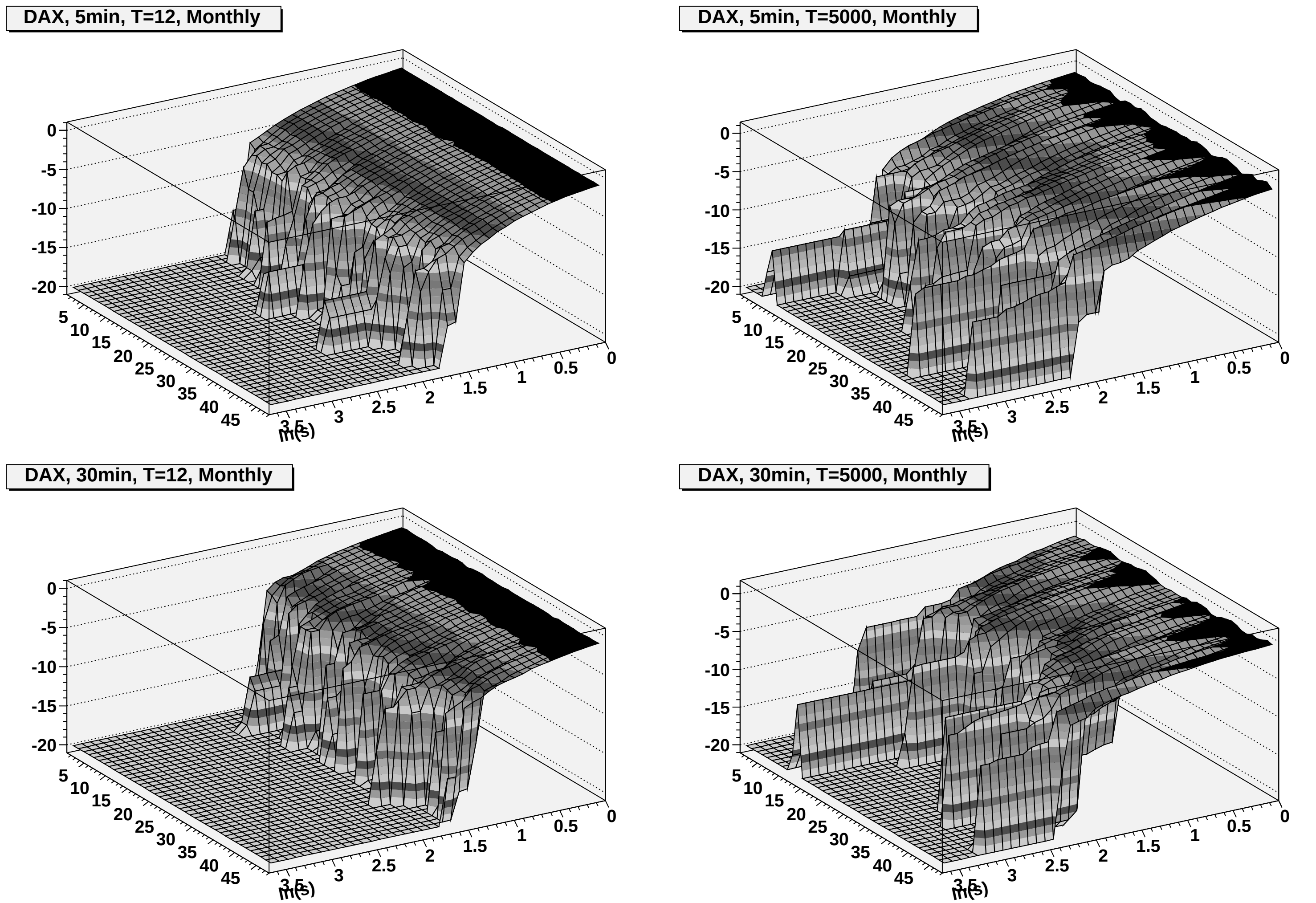


Figure 58: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for the BUX index using 5 and 30 minute frequency, and $T = 12$ and $T = 5000$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

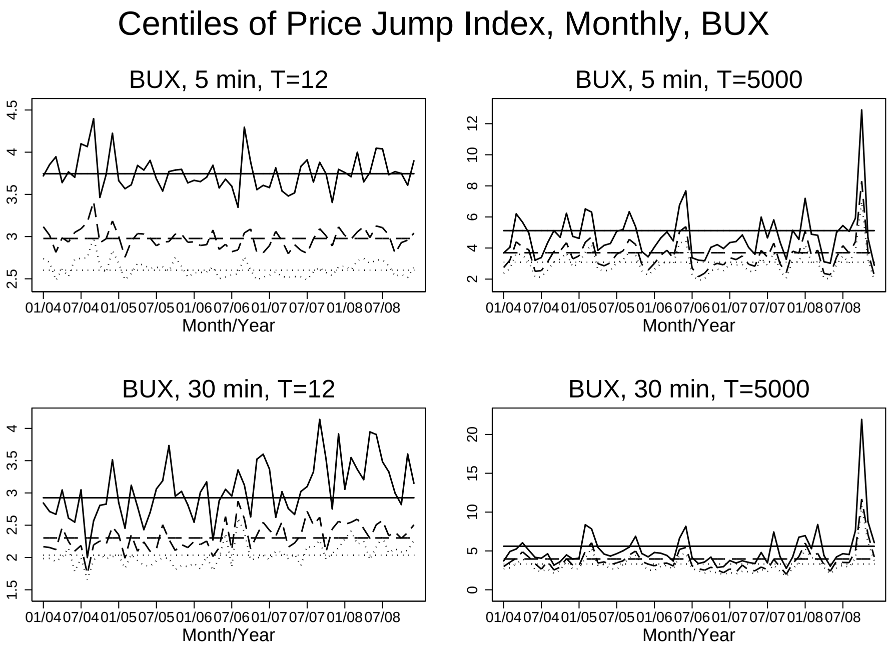


Figure 59: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for the WIG index using 5 and 30 minute frequency, and $T = 12$ and $T = 5000$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

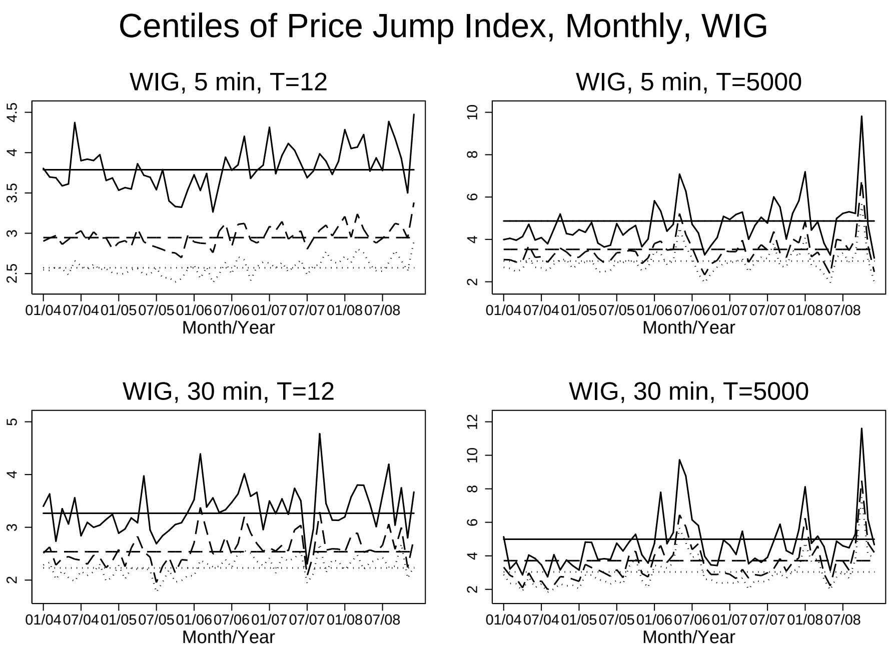


Figure 60: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for the DAX index using 5 and 30 minute frequency, and $T = 12$ and $T = 5000$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

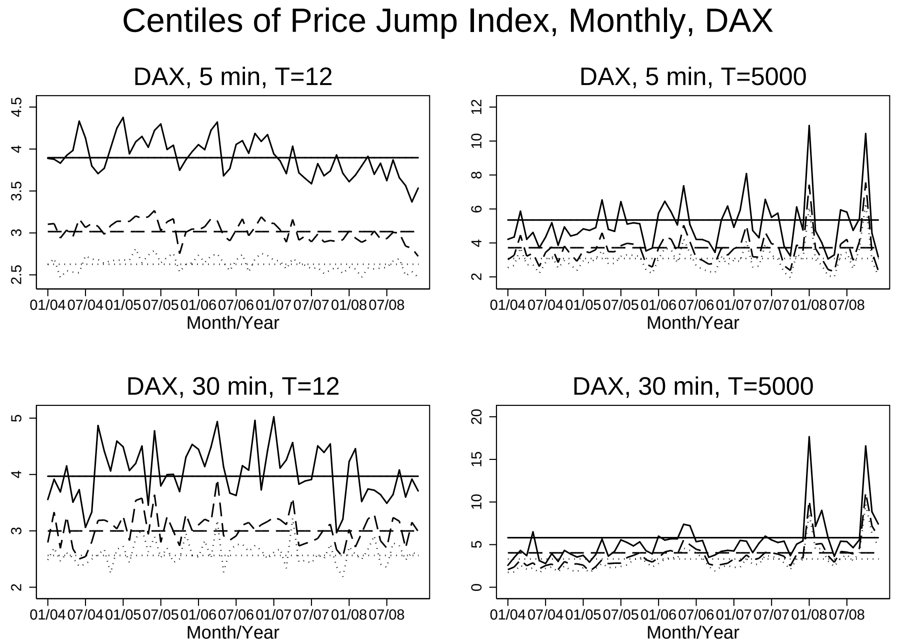


Figure 61: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for all four indexes using 5 minute frequency, and $T = 12$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

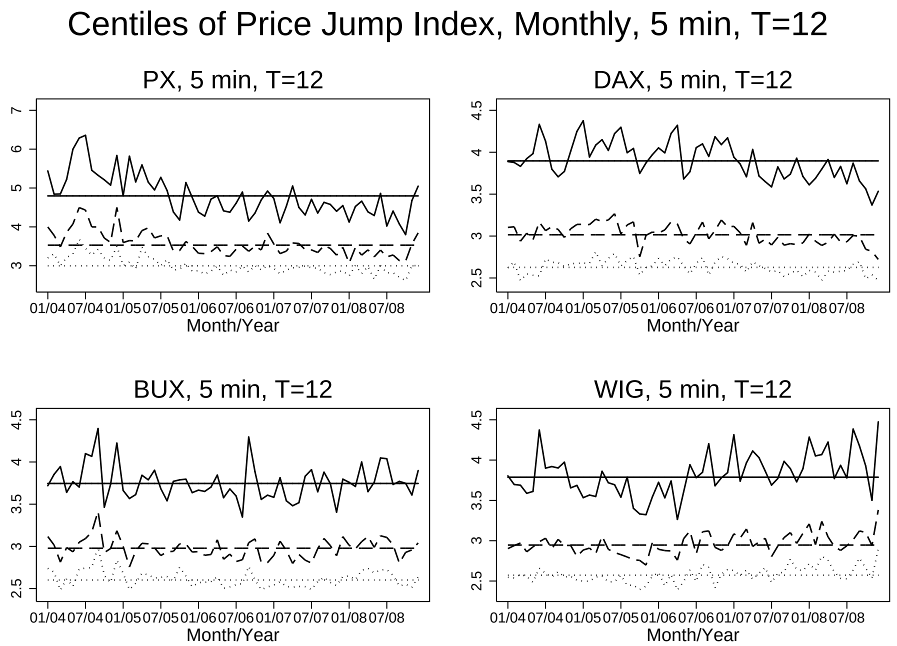


Figure 62: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for all four indexes using 5 minute frequency, and $T = 5000$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

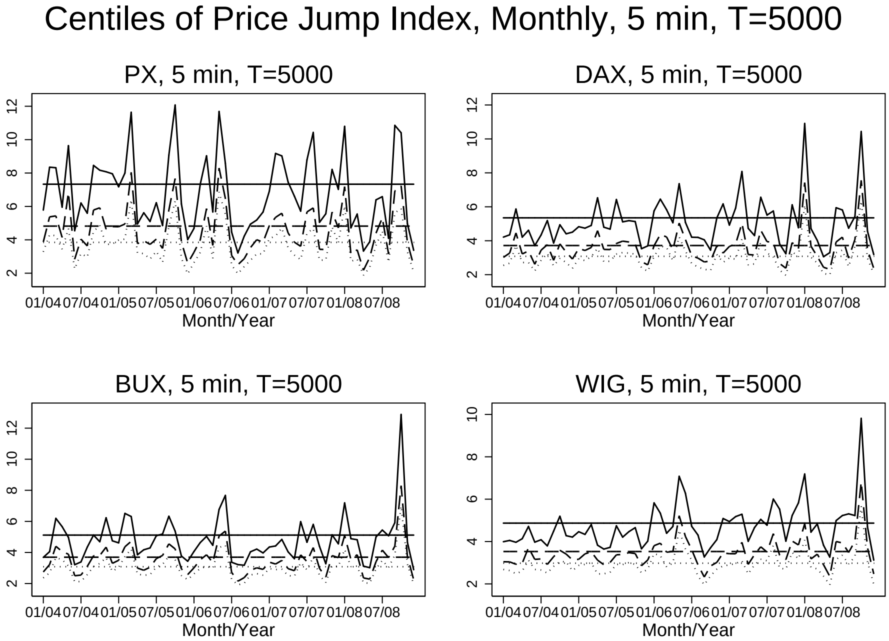


Figure 63: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for all four indexes using 30 minute frequency, and $T = 12$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

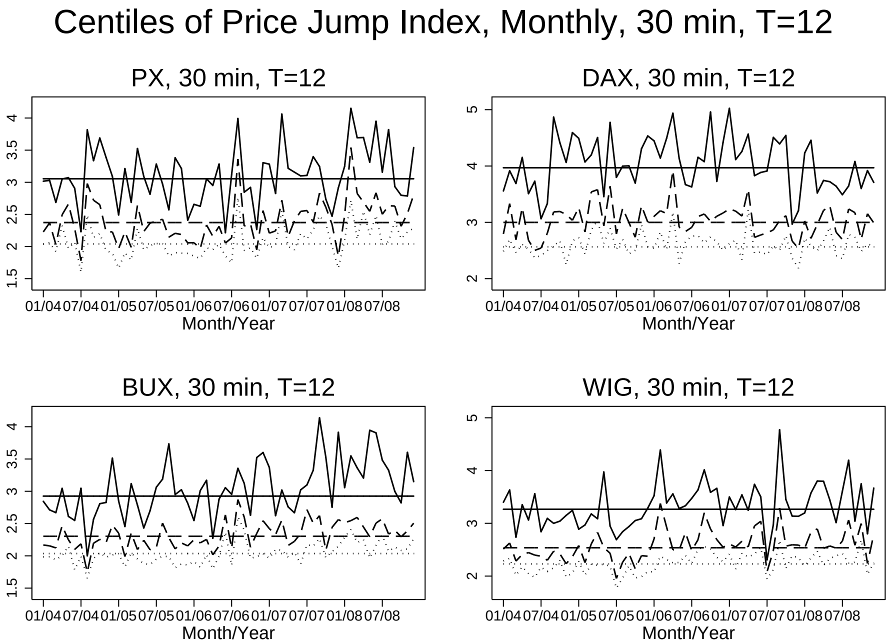


Figure 64: Centiles (99% – full line, 97% – dashed line, 95% – dotted line) of the price jump index for all four indexes using 30 minute frequency, and $T = 5000$. Straight lines are centiles calculated for the entire period spanning from 2004 to 2008, while the other three lines are calculated month by month.

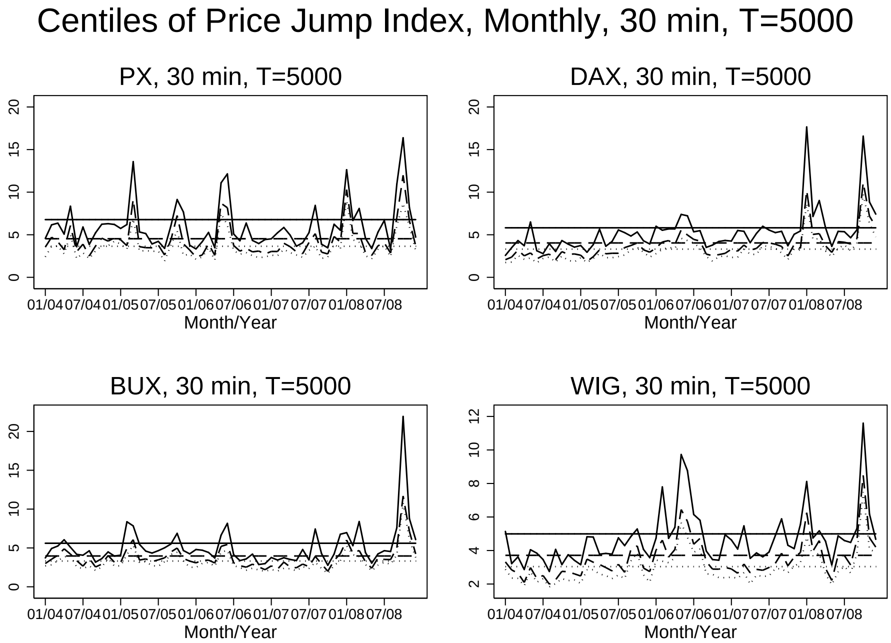


Figure 65: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis.

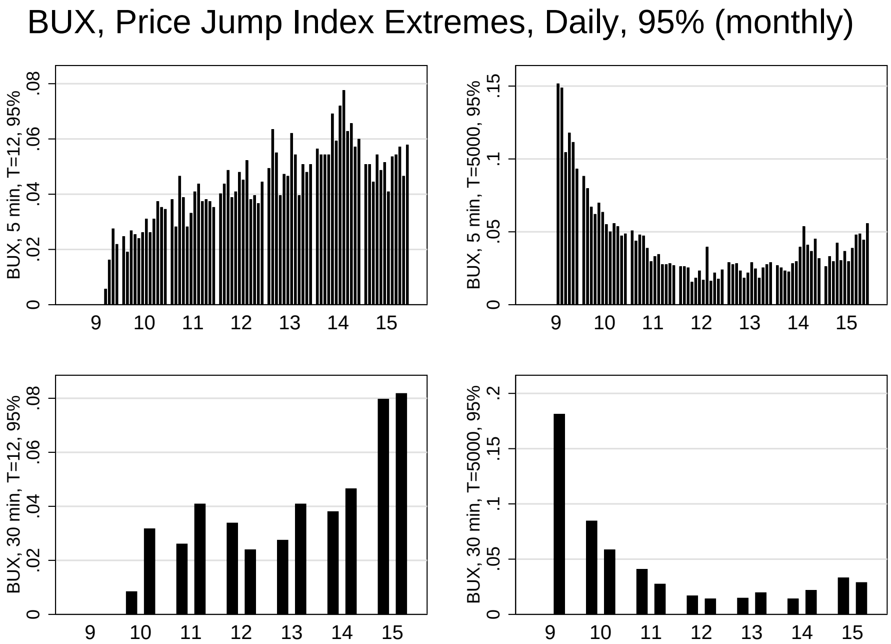


Figure 66: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis.

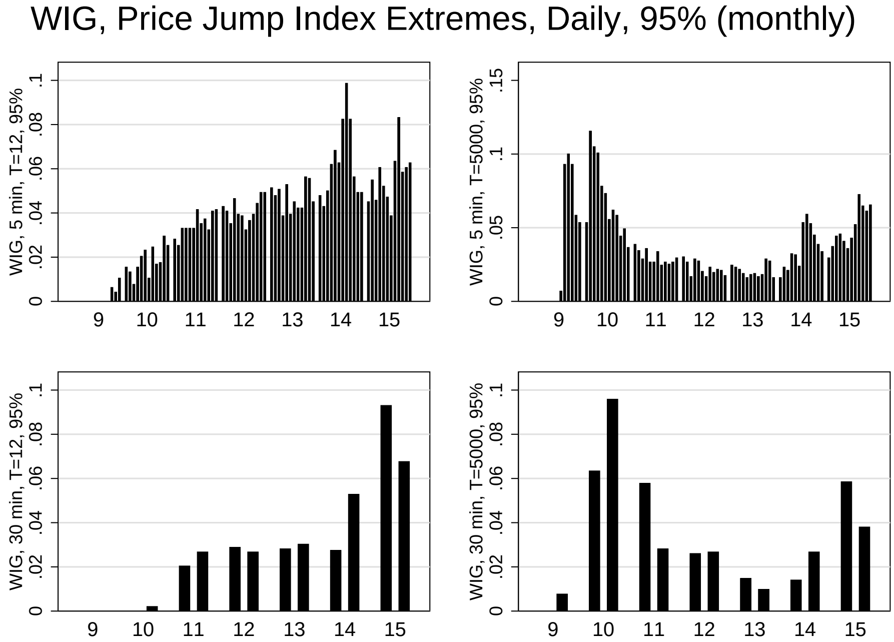


Figure 67: The distribution of the extreme values of the price jump index over a trading day for the DAX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis.

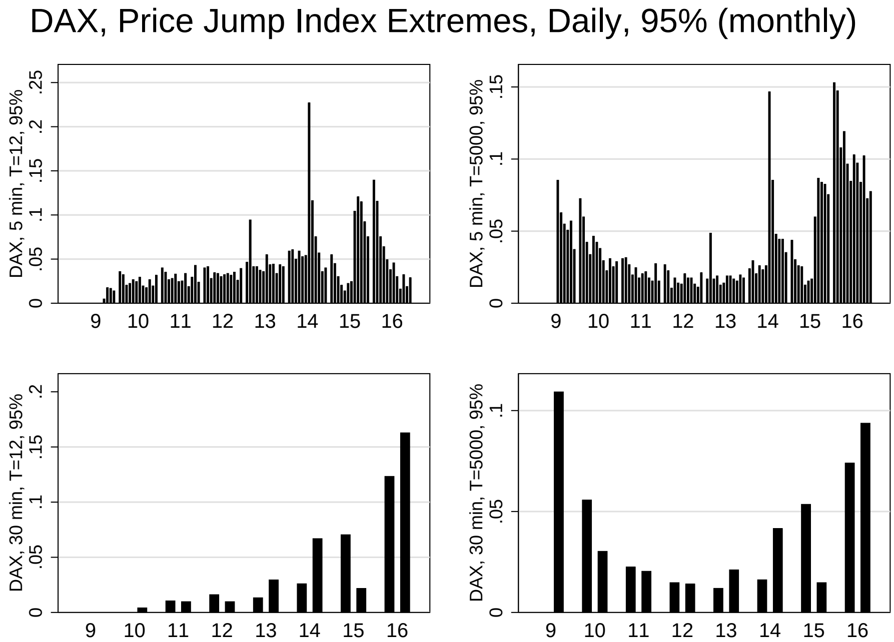


Figure 68: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated on monthly basis.

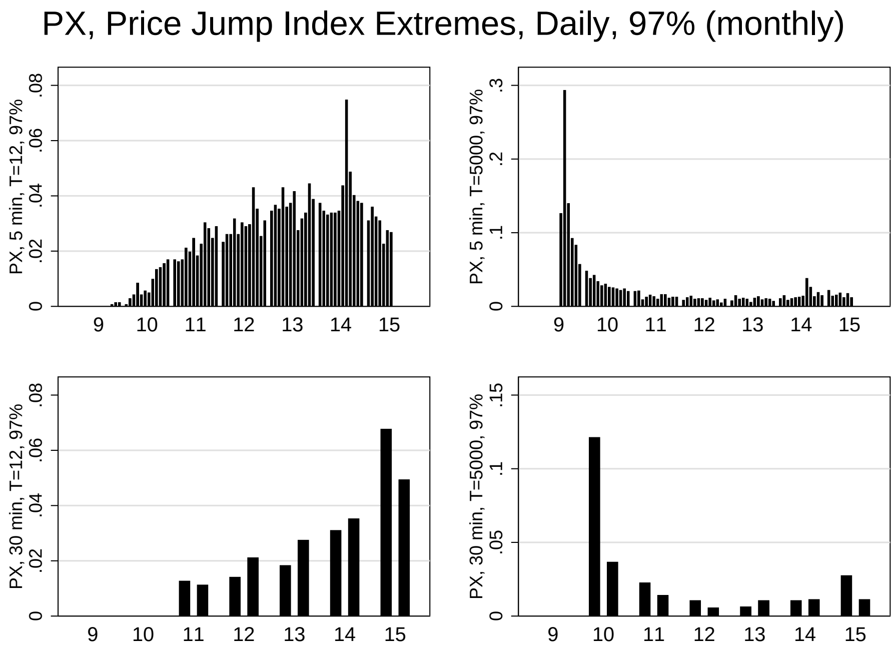


Figure 69: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated on monthly basis.

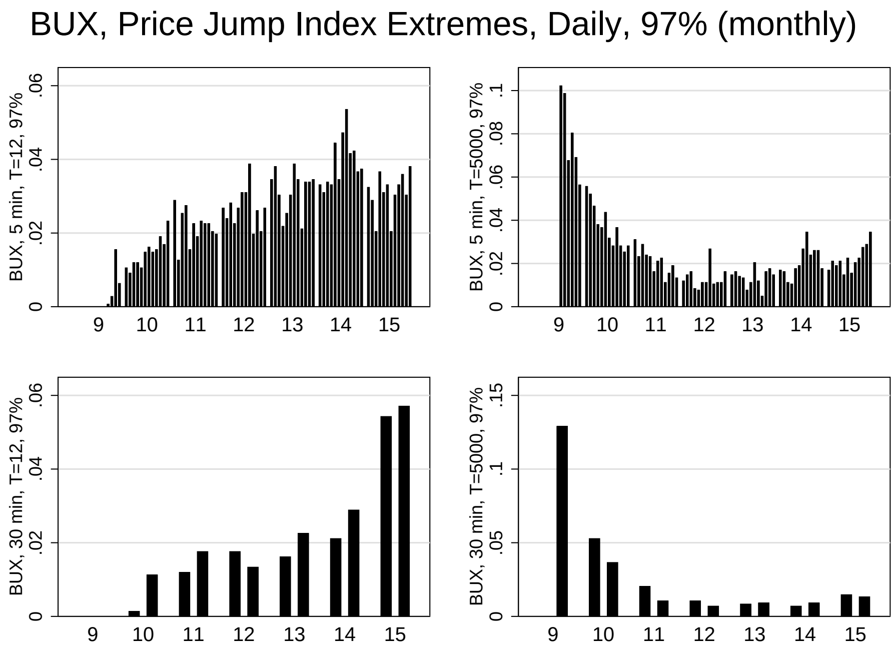


Figure 70: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated on monthly basis.

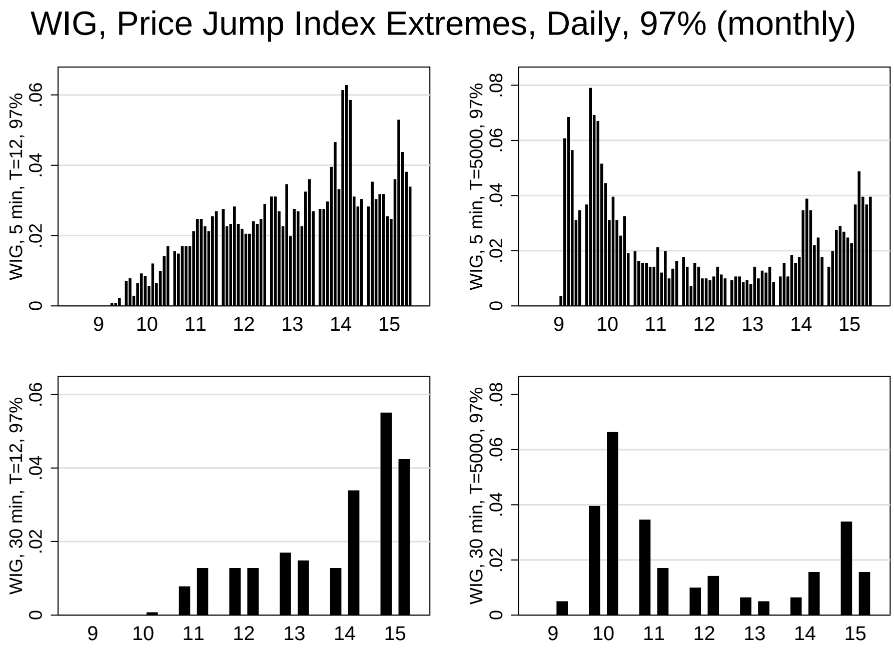


Figure 71: The distribution of the extreme values of the price jump index over a trading day for the DAX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated on monthly basis.

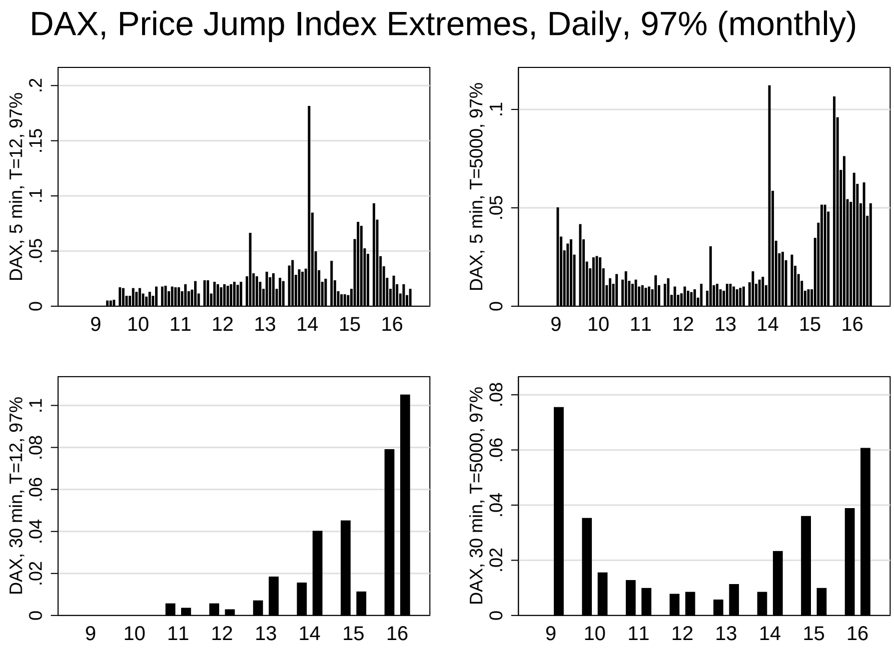


Figure 72: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated on monthly basis.

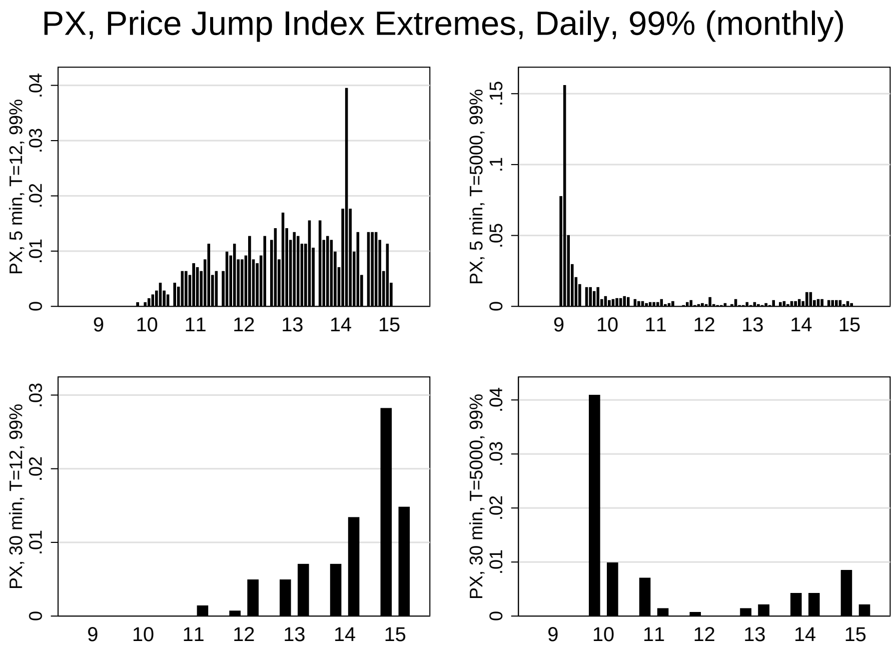


Figure 73: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated on monthly basis.

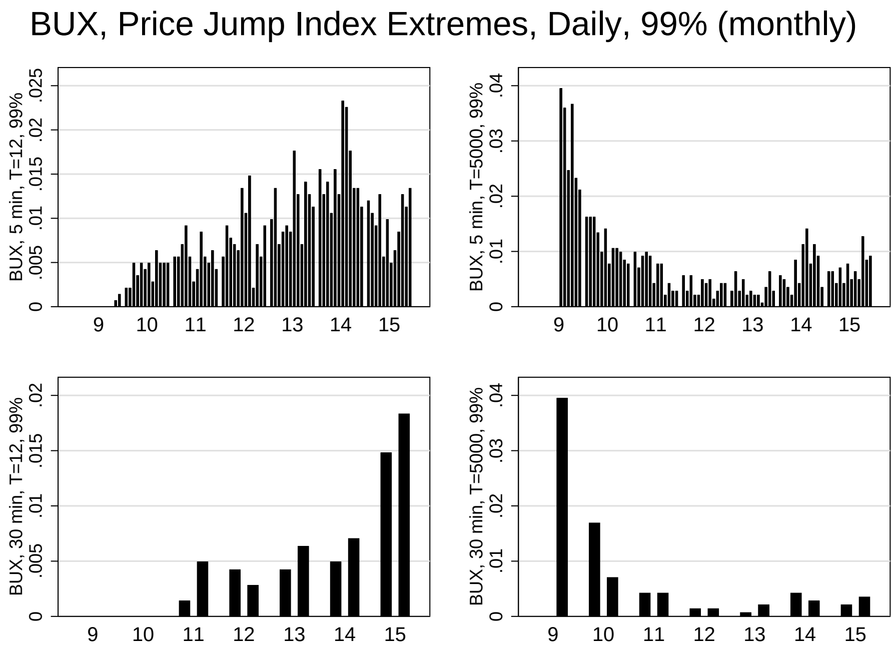


Figure 74: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated on monthly basis.

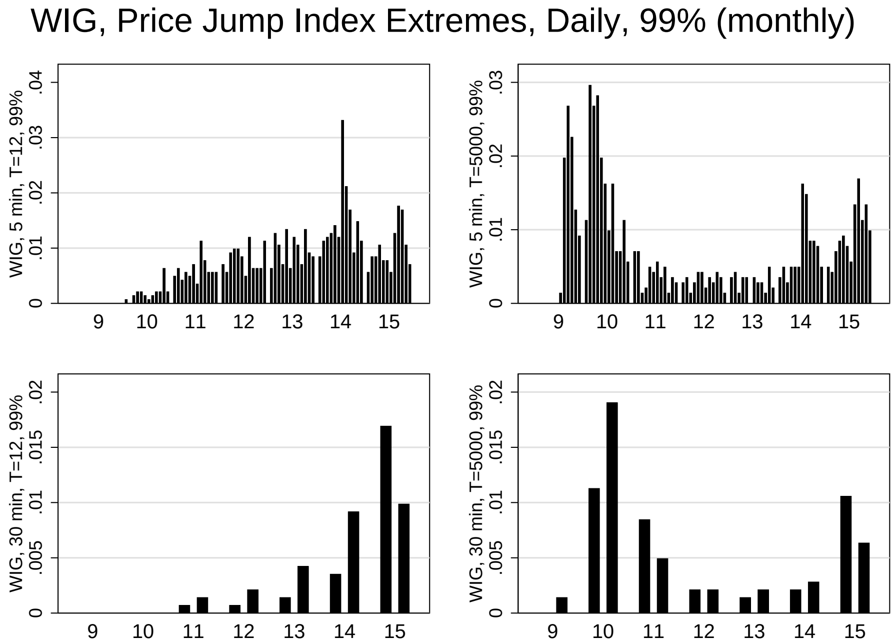


Figure 75: The distribution of the extreme values of the price jump index over a trading day for the DAX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated on monthly basis.

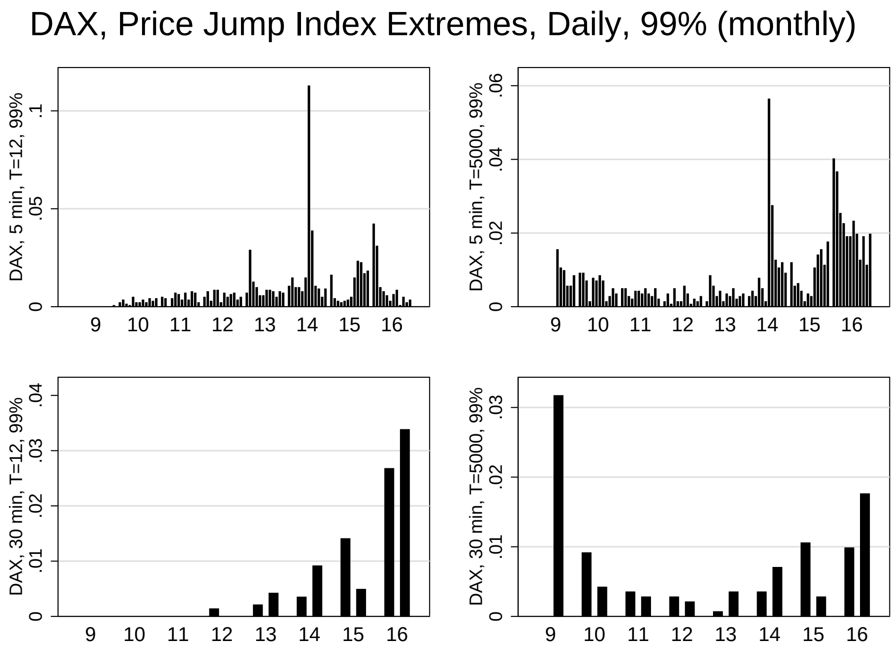


Figure 76: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated for the entire period together.

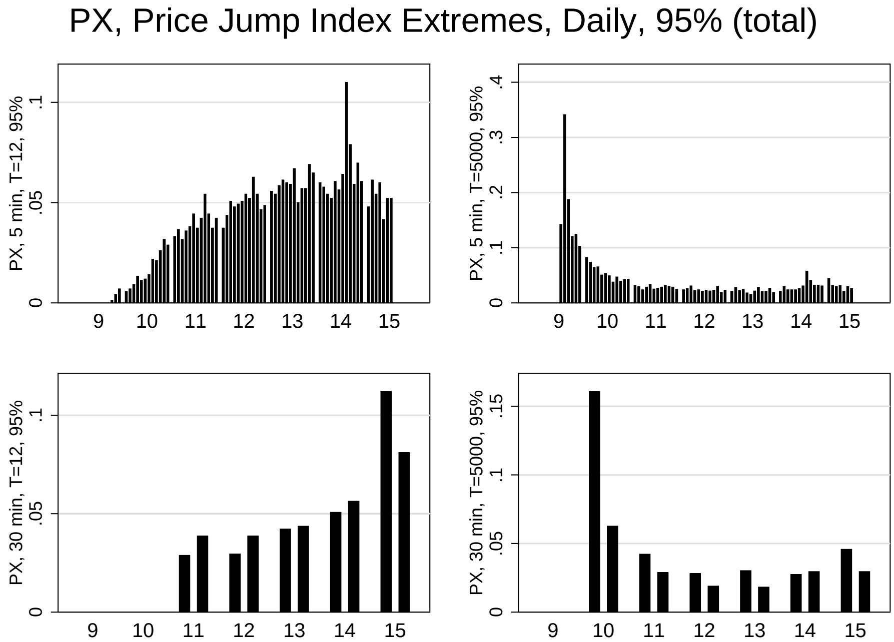


Figure 77: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated for the entire period together.

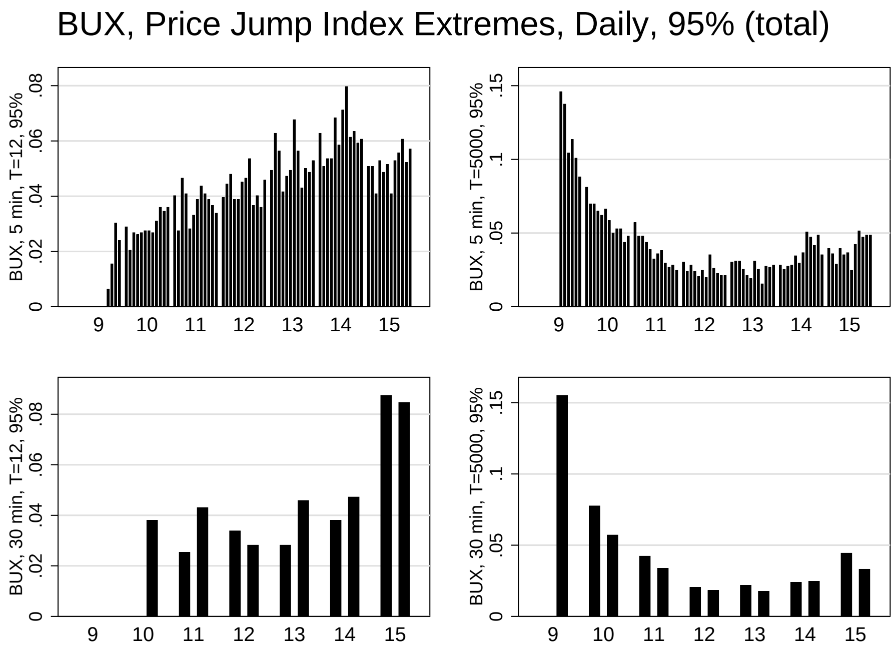


Figure 78: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated for the entire period together.

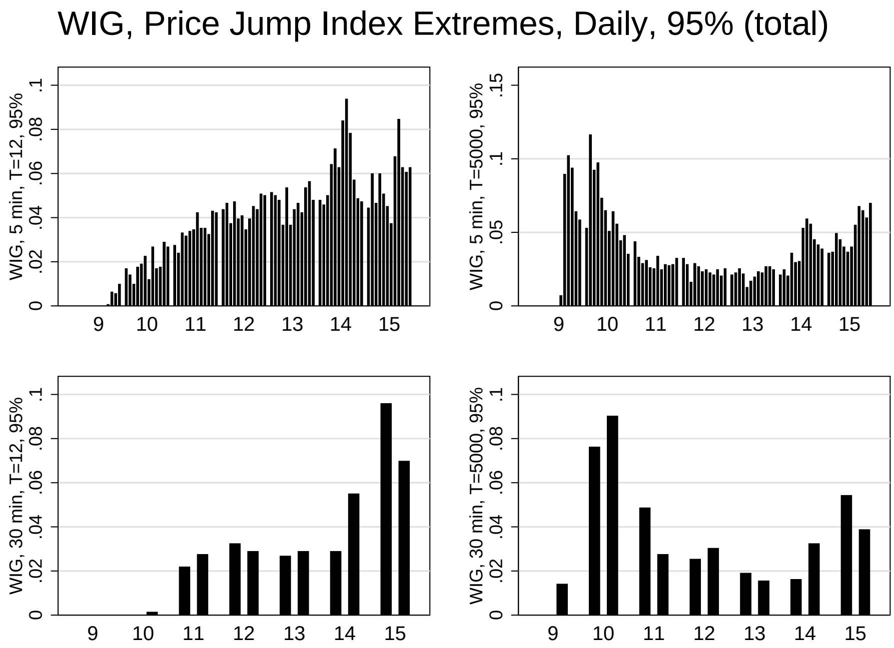


Figure 79: The distribution of the extreme values of the price jump index over a trading day for the DAX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated for the entire period together.

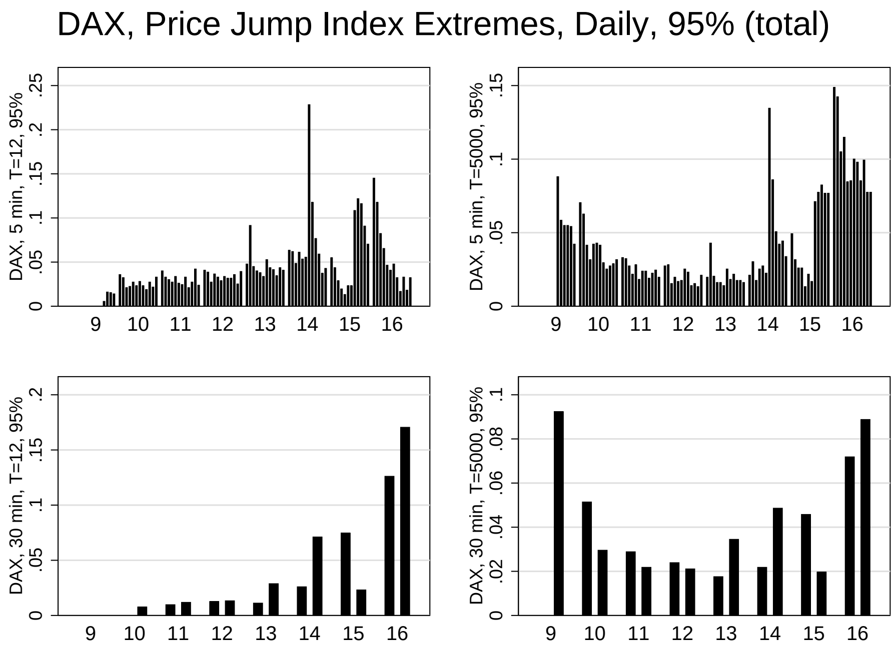


Figure 80: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated for the entire period together.

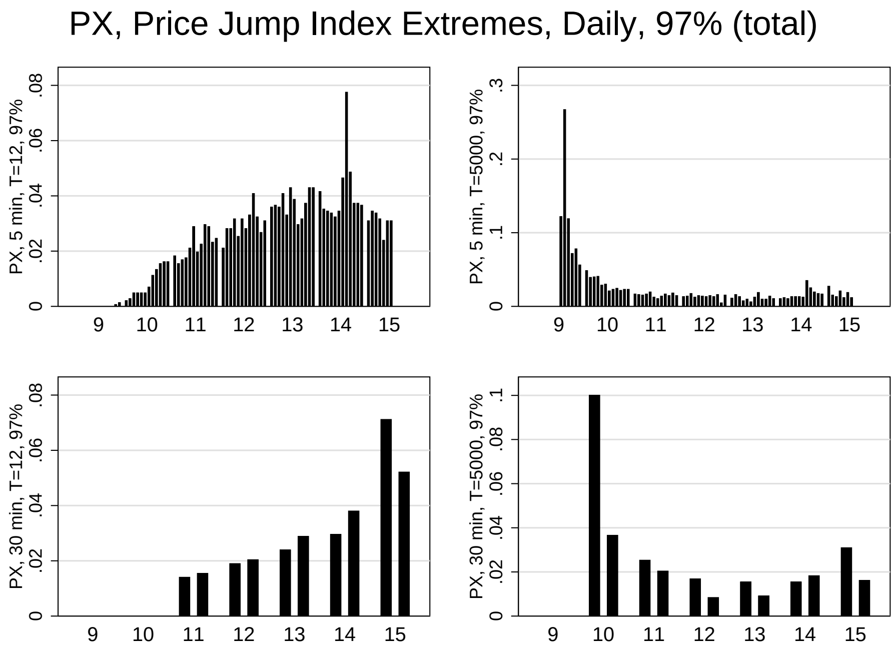


Figure 81: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated for the entire period together.

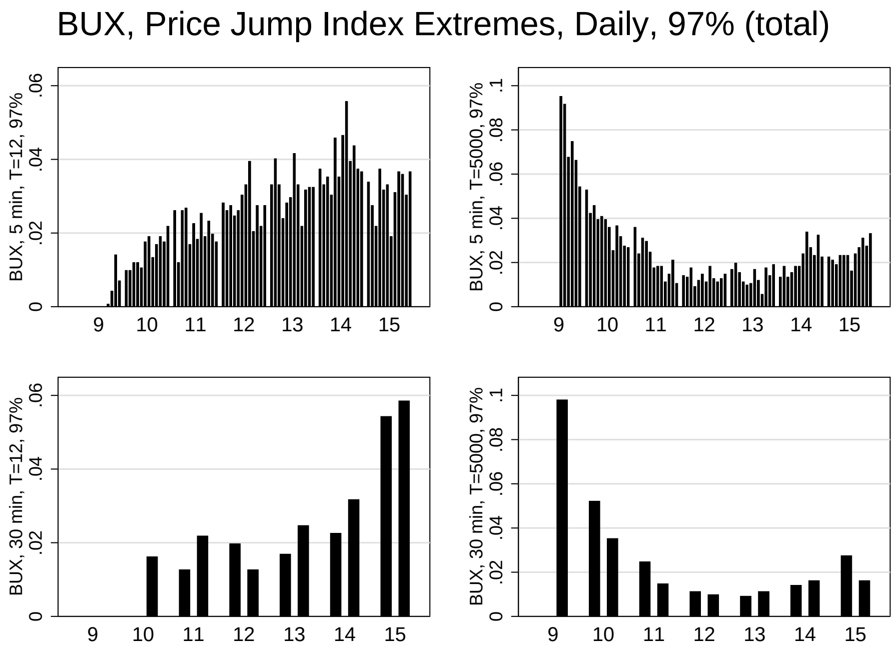


Figure 82: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated for the entire period together.

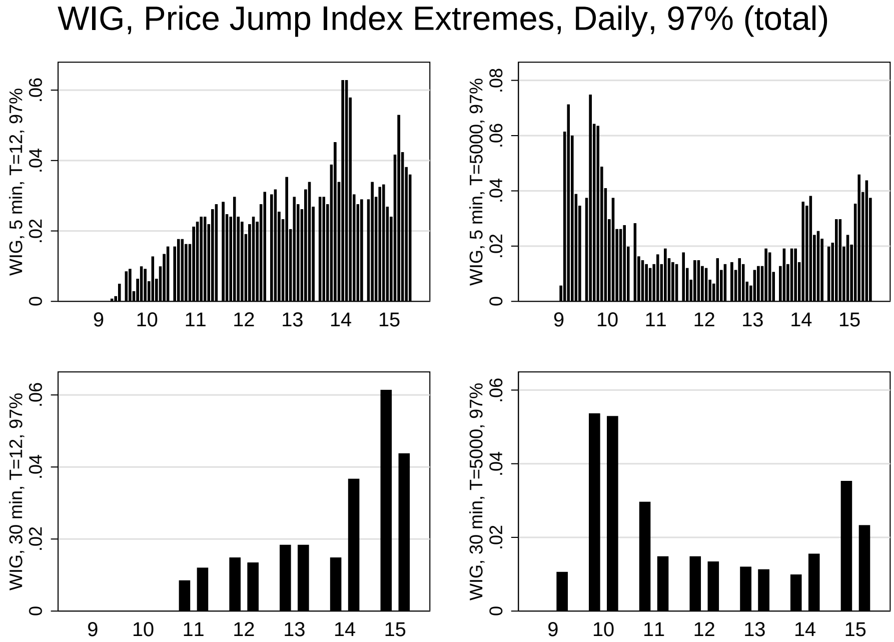


Figure 83: The distribution of the extreme values of the price jump index over a trading day for the DAX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 97% centile. Centiles are calculated for the entire period together.

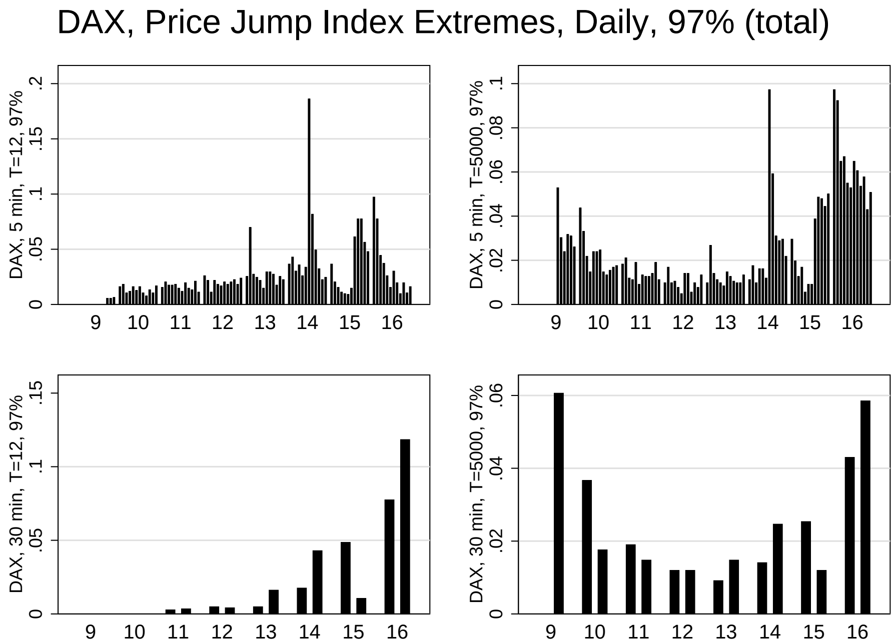


Figure 84: The distribution of the extreme values of the price jump index over a trading day for the PX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated for the entire period together.

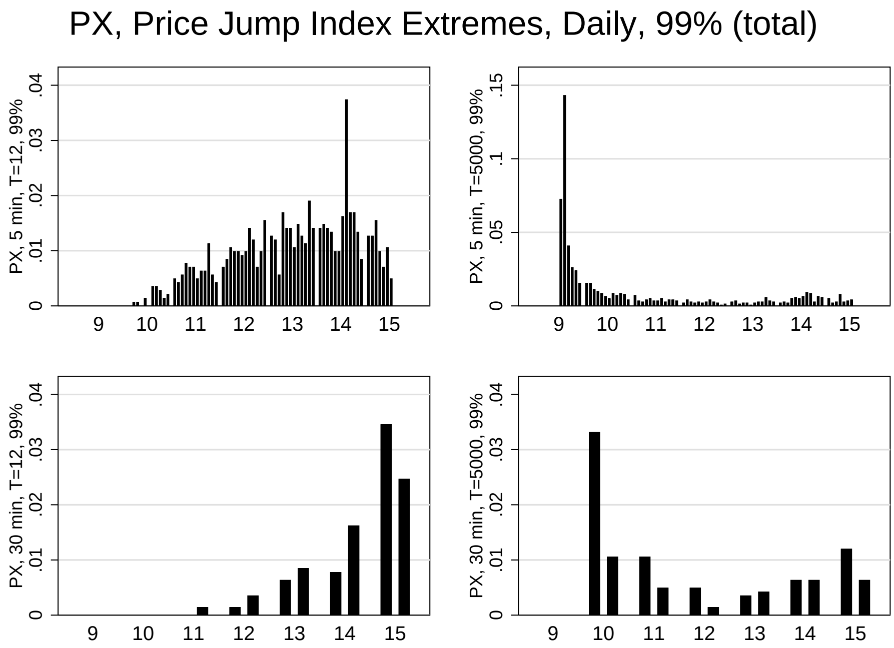


Figure 85: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated for the entire period together.

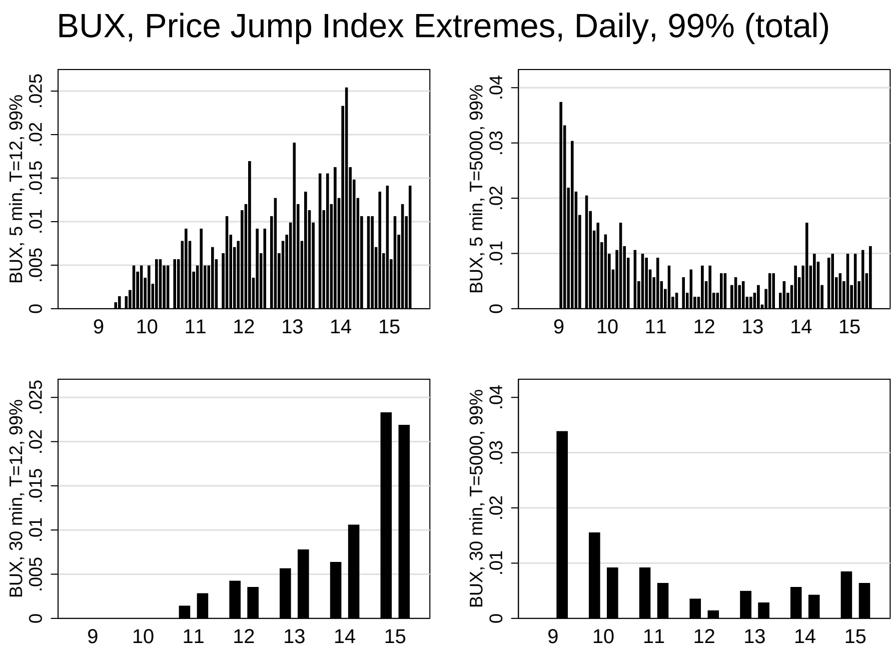


Figure 86: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated for the entire period together.

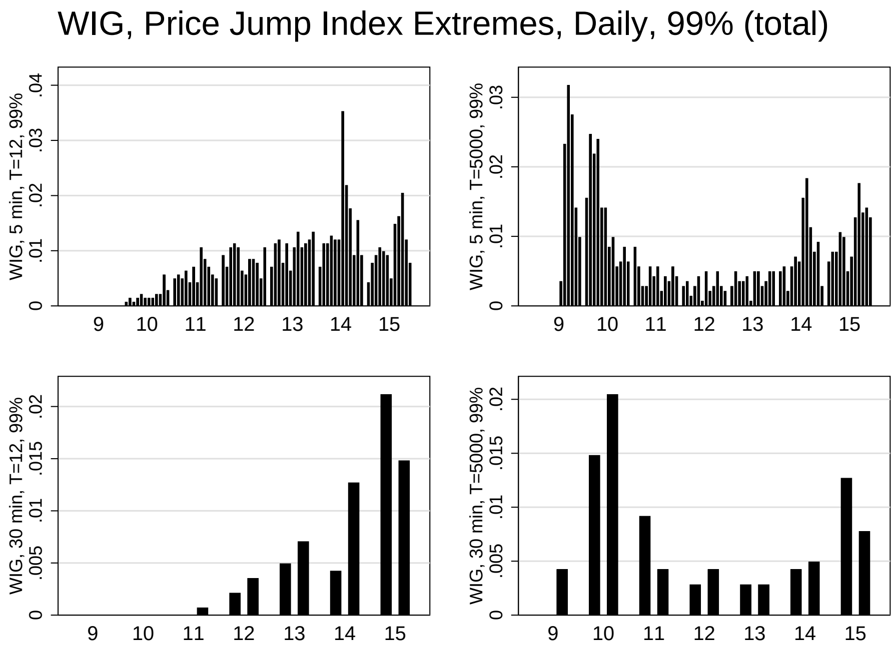


Figure 87: The distribution of the extreme values of the price jump index over a trading day for the DAX index using 5 and 30 minute frequency, and time window $T = 12$ and 5000. Extreme price jump index are those higher than 99% centile. Centiles are calculated for the entire period together.

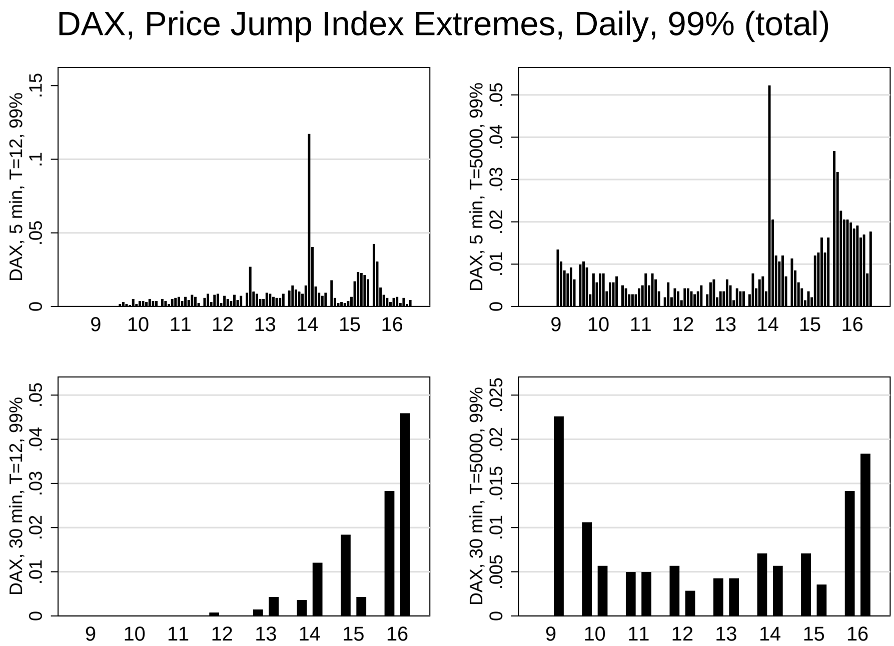


Figure 88: The distribution of the extreme values of the price jump index over a trading day for the BUX index using 5 minute frequency, and $T = 12$. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The period spans from 2004 to 2008, where 1 represents 01/2004 and 60 represents 12/2008.

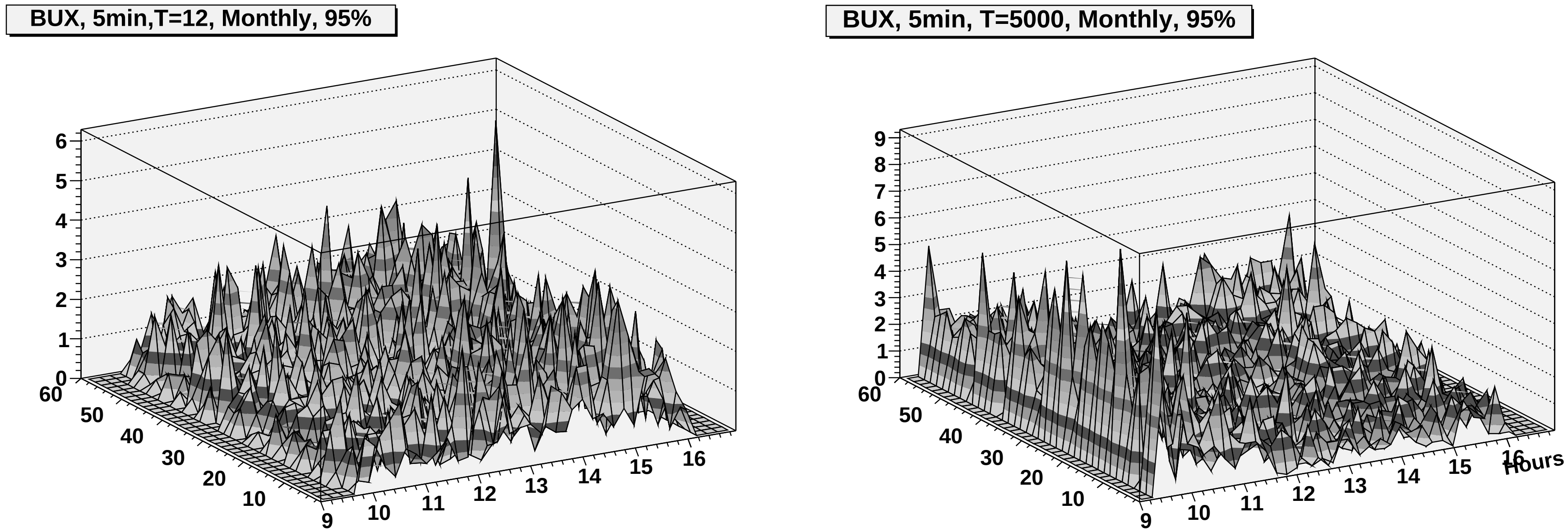


Figure 89: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 minute frequency, and $T = 12$. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The period spans from 2004 to 2008, where 1 represents 01/2004 and 60 represents 12/2008.

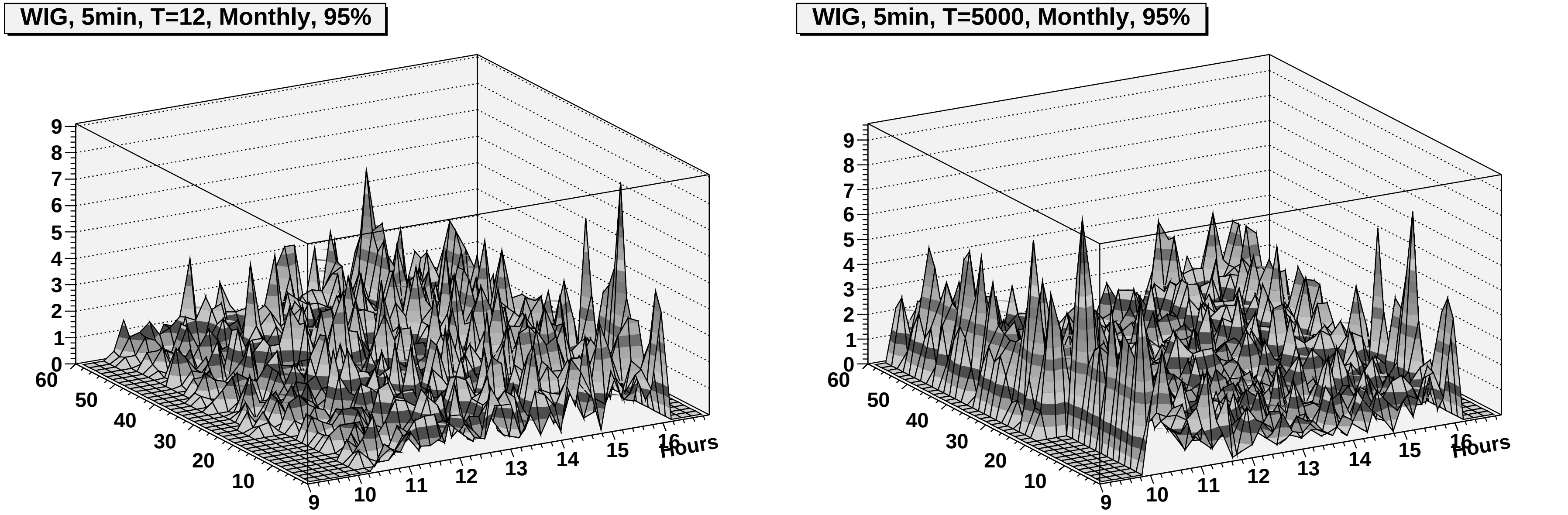


Figure 90: The distribution of the extreme values of the price jump index over a trading day for the WIG index using 5 minute frequency, and $T = 12$. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The period spans from 2004 to 2008, where 1 represents 01/2004 and 60 represents 12/2008.

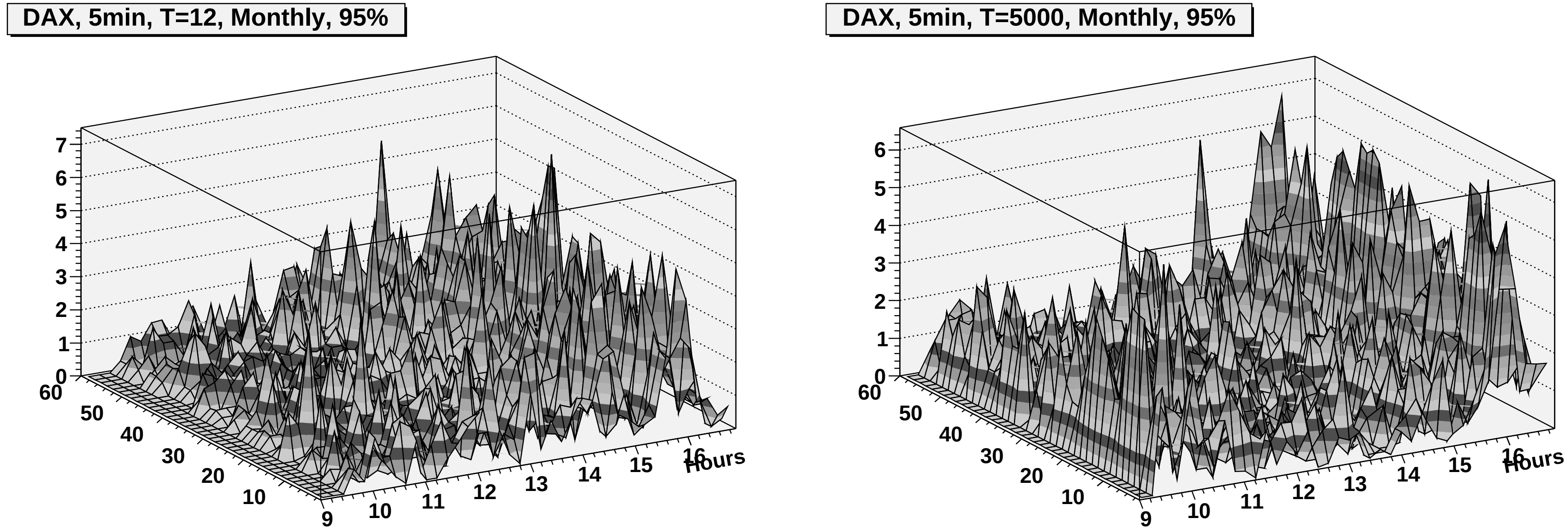


Figure 91: The distribution of the number of extreme price jump index per day for the BUX index using 5 and 30 minute frequency, and $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The time period spans from 2004 to 2008.

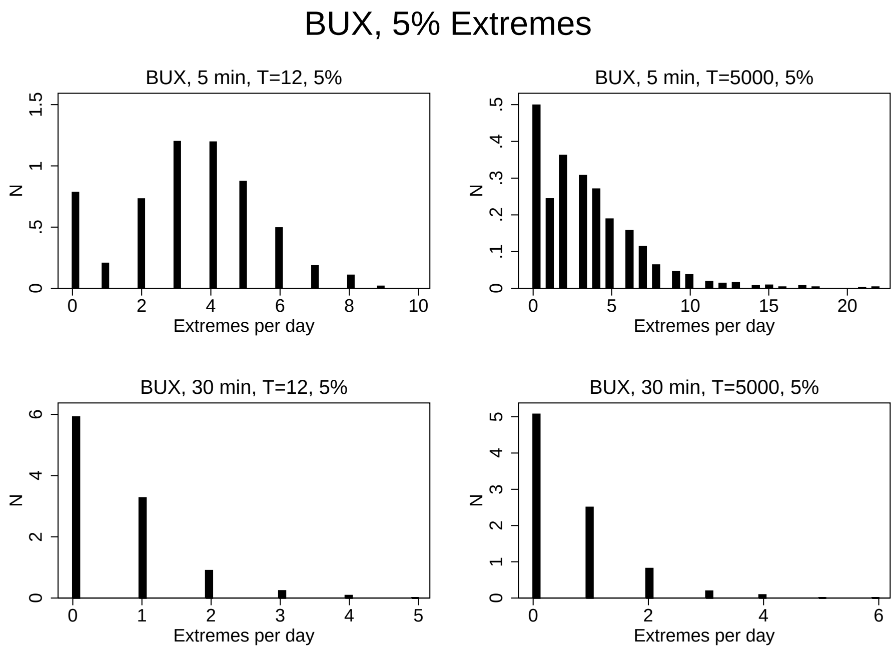


Figure 92: The distribution of the number of extreme price jump index per day for the WIG index using 5 and 30 minute frequency, and $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The time period spans from 2004 to 2008.

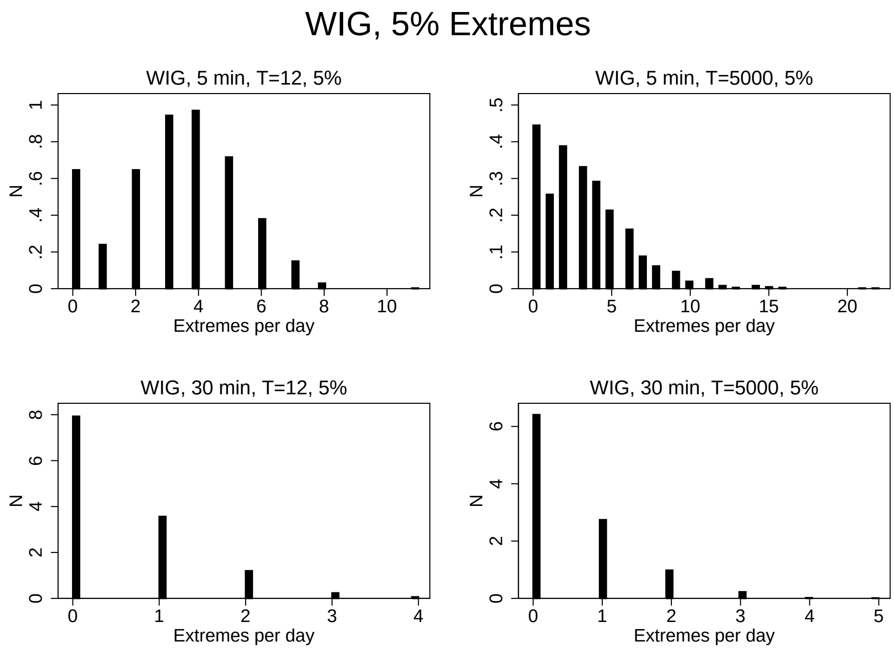
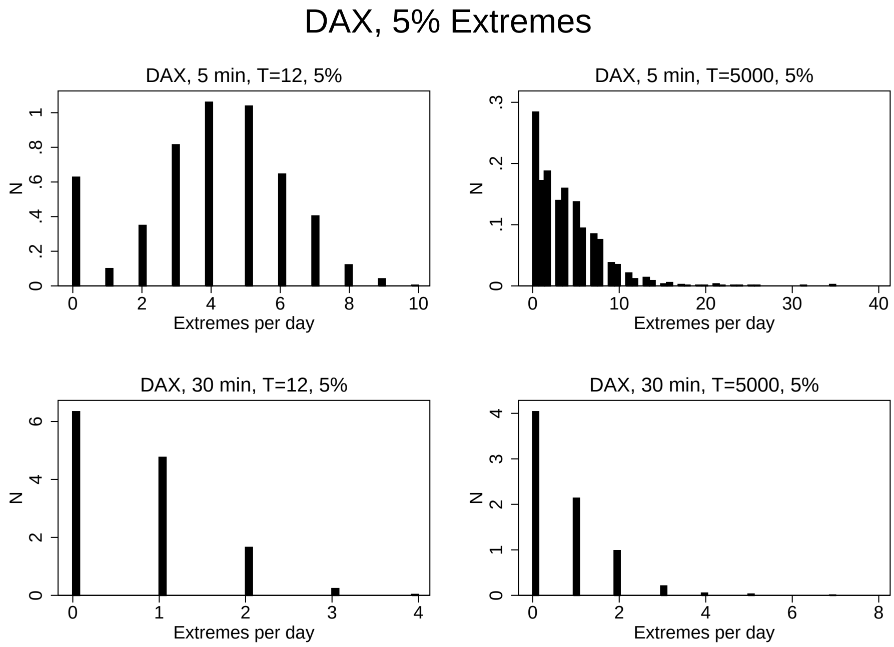


Figure 93: The distribution of the number of extreme price jump index per day for the DAX index using 5 and 30 minute frequency, and $T = 12$ and 5000. Extreme price jump index are those higher than 95% centile. Centiles are calculated on monthly basis. The time period spans from 2004 to 2008.



Normalized returns

The Figures 94, 95 and 96 show the log-log distribution for normalized returns of remaining three indexes using all four frequencies 5, 10, 15 and 30 minute.

Figure 94: Log-log distribution of the tail part of normalized returns for the BUX index using 5, 10, 15, and 30 minute frequencies. On the left hand side, the negative side of the distribution is depicted, while on the right hand side, positive normalized returns are shown. I have used following symbols: $T = 12$: $\bullet$, $T = 24$: $\diamondsuit$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$ and $T = 5000$: $\times$.

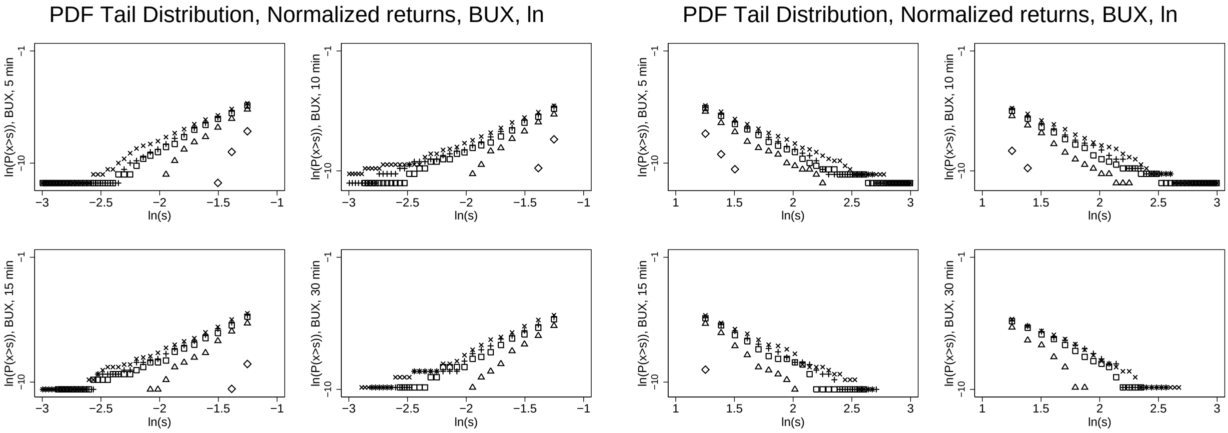


Figure 95: Log-log distribution of the tail part of normalized returns for the WIG index using 5, 10, 15, and 30 minute frequencies. On the left hand side, the negative side of the distribution is depicted, while on the right hand side, positive normalized returns are shown. I have used following symbols: $T = 12$: $\bullet$, $T = 24$: $\diamondsuit$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$ and $T = 5000$: $\times$.

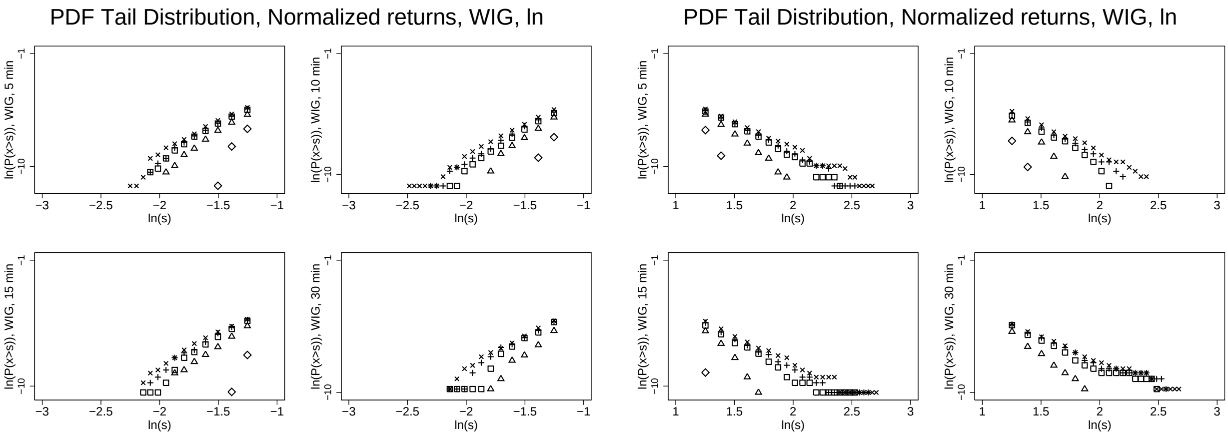
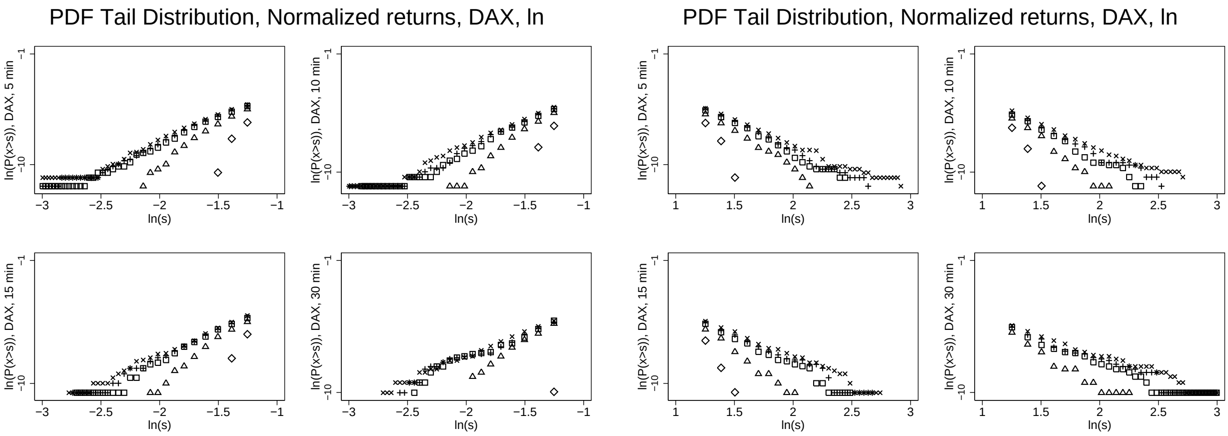


Figure 96: Log-log distribution of the tail part of normalized returns for the DAX index using 5, 10, 15, and 30 minute frequencies. On the left hand side, the negative side of the distribution is depicted, while on the right hand side, positive normalized returns are shown. I have used following symbols: $T = 12$: $\bullet$, $T = 24$: $\diamondsuit$, $T = 100$: $\triangle$, $T = 1000$: $\square$, $T = 2000$: $+$ and $T = 5000$: $\times$.



Price jump index on levels

The Figures 97, 98 and 99 show the central part of the price jump index for levels for remaining three indexes, using all four frequencies 5, 10, 15, and 30 minute, and all four time windows $T = 12$, 24, 100, and 2000.

The Figures 100 – 103 contain the central part of the price jump index for every frequency and all four indexes and all four time windows $T = 12$, 24, 100, and 2000.

Figure 97: The price jump index defined on levels for BUX using all four frequencies: 5, 10, 15, and 30 minute. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).

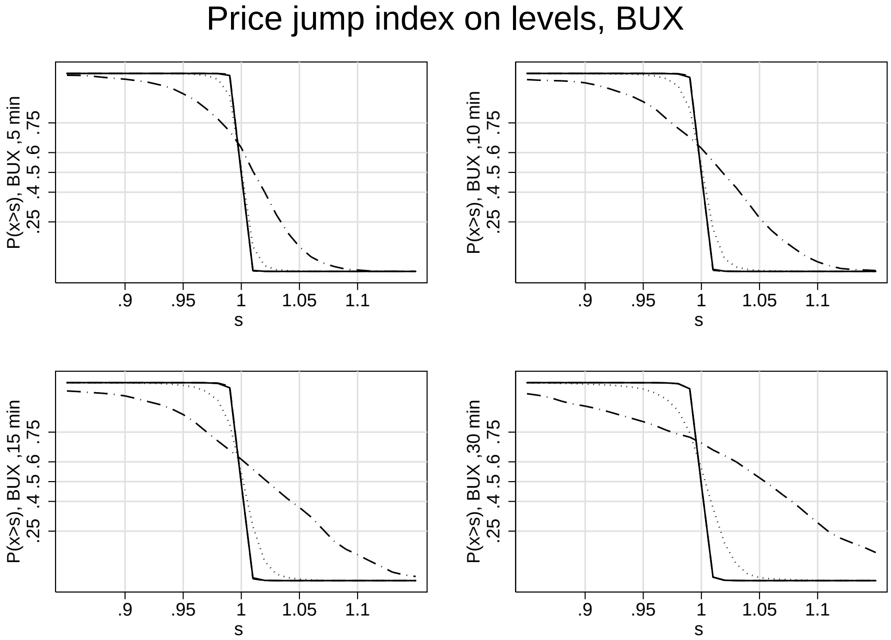


Figure 98: The price jump index defined on levels for WIG using all four frequencies: 5, 10, 15, and 30 minute. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).

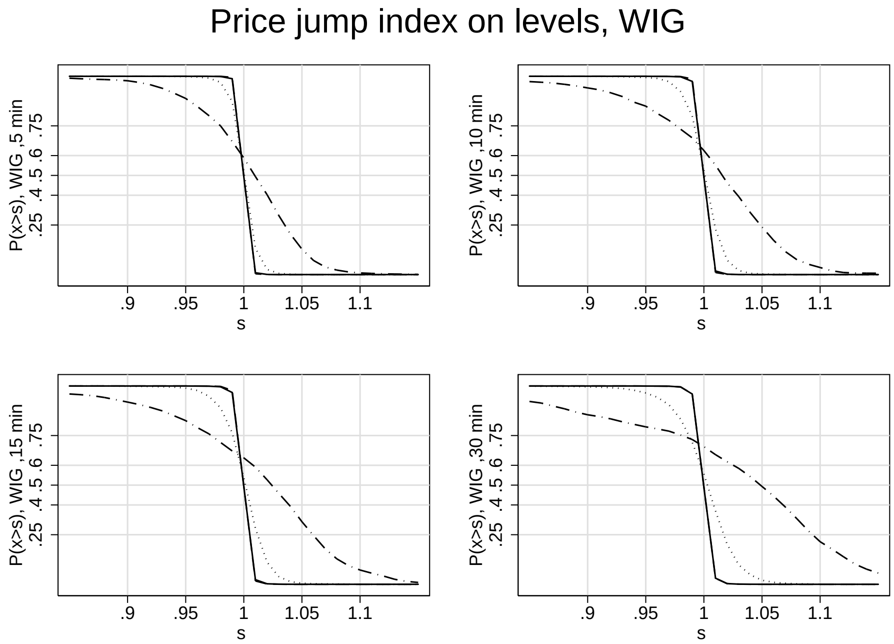


Figure 99: The price jump index defined on levels for DAX using all four frequencies: 5, 10, 15, and 30 minute. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).

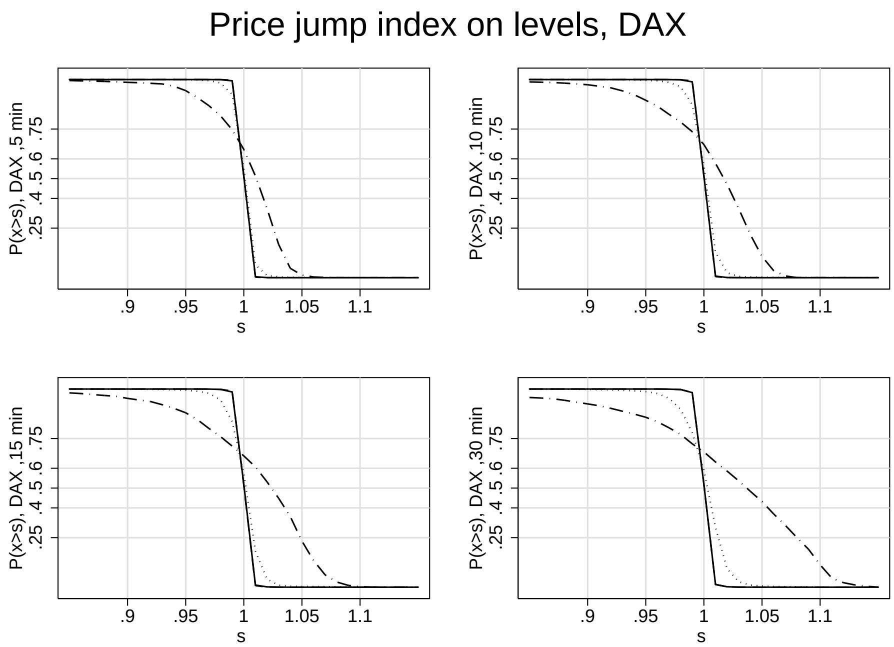


Figure 100: The price jump index defined on levels for all four indexes using 5 minute frequency. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).

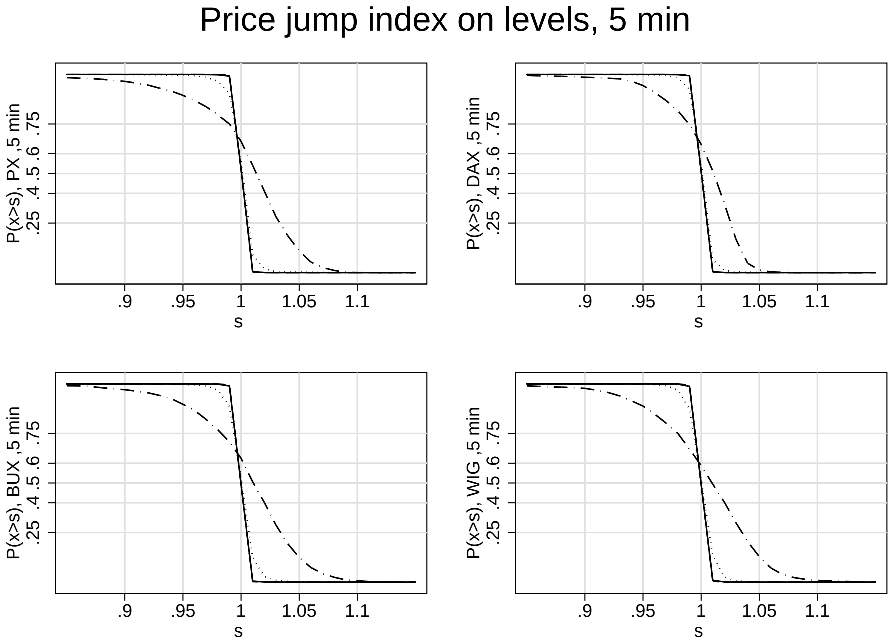


Figure 101: The price jump index defined on levels for all four indexes using 10 minute frequency. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).

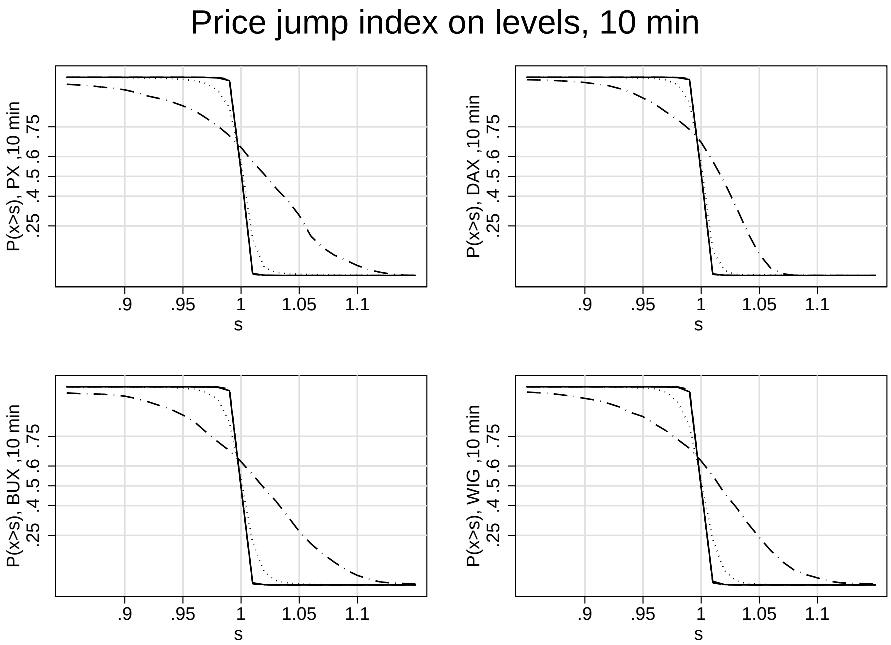


Figure 102: The price jump index defined on levels for all four indexes using 15 minute frequency. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).

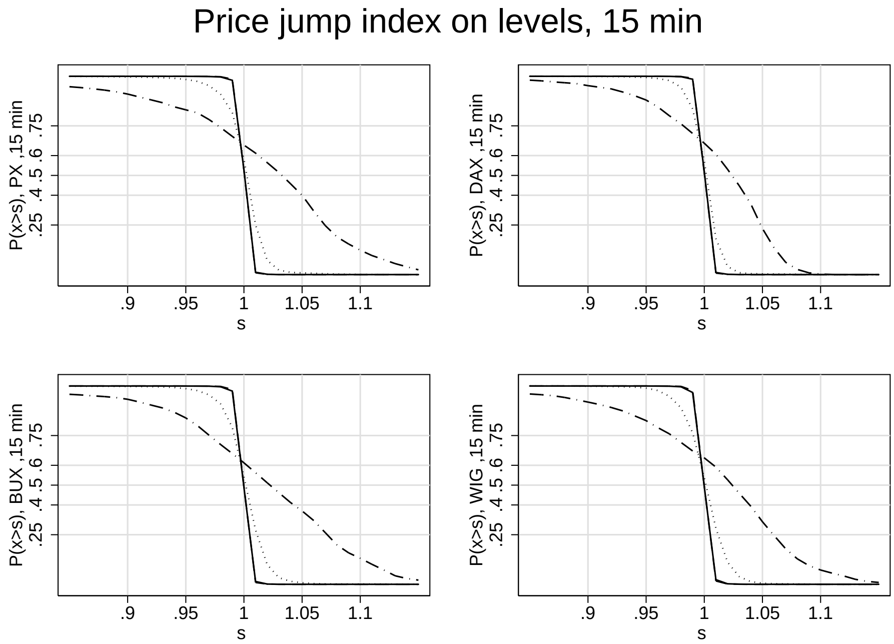
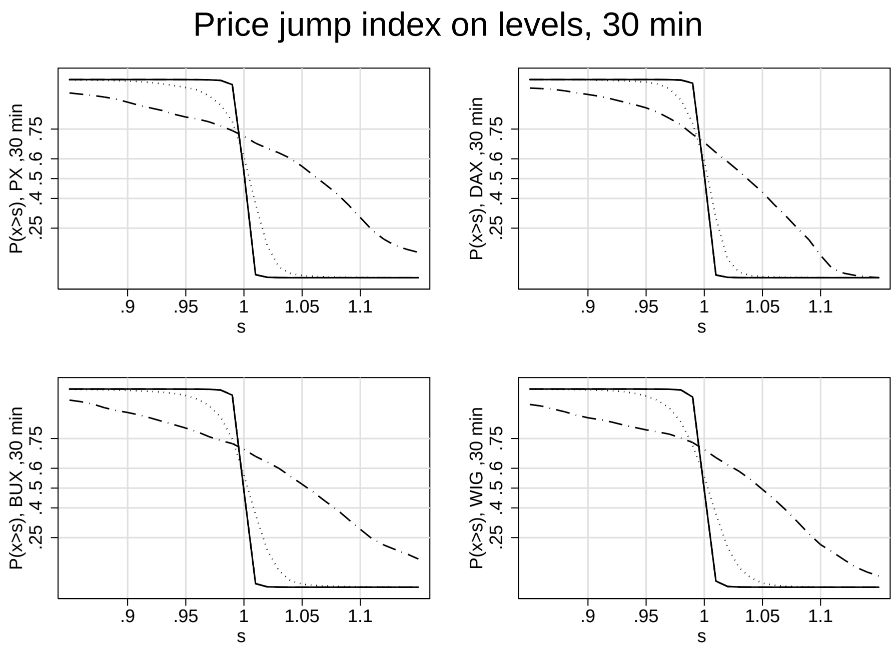


Figure 103: The price jump index defined on levels for all four indexes using 30 minute frequency. The lengths of the time window used for this plot are: $T = 12$ (dashed line), $T = 24$ (solid line), $T = 100$ (dotted line) and $T = 2000$ (dash-dotted line).



Extreme returns

The Figures 104 – 107 plot the critical values for every index using all four frequencies.

The Figures 108, 109 and 110 plot the critical values for all four indexes, using 10, 15 and 30 minute frequency.

The Figures 111 – 114 contain the distributions of the extreme returns over a trading day for all four indexes PX, BUX, WIG and DAX, where for every index, distributions for every frequency and all three threshold values are plotted.

The Figures 115 – 118 contains the distributions of extreme returns over a trading day for all four indexes PX, BUX, WIG and DAX, using first threshold values defined as 1% and 99% centiles and 5, 15 and 30 minute frequencies, and second, threshold values defined as 5% and 95% centiles and 5, 10, 15 and 30 minute frequencies.

The Figures 119, 120 and 121 show the distribution of extreme returns for each month in the period from 2004 to 2008 separately, using three remaining indexes and 10 minute frequency with thresholds equal to 1% and 99% centiles.

Figure 104: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures uses the PX index and all four frequencies. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

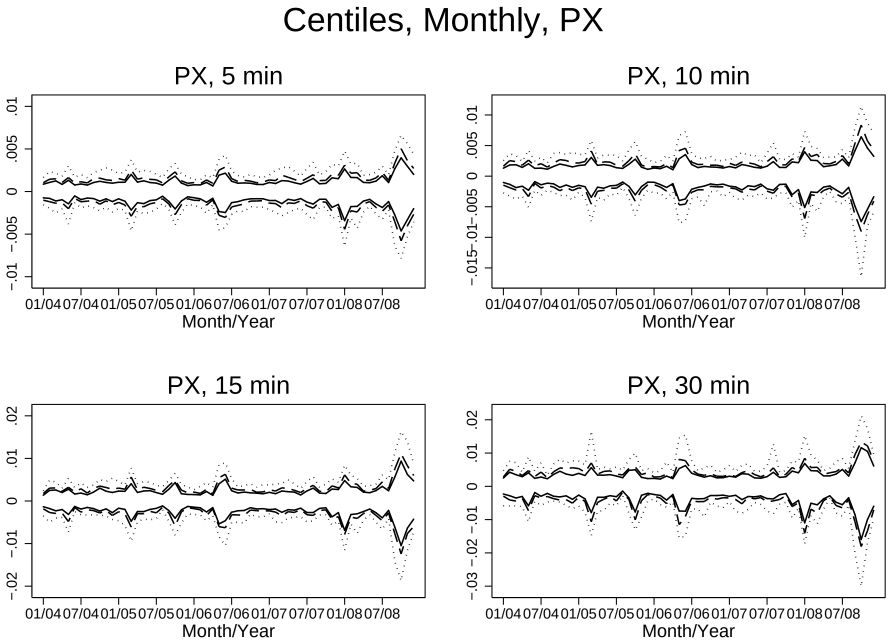


Figure 105: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures uses the BUX index and all four frequencies. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

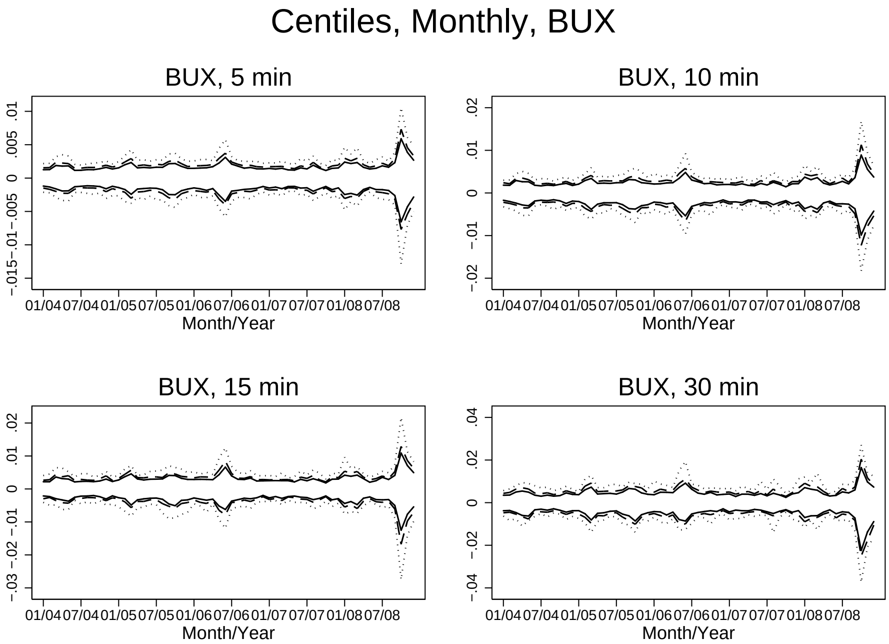


Figure 106: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures uses the WIG index and all four frequencies. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

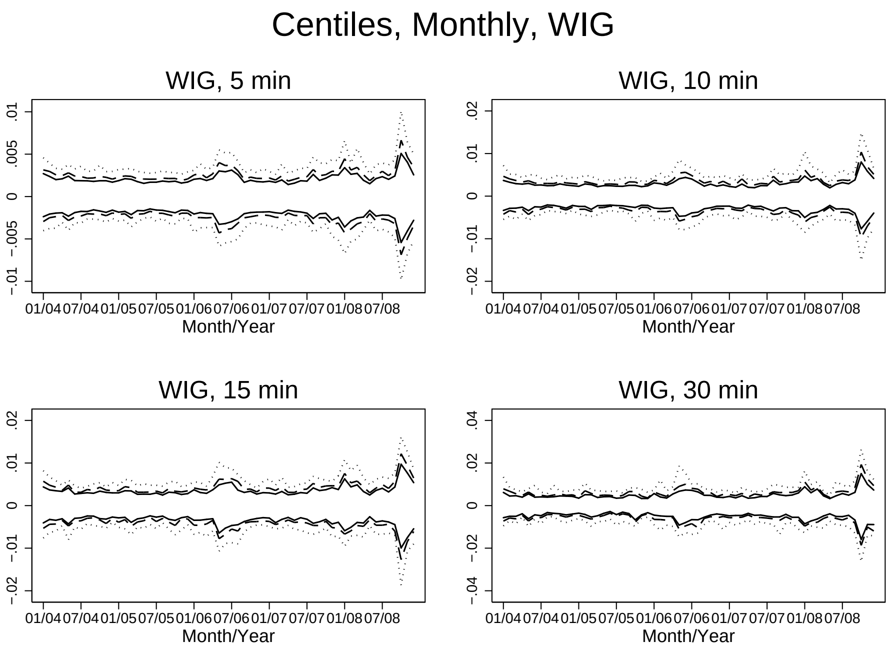


Figure 107: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures uses the DAX index and all four frequencies. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

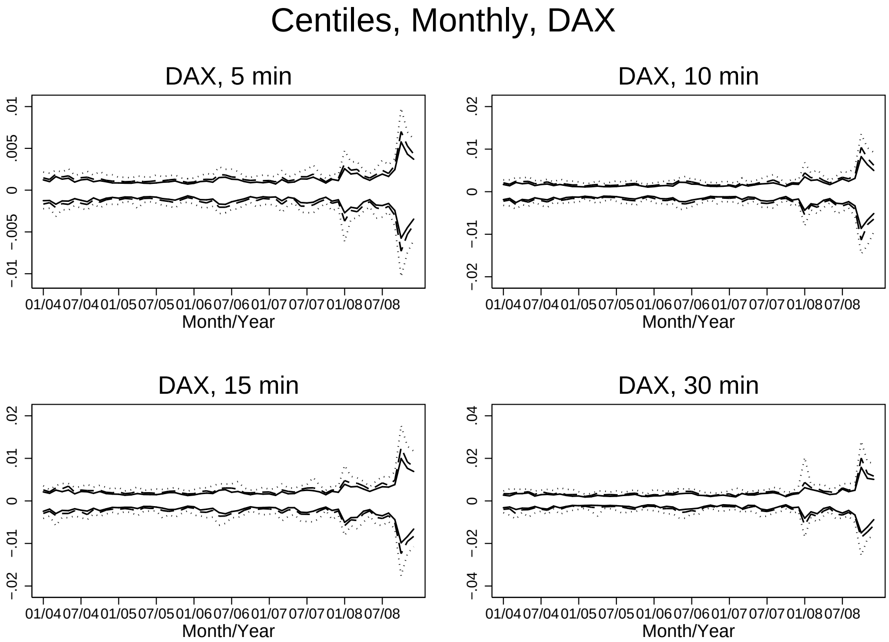


Figure 108: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures all four indexes: PX, BUX and WIG, and using 10 minute frequency. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

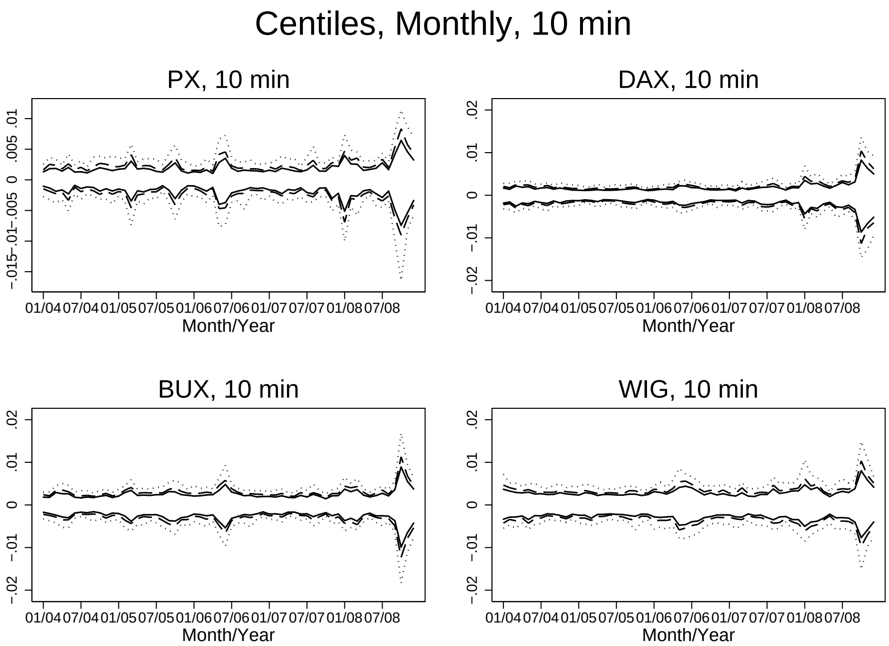


Figure 109: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures all four indexes: PX, BUX and WIG, and using 15 minute frequency. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

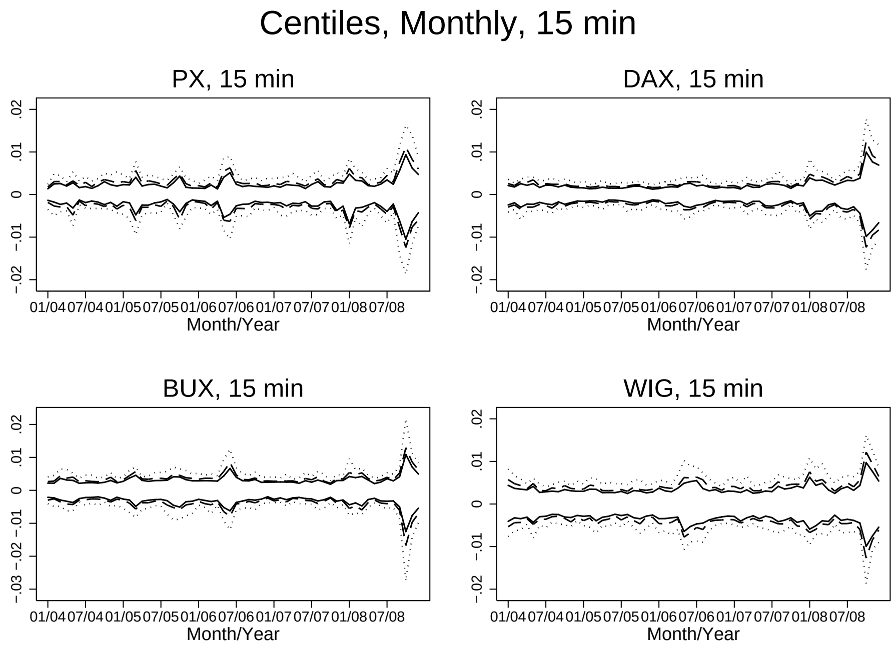


Figure 110: The Figure shows the threshold values used for calculating extreme returns. The values are defined as centiles calculated month by month. This Figure captures all four indexes: PX, BUX and WIG, and using 30 minute frequency. The value for centiles are the following pairs: 1% and 99% (dotted line), 3% and 97% (dashed line) and 5% and 95% (full line).

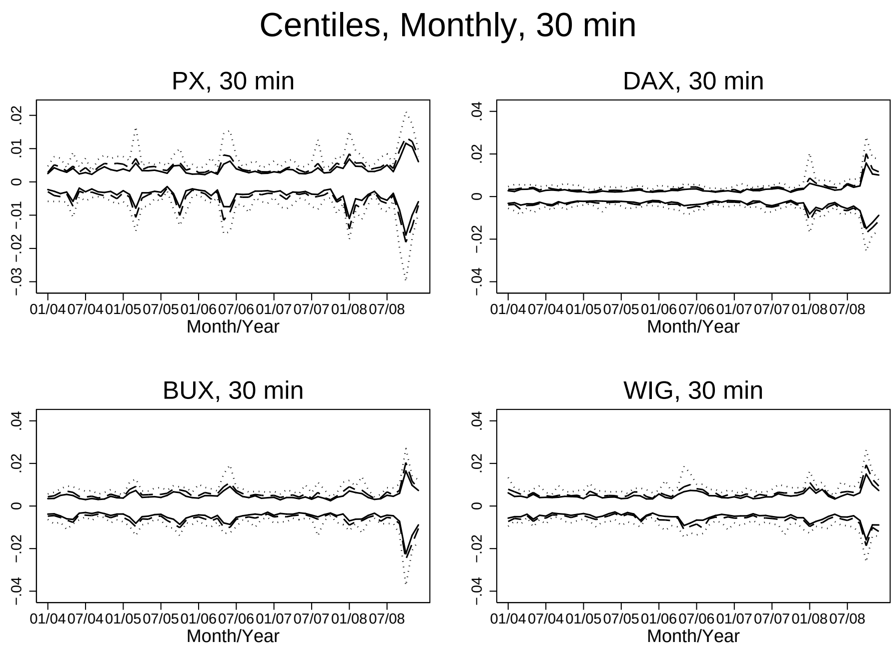


Figure 111: The distribution of extreme returns during a trading day for PX index, using all four frequencies and all three threshold values.

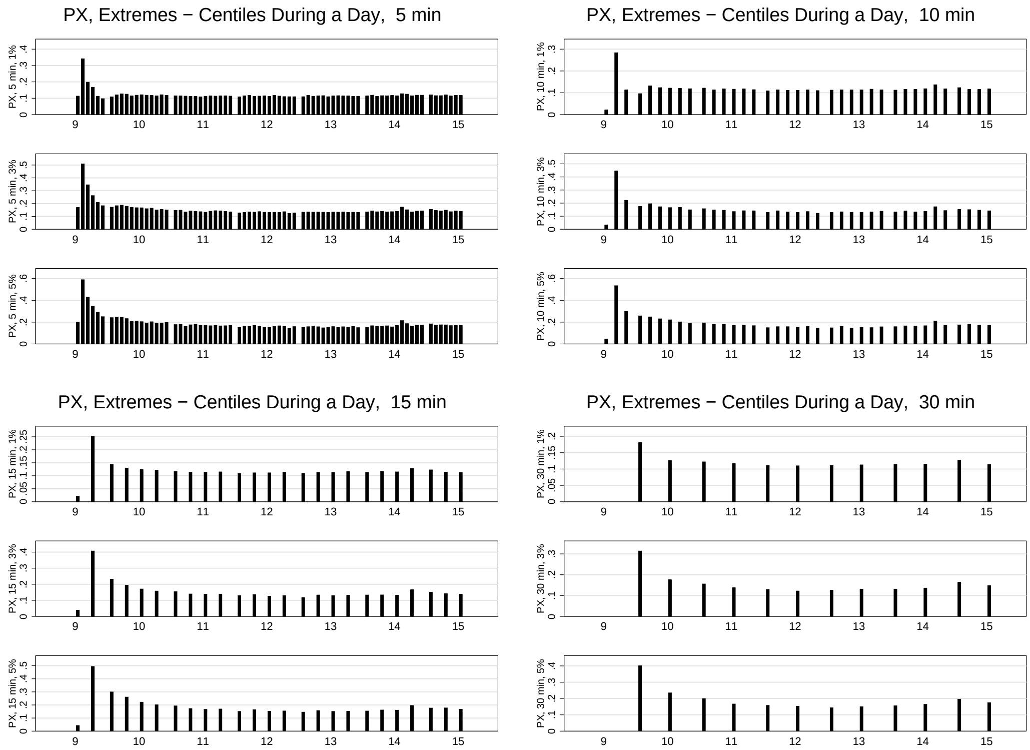


Figure 112: The distribution of extreme returns during a trading day for BUX index, using all four frequencies and all three threshold values.

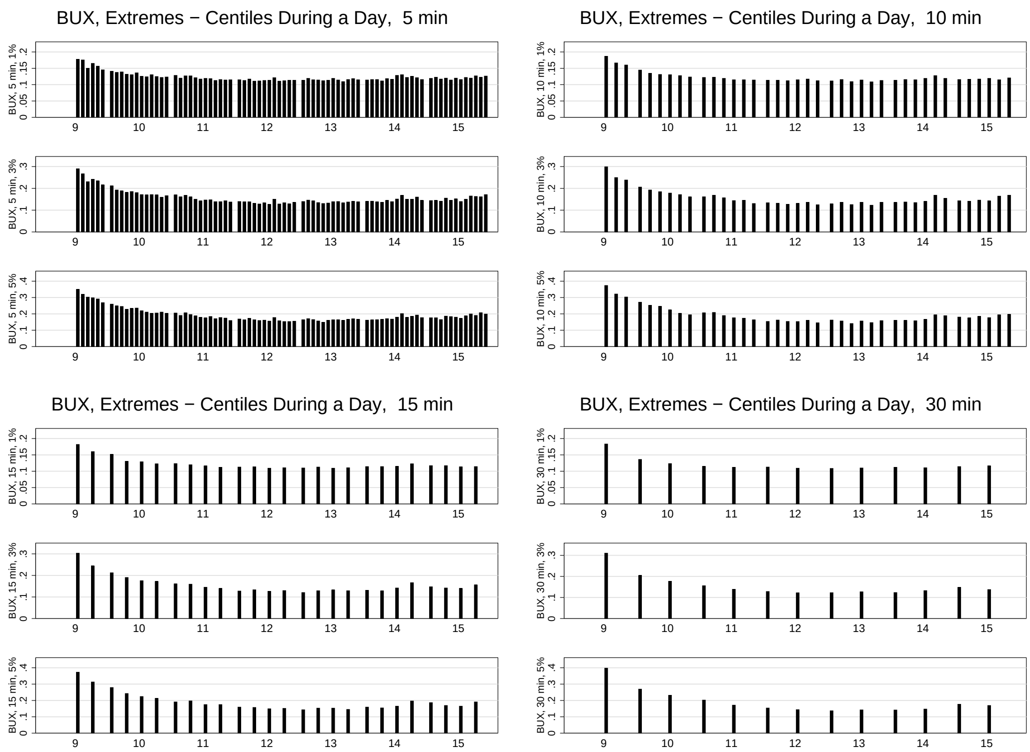


Figure 113: The distribution of extreme returns during a trading day for WIG index, using all four frequencies and all three threshold values.

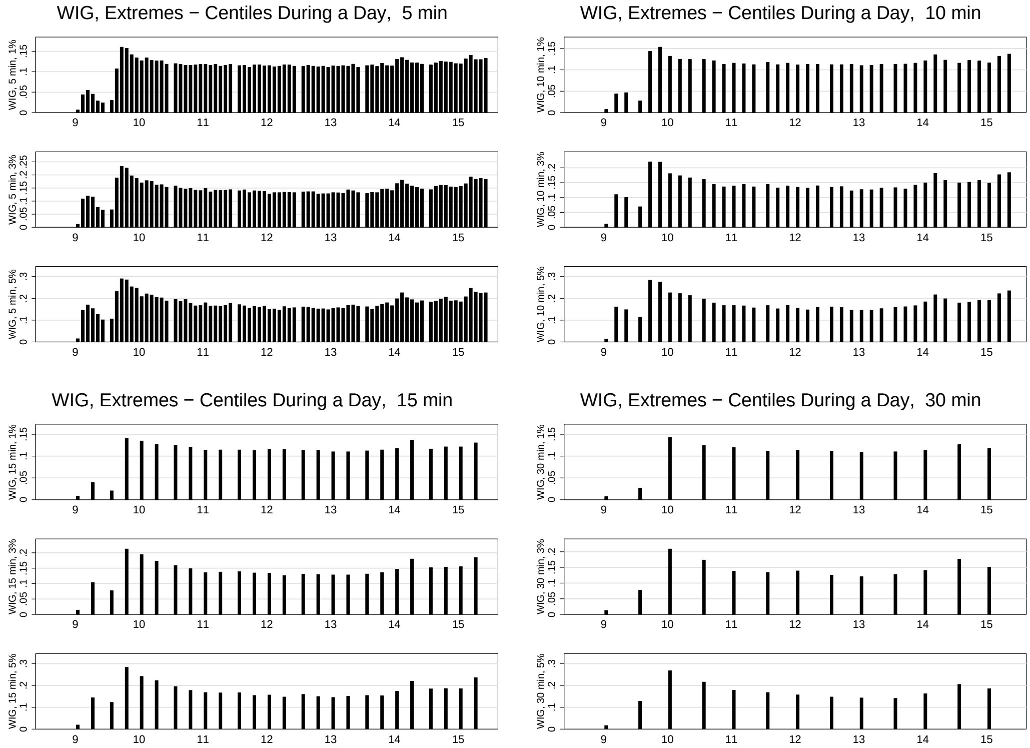


Figure 114: The distribution of extreme returns during a trading day for DAX index, using all four frequencies and all three threshold values.

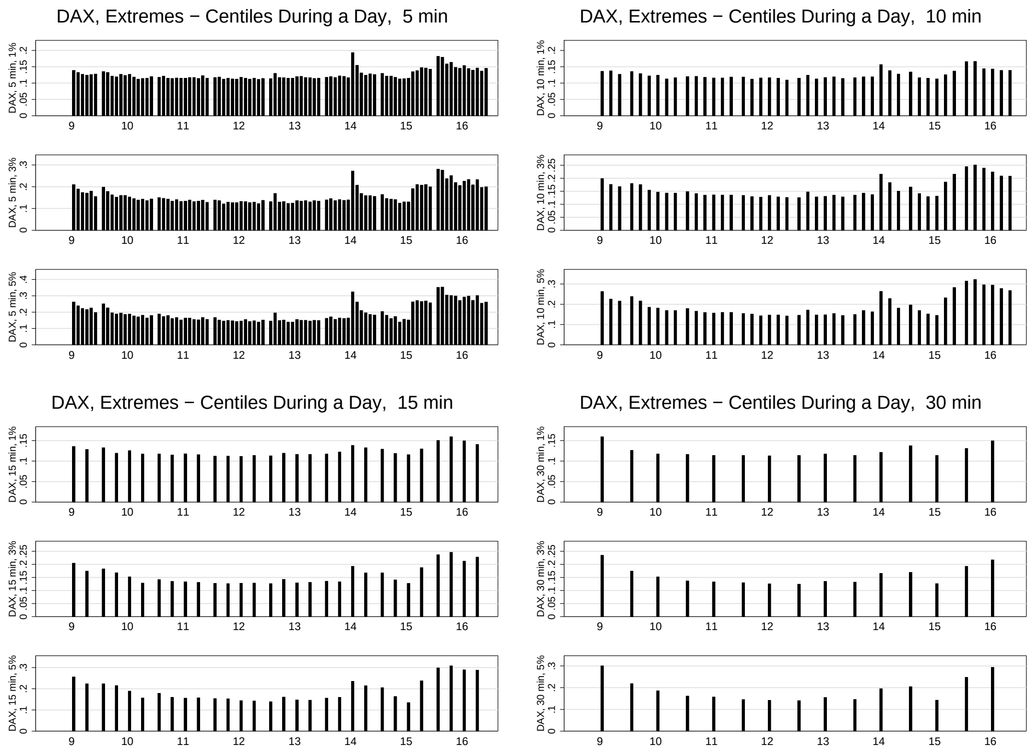


Figure 115: The distribution of extreme returns during a trading day for all four indexes PX, BUX, WIG and DAX, using 5 and 15 minute frequency and threshold values defined as 1% and 99% centiles.

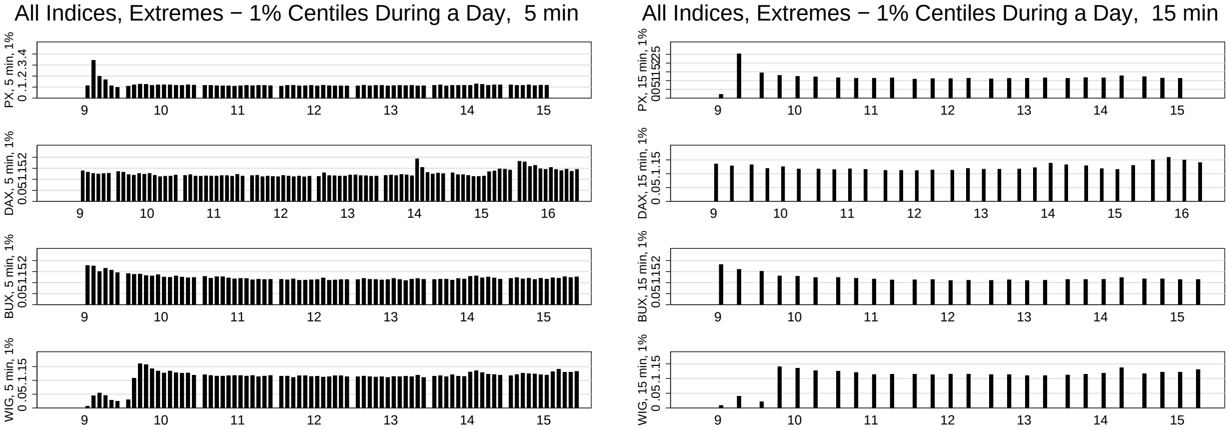


Figure 116: The distribution of extreme returns during a trading day for all four indexes PX, BUX, WIG and DAX, using 30 minute frequency and threshold values defined as 1% and 99% centiles.

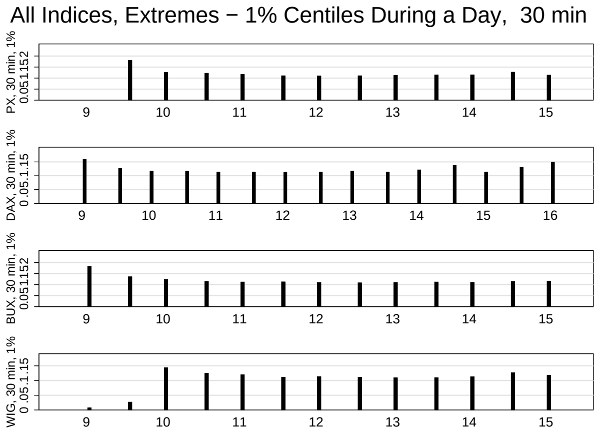


Figure 117: The distribution of extreme returns during a trading day for all four indexes PX, BUX, WIG and DAX, using 5 and 10 minute frequency and threshold values defined as 5% and 95% centiles.

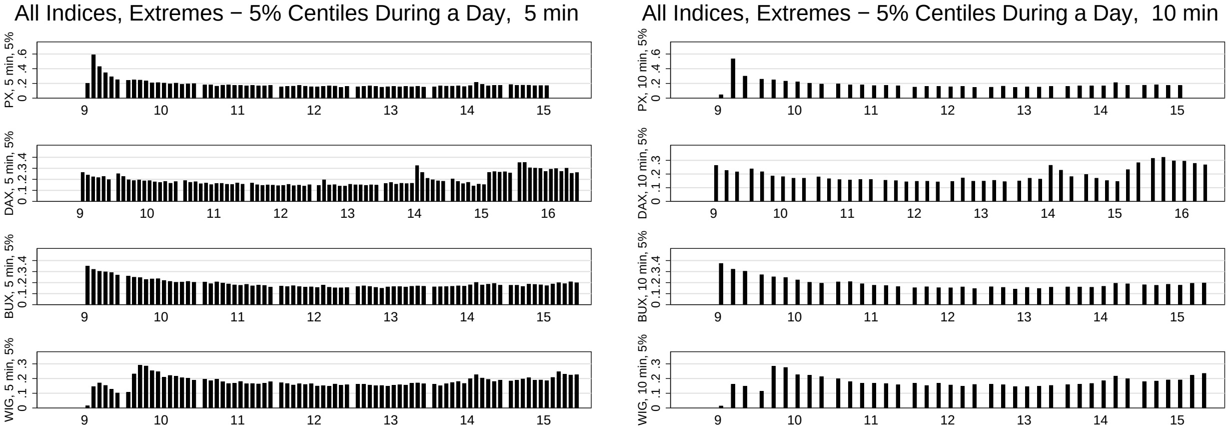


Figure 118: The distribution of extreme returns during a trading day for all four indexes PX, BUX, WIG and DAX, using 15 and 30 minute frequency and threshold values defined as 5% and 95% centiles.

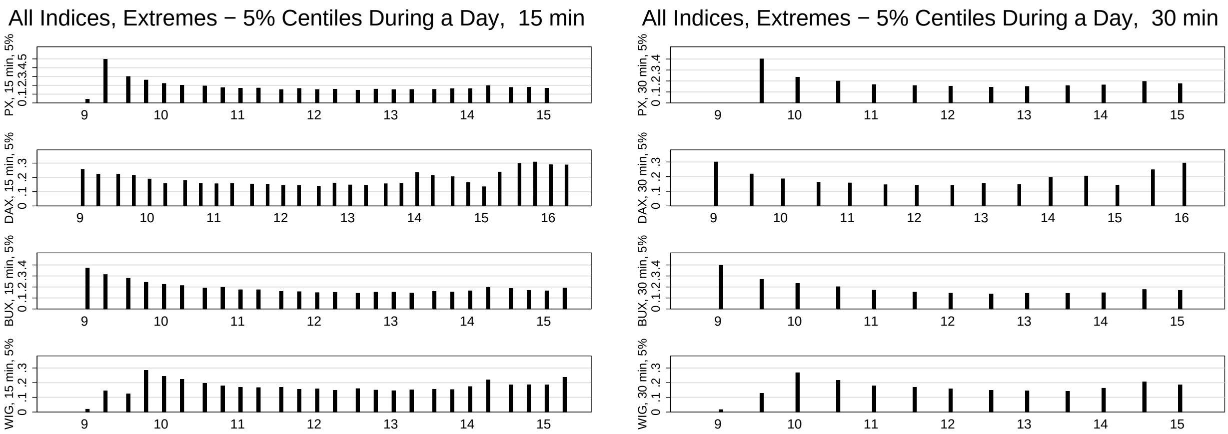


Figure 119: The distribution of extreme returns during a trading day for the BUX index using 10 minute frequency and thresholds defined as 1% and 99% centiles calculated month by month. The distribution is plotted for every month from 01/2004 to 12/2008, where 1 stands for 01/2004 and 60 for 12/2008.

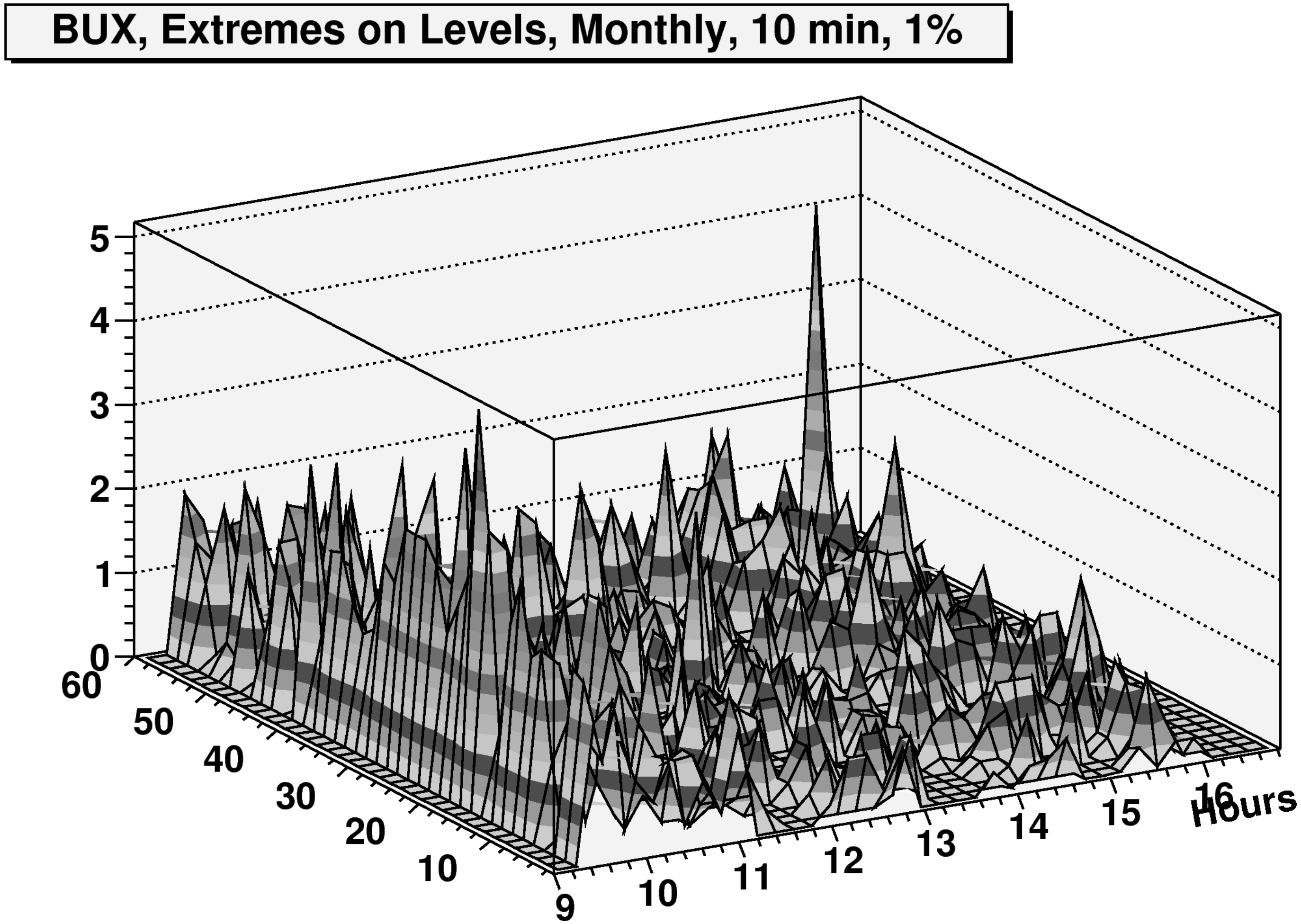


Figure 120: The distribution of extreme returns during a trading day for the WIG index using 10 minute frequency and thresholds defined as 1% and 99% centiles calculated month by month. The distribution is plotted for every month from 01/2004 to 12/2008, where 1 stands for 01/2004 and 60 for 12/2008.

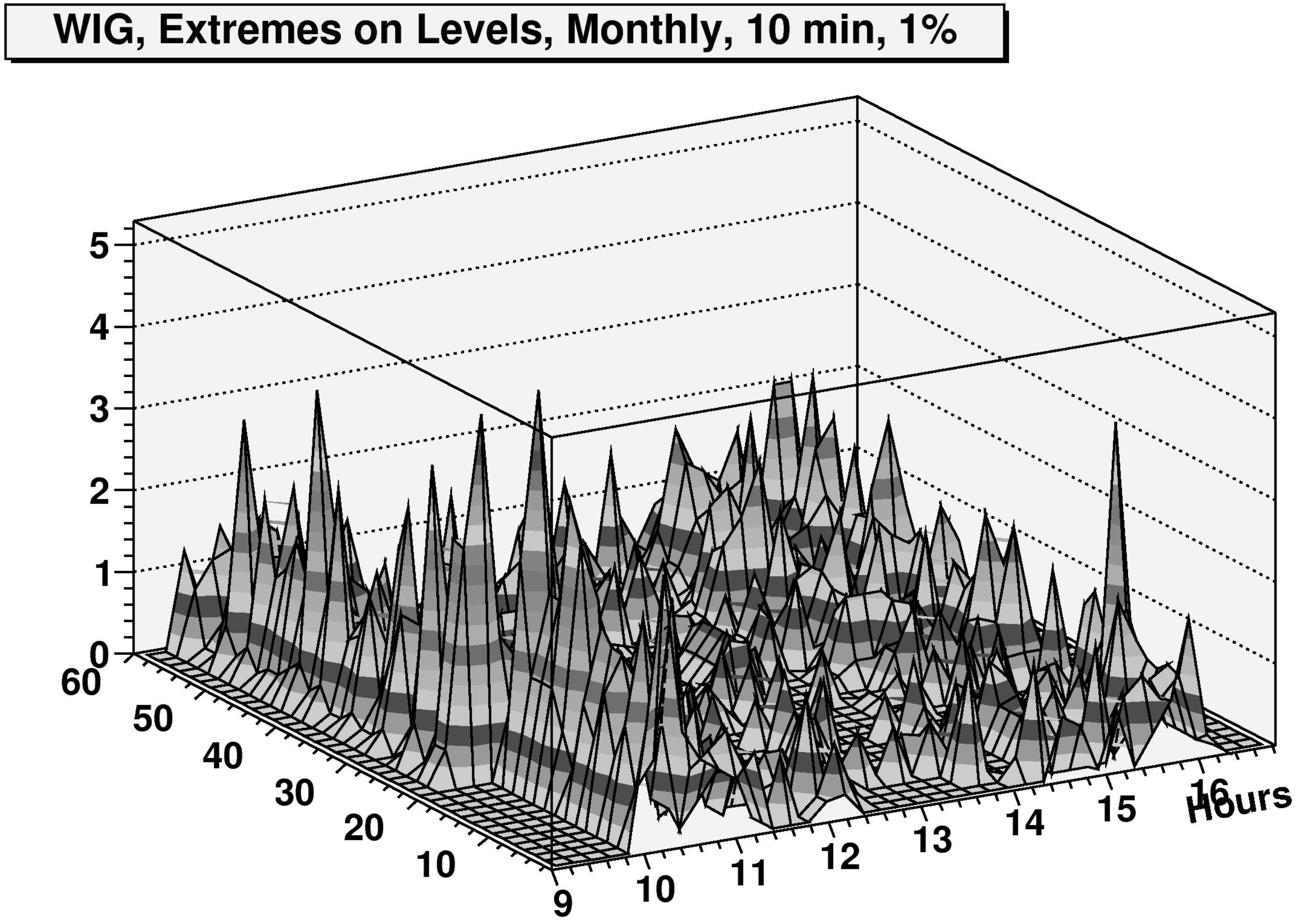
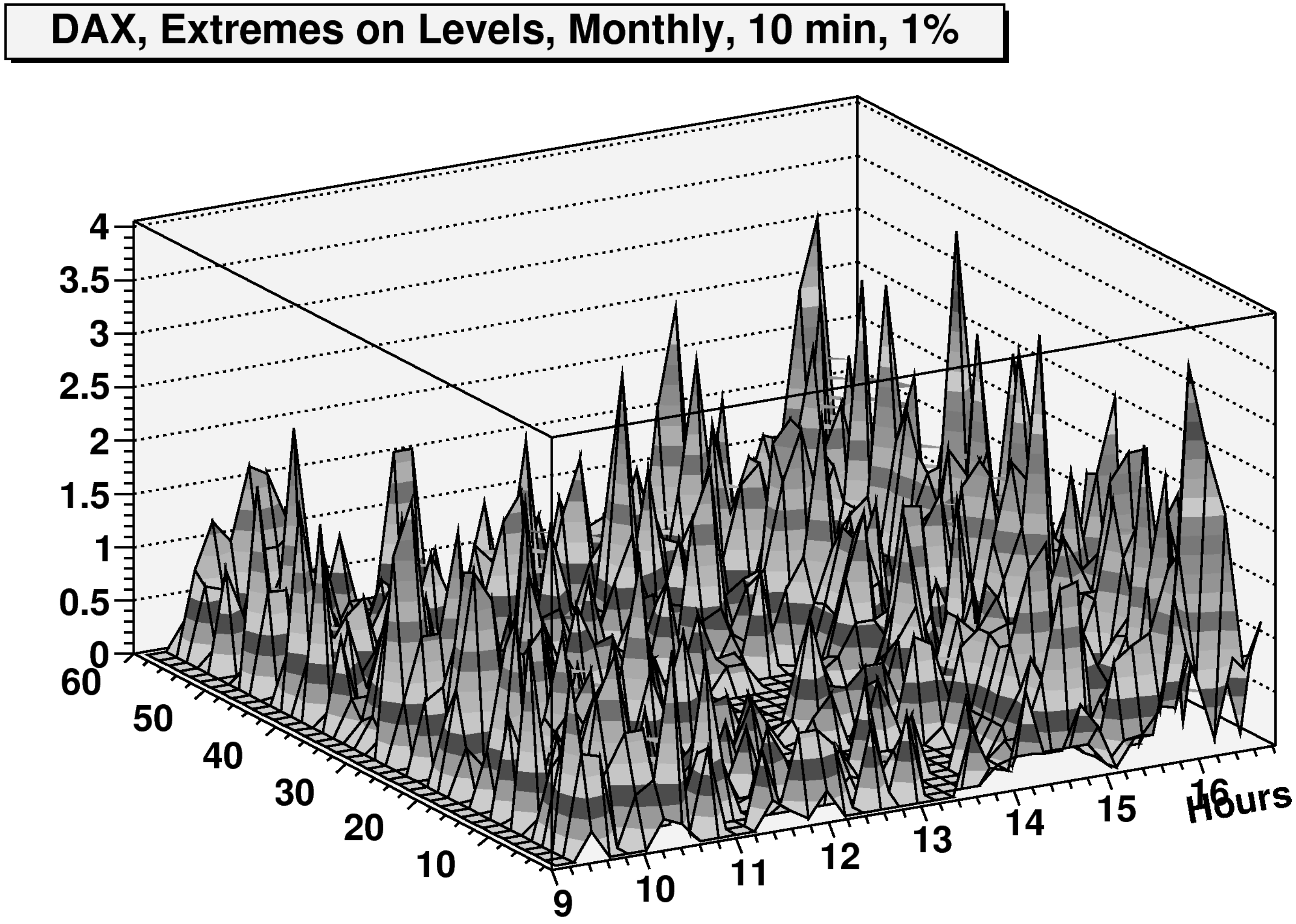


Figure 121: The distribution of extreme returns during a trading day for the DAX index using 10 minute frequency and thresholds defined as 1% and 99% centiles calculated month by month. The distribution is plotted for every month from 01/2004 to 12/2008, where 1 stands for 01/2004 and 60 for 12/2008.



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