

## 7 Economic Growth I - Motivation

### 7.1 Introduction

**Importance of sustained economic growth:** Great **absolute differences** in the standards of living measured by GDP per capita: <sup>1</sup>

- GDP p.c. of USA in 2007 (in 1996 prices) was \$42,897 (10<sup>th</sup> highest in the world)
- Russia - \$13,401, South Africa - \$10,483, China - \$8,511, India - \$3,825, Dem. republic of Congo - \$390, Liberia - \$386 (i.e. \$1.06 a day)
- corresponding differences in nutrition, literacy, infant mortality, life expectancy and other measures of well-being
- Czech Republic - \$21,929, Slovak Republic - \$17,284

Small **differential in growth rates** implies huge differences in final outcomes when compounded over long periods of time (centuries):

- With GDP p.c. \$3,300 in 1870, US was growing at average rate 1.886% per year in period 1870 - 2007<sup>2</sup>
- Though experiment 1: If the growth rate would be 0.886%, then GDP p.c. would be \$11,049 (i.e. 26% of the actual value)
  - similar GDP p.c. level to Cuba, Mexico and Turkmenistan
- Though experiment 2: If the growth rate would be 2.886%, then GDP p.c. would be \$162,664 (i.e. 3.8 times the actual value)

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<sup>1</sup>Data are from version 6.3 of Penn World Tables, <http://pwt.econ.upenn.edu>.

<sup>2</sup>Let  $y_0$  be the GDP p.c. at year 0,  $y_T$  the GDP p.c. at year  $T$ , and  $x$  the average annual growth rate over that period. Then,  $y_T = (1 + x)^T y_0$ . We can compute  $x$  by taking logarithms, getting  $\ln y_T - \ln y_0 = T \ln(1 + x) \approx Tx$ , or  $x \approx (\ln y_T - \ln y_0)/T$ .

Even in the horizon of **2 generations**, growth rates matter:

- If the Czech Republic would grow at the same average rate as throughout the period 2000-2007 (i.e. 3.9%), in 30 years it would triple its real GDP p.c.
- However, if the Slovak Republic would grow at the same average rate as throughout the period 2000-2007 (i.e. 4.7%), in 30 years it would attain 4 times its real GDP p.c. and it would "catch on" the Czech Republic.

## 7.2 World Distribution of Income and Growth Rates

High **cross-country dispersion** in the level of income - GDP p.c., persistent with time

- **Figure 1** distribution of GDP p.c. in 1960 across 113 countries from the Penn World Tables 6.1.
  - richest country - Switzerland (\$15,000), poorest - Tanzania (\$381)
  - wealthiest countries: OECD + Latin America (Venezuela, Argentina); poorest countries: Africa (Tanzania, Uganda) and Asia (China, India, Indonesia)
- **Figure 2** - distribution of GDP p.c. in 2000 across 150 countries from the Penn World Tables 6.1.
  - richest country - Luxembourg (\$44,000), poorest - Tanzania (\$482)
  - wealthiest countries: OECD + East Asia (Taiwan, Japan, Singapore); poorest countries: sub-Saharan Africa (Tanzania, Uganda); Latin America + Asia: mid-range
- **Comparison:**
  - similar cross-country dispersion of income over this period
  - mean of GDP p.c. in 2000 was 2.5 higher than in 1960 (compare \$8,490 and \$3,390)
  - change of relative position of countries (drop of Argentina, Venezuela, Israel or RSA; rise of China, India, Singapore) due to **differences in the rate of economic growth**
- **Figure 3** - distribution of growth rate of GDP p.c. from 1960 - 2000.
  - range from  $-3.2\%$  for the Democratic Republic of Kongo to  $6.4\%$  for Taiwan
  - growth miracles: Singapore ( $6.2\%$ ), South Korea ( $5.9\%$ ), Hong Kong ( $5.4\%$ ), Thailand, Japan (after WWII), China, Ireland
  - growth disasters: sub-Saharan Africa (Niger, Angola, Madagascar, Nigeria, Rwanda) + Latin America (Venezuela, Bolivia, Peru, Argentina)

### 7.2.1 Convergence: Do the poor countries catch up rich countries, i.e. do they tend to grow faster?

(+ rationale)

- **Unconditional convergence:**  $\Delta \ln y_{2000-1960} = \alpha + \beta \ln y_{1960}$ 
  - Figure 4, based on Penn World Tables data, shows that average growth rate over the period 1960-2000 has little (and slightly positive) correlation with initial level of GDP p.c.
- **Conditional convergence:**  $\Delta \ln y_{2000-1960} = \alpha + \beta \ln y_{1960} + \gamma \mathbb{X}_{1960}$ , where  $\mathbb{X}_{1960}$  is a set of country-specific controls (education, fiscal and monetary policy, competition level, etc.) - we compare countries with similar starting characteristics
  - After conditioning on the underlying characteristics, the countries with lower initial income tend to grow faster than their rich counterparts. For illustration, see Figure 5 for evidence of convergence within OECD countries and Figure 6 for the convergence among US states (both with apparent negative correlation).

**What are the factors behind the differences in economic growth, and how can we control them?**

- government policies with effects on long-term growth
- evaluation framework = models

## 7.3 Stylized Facts - Building Blocks of Models

**Kaldor (1963) - balanced growth in the long run**

1. Output per worker  $Y/L$  (GDP p.c.) grows over time and the growth rate does not tend to diminish
2. Physical capital per worker  $K/L$  grows over time
3. The capital to output ratio  $K/Y$  is nearly constant  $\Rightarrow$  capital and output grow at the same rate
4. The return to capital ( $r$ ) is roughly constant
5. The income shares of labor and capital ( $wL/Y$  and  $rK/Y$ ) stay roughly constant
6. The level as well as the growth rate of output per worker differs substantially across countries.

$\Rightarrow$  applies to **developed countries**  
 $\Rightarrow$  explained by Solow model

## 7.4 Figures

Figure 1: Histogram for GDP p.c. in 1960 (reproduced from Barro, 2003). The data for 113 countries are taken from Penn World Tables 6.1. Representative countries within each group are labeled.

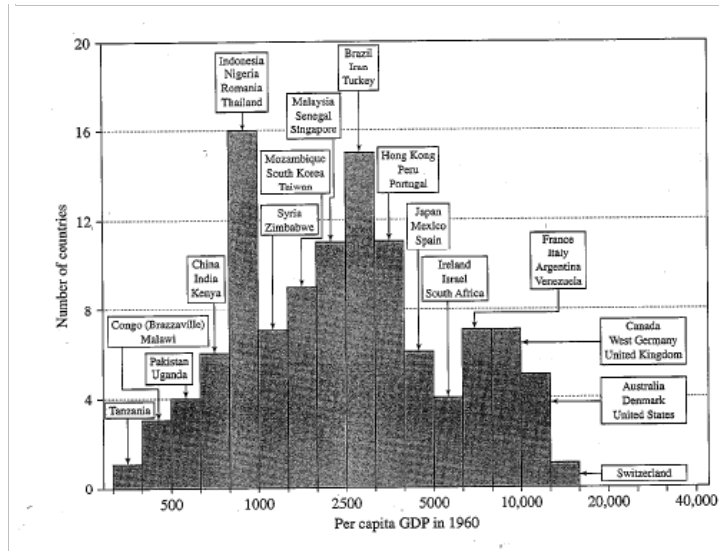


Figure 2: Histogram for GDP p.c. in 2000 (reproduced from Barro, 2003). The data for 150 countries are taken from Penn World Tables 6.1. Representative countries within each group are labeled.

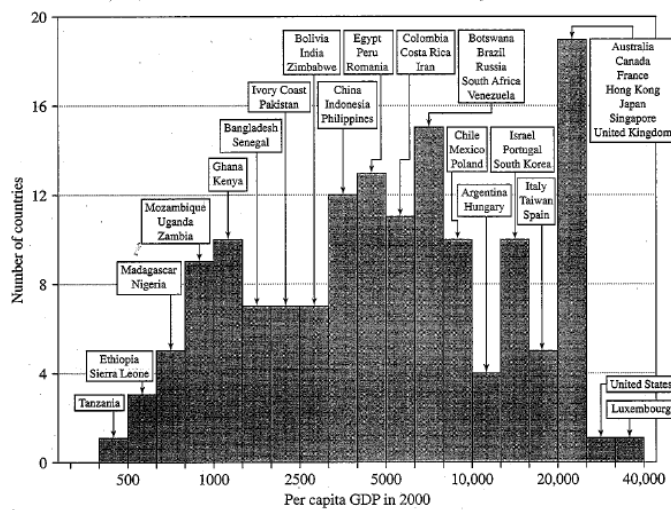


Figure 3: Histogram for growth rates of GDP p.c. from 1960-2000 (reproduced from Barro, 2003). The data for 150 countries are computed from the values of GDP p.c. shown in Figures 1 and 2. Representative countries within each group are labeled.

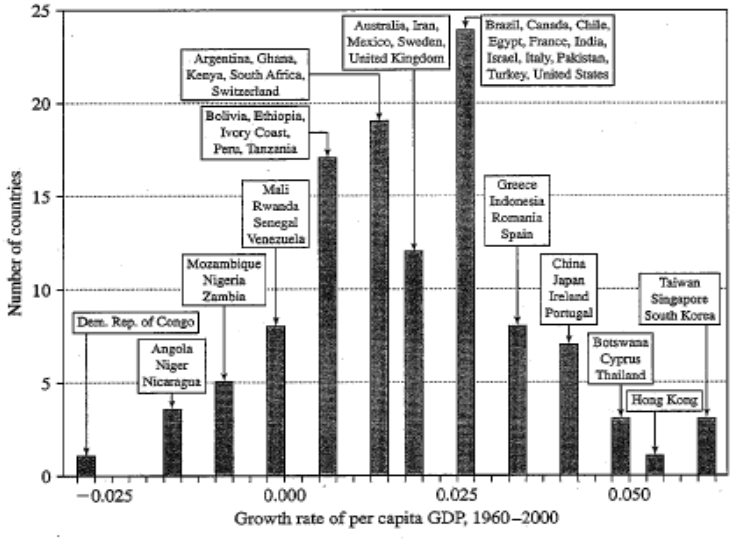


Figure 4: Convergence of GDP across countries: Growth rate from 1960 to 2000 over the initial level of real GDP p.c. for 114 countries (reproduced from Barro, 2003).

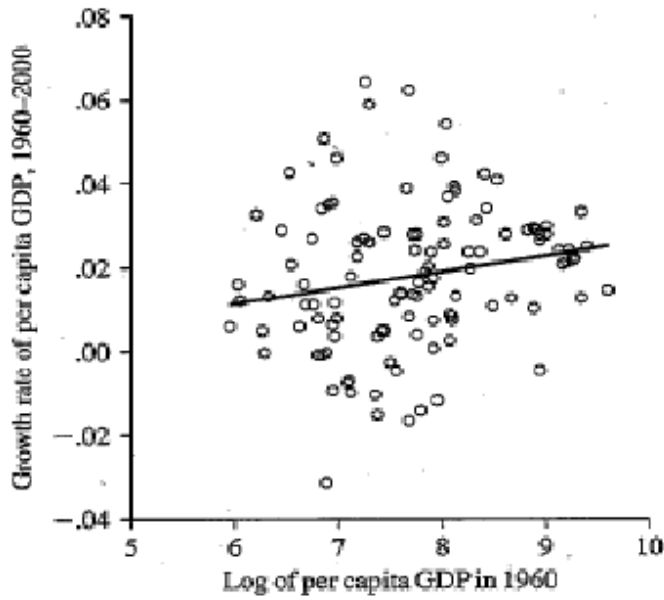


Figure 5: Convergence of GDP across OECD countries: Growth rate from 1960 to 2000 over the initial level of real GDP p.c. for 18 countries (reproduced from Barro, 2003).

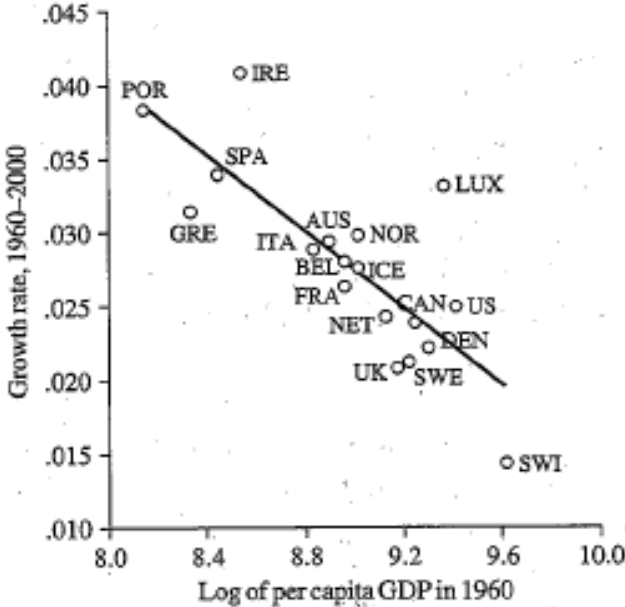
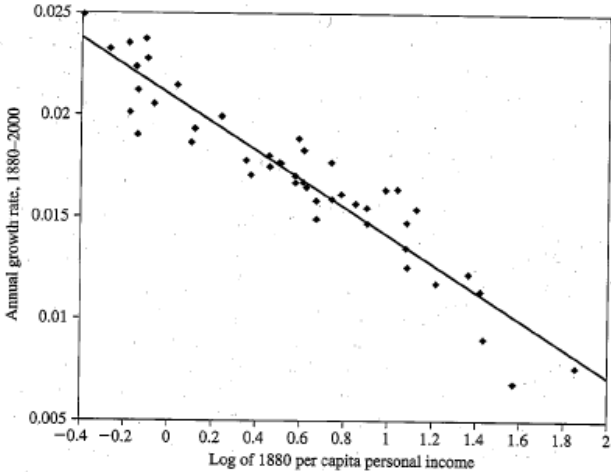


Figure 6: Convergence of personal income across US states: Growth rate of personal income from 1880 to 2000 over the initial level of personal income for 47 states (reproduced from Barro, 2003).



## 8 Economic Growth II - Solow model

- 3 main sources of growth - capital, people, technolog. progress
- SOLOW: how these three interact and affect national output; build up in steps

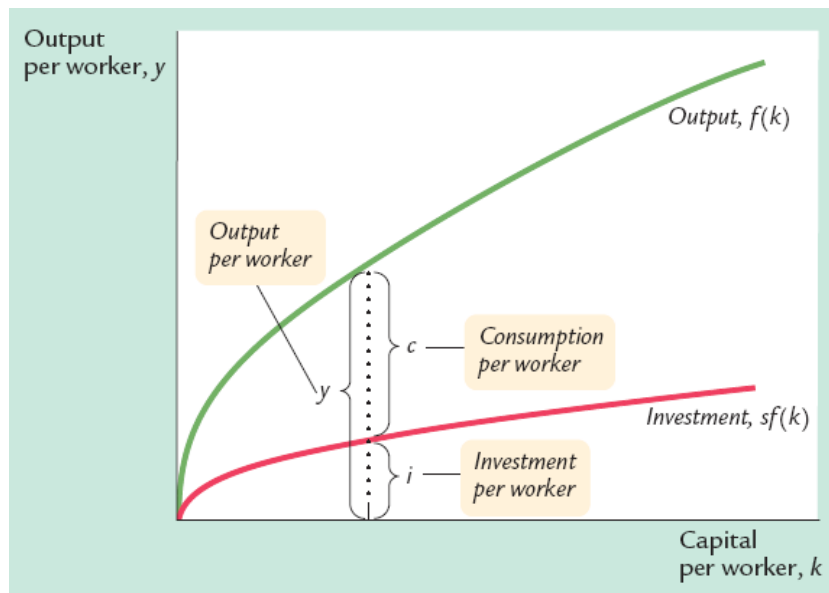
### 8.1 STEP 1: Accumulation of capital

#### 8.1.1 Assumptions of the model:

- **Neoclassical production function:**  $Y = F(K, AL)$ 
  - firms hire people ( $L$ ) and capital ( $K$ ) to produce good ( $Y$ ) by production technology ( $F(\cdot)$ )
  - have access to knowledge ( $A$ ) that makes the production more effective (labor-augmenting or Harrod-neutral technological change)
  - let's assume (for a while) that both labor force and efficiency of production are constant over time
  - Ass.1: Constant returns to scale - i.e.  $zY = F(zK, zAL)$ 
    - allows us to analyze the per capita quantities: output per effective worker  $y = Y/AL$  and capital per effective worker  $k = K/AL$
    - take  $z = 1/AL \Rightarrow Y/AL(= y) = F(K/AL, 1)(= f(k))$
  - Ass.2: Marginal product is positive and diminishing
    - applies also for transformed function -  $f'(k) > 0, f''(k) < 0$
  - Ass.3: Inada conditions + essentiality
- output is divided between consumption and investment:  $y = c + i$
- HHs save a constant fraction of their income  $s \in (0, 1)$ :  $i = sy, c = (1 - s)y$

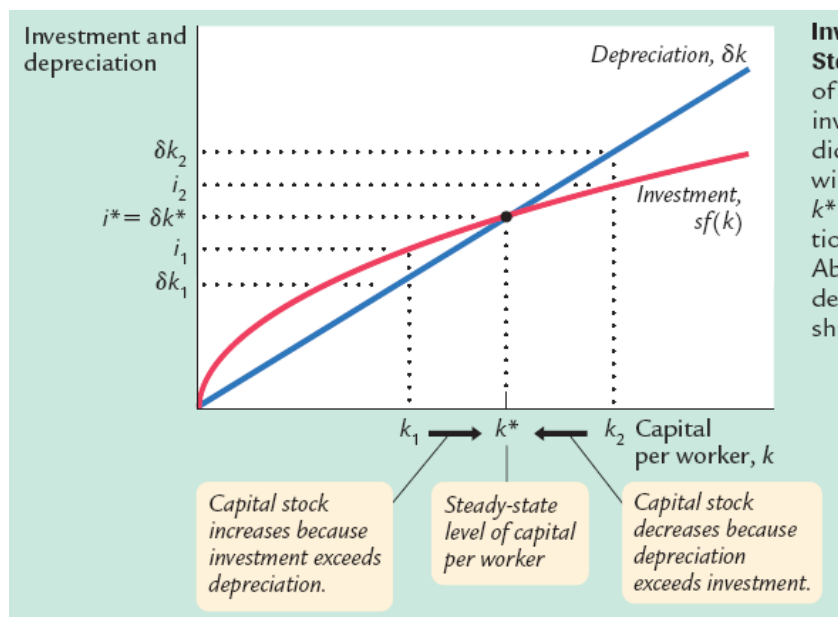
#### 8.1.2 Basic analysis:

- capital stock of economy changes over time
  - increases due to investment - new plants and equipment
  - decreased due to depreciation - wearing out of capital
- Investment:  $i = sf(k)$
- Depreciation: fraction  $\delta$  of capital stock "disappears"  $\delta k$



- **change of capital stock** = investment - depreciation

$$\Delta k = sf(k) - \delta k$$



- **STEADY STATE:** there exist single capital stock  $k^*$  for which amount of depreciation equals the invested amount

$$\exists! k^* : \Delta k = 0 \text{ or } sf(k) = \delta k$$



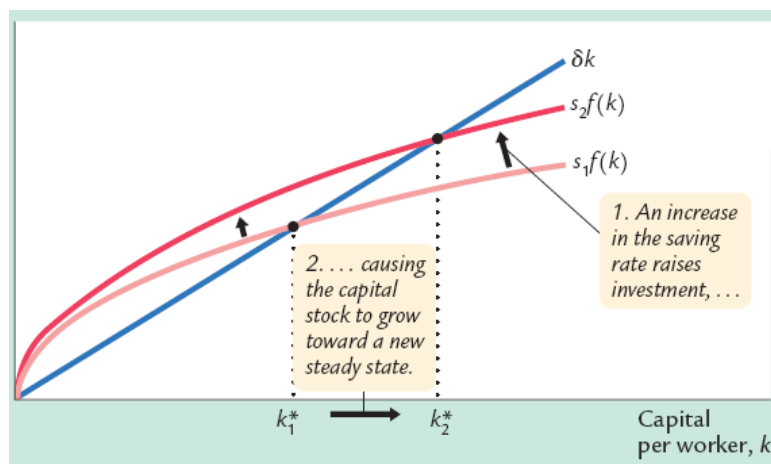
- if economy is in steady-state, it will stay there
- if economy starts with any other level of capital, it will converge to steady state (stable equilibrium)

- **Prediction of model:**

- in the long run, all economies will converge to their respective steady state
- if country starts from relatively lower level of capita per person, it will grow faster (Japan, Germany after WWII)

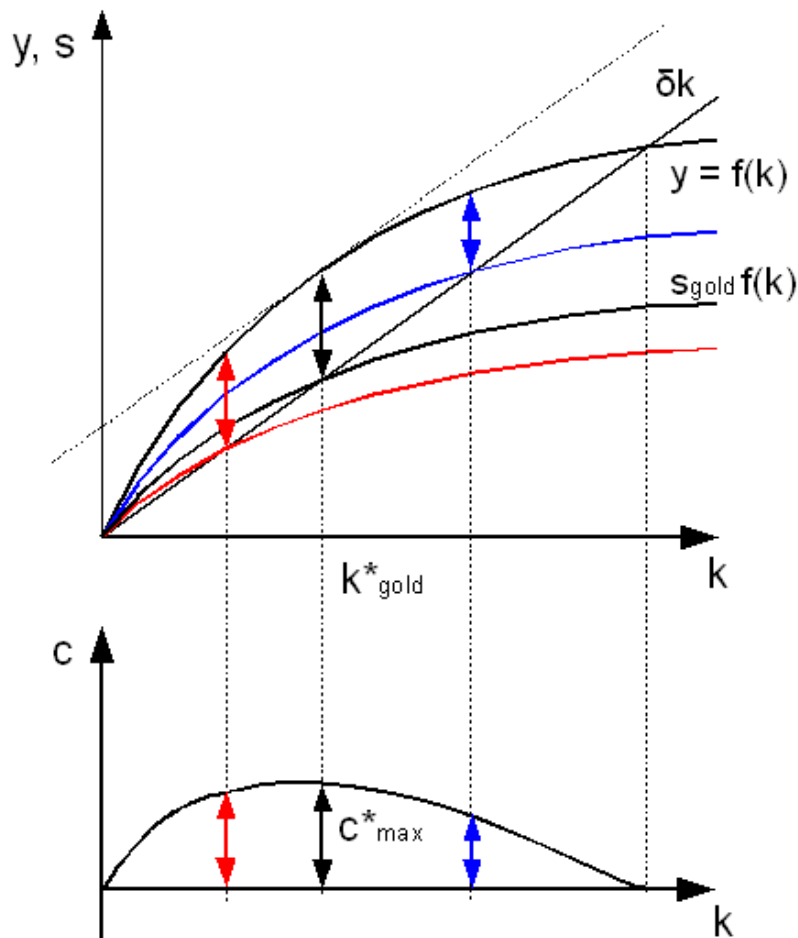
- **Effect of savings:** as key determinant of capital stock

- higher saving rate => higher steady state level of capital and output per capita
- increase in saving rate => temporary increase in growth rate of economy



### 8.1.3 Golden Rule level of capital:

- different saving rates lead to different steady states - with corresponding steady state level of capital, output and consumption.
- Questions: How do we compare these different steady states? What is optimal from HH's point of view?
- Answer: Choose saving rate (and corresponding capital level) that **maximizes the consumption**.



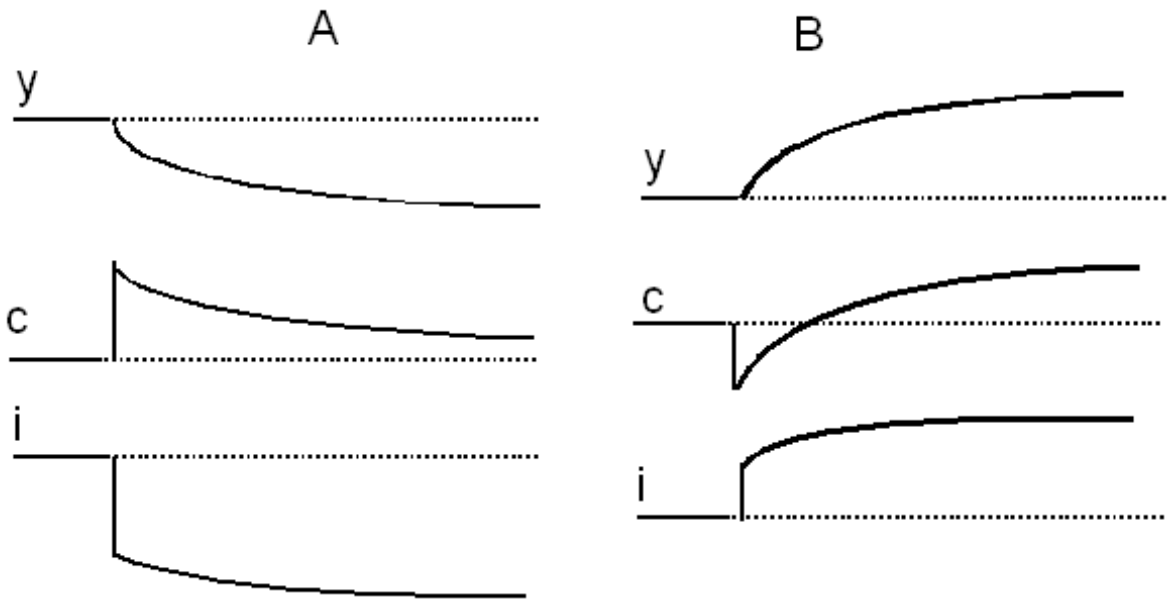
- Computation:
  - $y = c + i$
  - in the steady state:  $i = sf(k^*) = \delta k^*$
  - therefore, consumption can be expressed as  $c^* = f(k^*) - \delta k^*$

– Maximum:

$$\frac{\partial c^*}{\partial k^*} = 0 \Big|_{k^*=k_{gold}^*} \rightarrow f'(k_{gold}^*) - \delta = 0$$

• Intuition:

- small  $s \Rightarrow$  small  $k^* \Rightarrow$  small  $y^* \Rightarrow$  small consumption
- high  $s \Rightarrow$  high  $k^* \Rightarrow$  high  $y^*$  and high depreciation  $\delta k^* \Rightarrow$  high investment needed to cover for depreciation  $\Rightarrow$  small consumption



• **Transition:** what are the costs of transition to optimal steady state?

- A. starting with **too much capital**  $\Rightarrow$  POLICY = reduce saving rate
  - \* investment drops immediately
  - \* consumption jumps up - keeps over the initial level over all transition time
- B. starting with **too low capital**  $\Rightarrow$  POLICY = increase saving rate
  - \* investment partially jumps up
  - \* consumption jumps down - first lower than initial level  $\Rightarrow$  then increases
- tradeoff among welfare of different generations - decision depends on the weight that policy makers put on different generations

## 8.2 STEP 2: Population growth

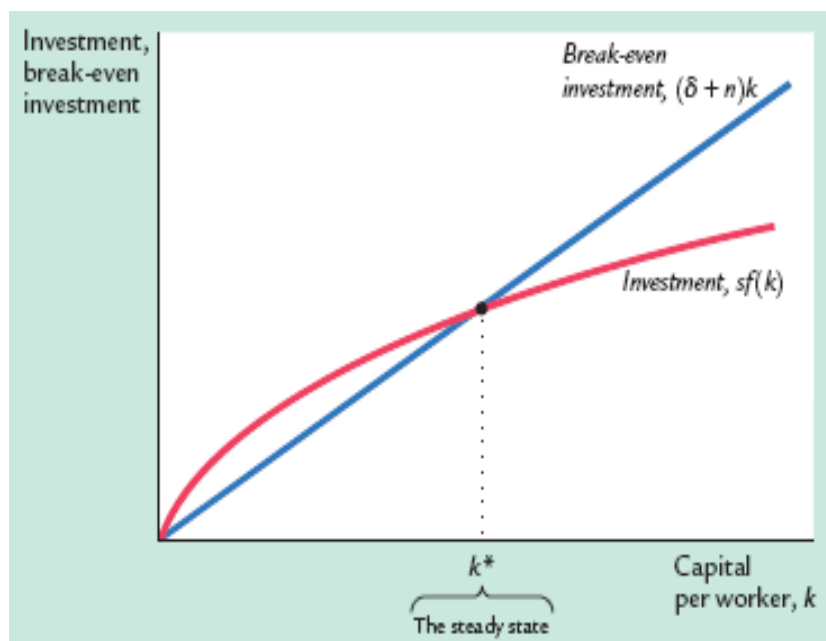
- capital accumulation alone cannot explain sustained growth
  - eventually converge to steady state => capital and output p.c. are constant
  - with ass. on constant labor force => constant total output and capital
- assumption: population grows at constant rate  $n$

### 8.2.1 Effect of population growth:

- with increasing population, capital per worker is decreasing
  - \* capital is distributed among larger population of workers
  - \* similar effect as depreciation
- new condition for steady state: investment = replacement of depreciation + capital for new workers

$$\Delta k = sf(k) - (\delta + n)k$$

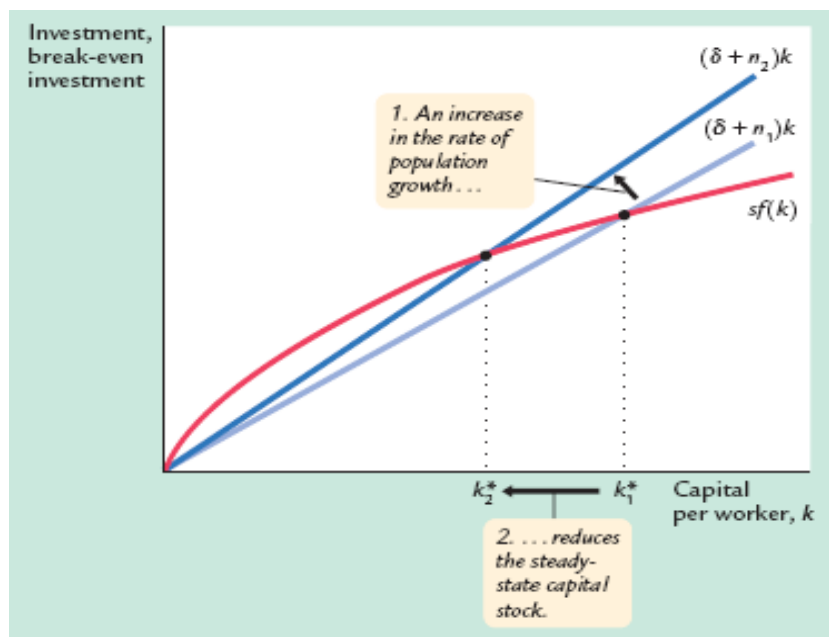
$$\Delta k = 0 \Leftrightarrow sf(k^*) = (\delta + n)k^*$$



1. partial explanation of sustained economic growth
  - output and capital per worker is constant
  - total output and capital grow at rate  $n$
2. higher population growth => lower level of GDP per capita

- consistent with empirical data
  - possible reverse causation
3. new equation for Golden Rule level of capital

$$MPK = f'(k) = \delta + n$$



## 8.3 STEP 3: Technological growth

### 8.3.1 Efficiency of labor

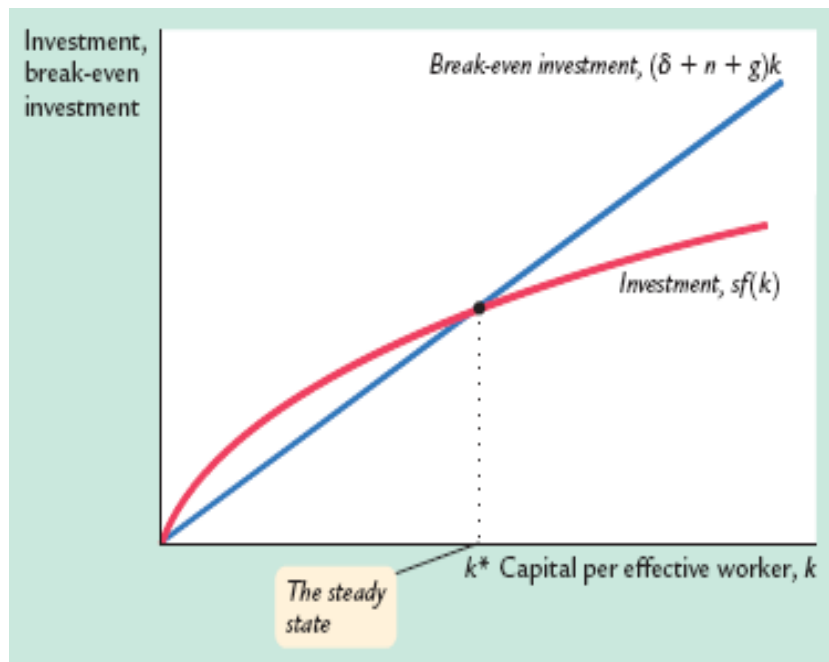
- Assumption: technological growth causes the efficiency  $A$  to grow **at constant rate  $g$** 
  - labor-augmenting technological progress
  - number of effective workers grows approx. at rate  $n + g$

### 8.3.2 Effects of technological growth

- analogous to population growth

$$\begin{aligned}\Delta \hat{k} &= sf(\hat{k}) - (\delta + n + g)\hat{k} \\ \Delta \hat{k} &= 0 \Leftrightarrow sf(\hat{k}^*) = (\delta + n + g)\hat{k}^*\end{aligned}$$

1. explanation of sustained economic growth



- total output and capital grow at rate  $n + g$  (like effective workers)
- output and capital per worker is growing at rate  $g$

2. new equation for Golden Rule level of capital

$$MPK = f'(\hat{k}) = \delta + n + g$$

## 8.4 Endogenous growth models

- Solow-Swan model: s.s. growth rate of  $Y/L = \frac{\Delta Y/L}{Y/L} = g$
- $g$  - rate of technological progress, **EXOGENOUSLY** given + assumed to be positive and constant
- want to have growth **ENDOGENOUS**, i.e. we are able to explain it as the outcome of the decisions of agents within the model

Possible solutions:

- **AK models** - abandon diminishing returns to capital (DRC)
  - broad definition of capital (physical + **human**)
    - \* 1 sector: production of goods - basic model
    - \* 2 sectors: production of both goods and (human) capital - education
  - **learning-by-doing** + **spillovers** of knowledge

\* individual firms - DRC, aggregate level - CRC/IRC

- **R&D models** Advances in technology level determined by purposeful activity (explicitly model determinants of  $g$ )
  - expanding **variety** of products
  - **quality** improvements of existing products

#### 8.4.1 Basic AK model

- production function:  $Y = AK \Rightarrow MPK = A > 0$
- abolished diminishing product of capital -  $A$  is positive constant  $\Rightarrow$  constant return to capital (CRC)
- accumulation of capital:

$$\begin{aligned}\Delta K &= sY - \delta K \\ \frac{\Delta K}{K} &= \frac{\Delta Y}{Y} = sA - \delta\end{aligned}$$

- as long as  $sA > \delta$ , economy grows forever
- saving decision alone leads to permanent growth

Is CRC reasonable assumption?

- NO, if we assume classical definition of capital - stock of plants and equipment.
- YES, if we consider broad definition of capital - including knowledge (know-how).

#### 8.4.2 2 sector model of Human capital

- 2 sectors: production of output (firms) and production of education or human capital (universities)
- production function - CRS

$$Y = F(K, (1 - \mu)EL)$$

- accumulation of capital (physical + human):

$$\begin{aligned}\Delta E &= g(\mu)E \\ \Delta K &= sY - \delta K\end{aligned}$$

- $\mu$  - fraction of labor force in universities
- $g$  - production of new knowledge, dependent on share of labor force in that sector
- persistent growth attained endogenously - production of knowledge on universities will explain  $g$  - otherwise similar to Solow
- two societal decision variables -  $s$  and  $\mu$