OVS452 + 5EN 253
VSE NF, Spring 2010
Lecture Notes #2
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3 General equilibrium model of national income

3.1 Concept of equilibrium - Clasic model

General concept = steady-state (i.e. state of rest), no market clearing needed

Walrasian general equilibrium = demand equals to supply on all the markets of the economy (goods and services, labour and financial) simultaneously
Assumptions:

- rationally behaving agents who maximize utility / profits
- fully competitive markets and fully flexible prices
- full information of all agents
- stable expectations

Above stated assumptions are obviously unrealistic: irrational behavior, price rigidities, existence of monopolies and oligopolies, asymmetric information. Yet, classical model is important benchmark, hypothetical long-term outcome.

3.2 Aggregate supply - output of economy

The level of GDP depends on

1. quality and quantity of resource (inputs) = factors of production
2. ability to turn inputs into goods and services = production function

3.2.1 Factors of production

- **labor** L: time that people spend working
- **capital** K: everything that is replaceable and used in production process (machinery, tools, offices, etc.)

Simplifications we are making:

- do not consider quality differences or specialization
- do not consider quality other potential factors of production (e.g. land)
- we take supply of factors as given ($K = \bar{K}, L = \bar{L}$) - do not analyze how it is created and how it evolves over time
- assume full utilization, BUT in reality unemployment

3.2.2 Production function

$Y = F(K, L)$ - F represents technology which turns inputs into output

Assumptions about production function (neoclassical):

1. **constant returns to scale** (CRS):

$$F(cK, cL) = cF(K, L) \quad \text{for all } c \geq 0$$

   - Ex.: 4kg of oranges = 1l of juice, 8kg of oranges = 2l of juice

2. **positive and diminishing marginal product** of inputs:
   - what happens if we add one more unit of just one of the factors?

$$\begin{aligned} F(K, L) &< F(K+1, L) < F(K+2, L) \\ F(K+1, L) - F(K, L) &> F(K+2, L) - F(K+1, L) \end{aligned}$$

   - mathematically expressed:

$$\frac{\partial F}{\partial K} > 0,\ \frac{\partial F}{\partial L} > 0; \quad \frac{\partial^2 F}{\partial K^2} < 0,\ \frac{\partial^2 F}{\partial L^2} < 0$$

   - Ex.: tractors on the same field

3. **Essentiality** and **Inada conditions**:
   - both inputs are essential for production - i.e. $F(K, 0) = F(0, L) = 0$
   - Inada conditions describe the extremes of marginal productivity

$$\begin{aligned} \lim_{K \to 0} \frac{\partial F}{\partial K} &= \lim_{L \to 0} \frac{\partial F}{\partial L} = \infty \\ \lim_{K \to \infty} \frac{\partial F}{\partial K} &= \lim_{L \to \infty} \frac{\partial F}{\partial L} = 0 \end{aligned}$$

Example 1: **Cobb-Douglas production function:** defined as

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}, \quad \alpha \in [0, 1]$$

where $A > 0$ is a parameter that captured effectivity of production.

Let us check the properties of production function:

ad 1. $F(zK, zL) = A(zK)^{\alpha}(zL)^{a-\alpha} = zAK^{\alpha}L^{1-\alpha} = zF(K, L)$

ad 2. $MPK = \dfrac{\partial F(K, L)}{\partial K} = A\alpha K^{\alpha-1}L^{1-\alpha} > 0$

$$MPL = \frac{\partial F(K, L)}{\partial L} = A(1-\alpha)K^{\alpha}L^{-\alpha} > 0$$

$$\frac{\partial^2 F(K, L)}{\partial K^2} = A\alpha \underbrace{(\alpha - 1)}_{<0} K^{\alpha-2}L^{1-\alpha} < 0$$

$$\frac{\partial^2 F(K, L)}{\partial L^2} = A(1-\alpha) \underbrace{(-\alpha)}_{<0} K^{\alpha}L^{-\alpha} < 0$$

3.3 Distribution of national income

- labor is provided by HHs - traded on labor market
- capital is provided primarily from savings of HHs (in reality capital is owned by firms) - traded on financial market
- firms are owned by HHs

=> national income is divided between labor and capital

3.3.1 Factor prices

- **wage** that workers earn + **rent** paid to the owners of capital
- we assume supply of factor is fixed
- demand for factors => **firm's decision problem:**
  - competitive firm => small => take prices of goods & services P as well as labor W and capital R as given
  - **profit maximization:**

    $$\max_{L,K} \pi = P \times Y - W \times L - R \times K$$

  - solve using **marginal product of labor**: product of one more unit of labor, diminishing

    $$\Delta\pi = P \times MPL - W \quad \Rightarrow \quad \text{hire} \Leftrightarrow W \leq MPL$$

– hire new people until $P \times MPL = W$ or $MPL = \frac{W}{P}$ (real wage)

– same logic with **marginal product of capital**

– rent new capital until $P \times MPK = R$ or $MPL = \frac{R}{P}$ (real rent)

3.3.2 Division of national income

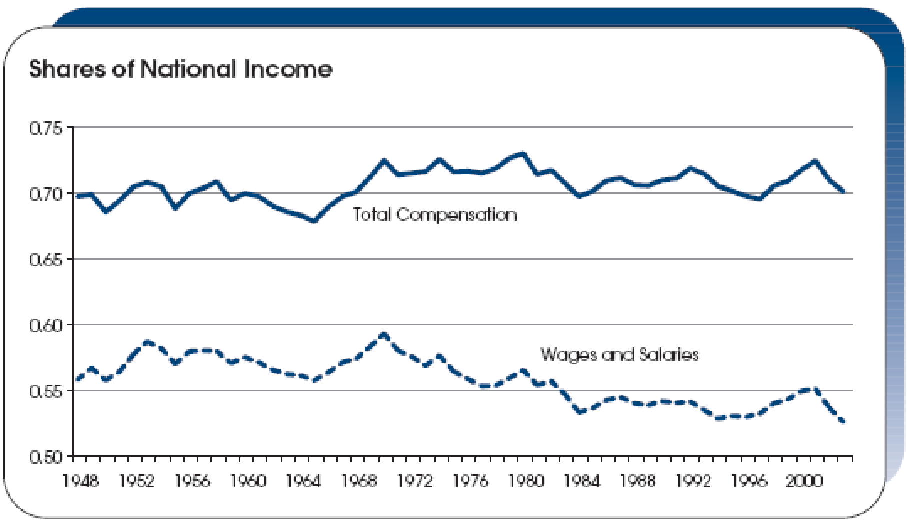
- we assume that all firms in the economy are homogenous ("the same") and profit maximizing $=> W = MPL,\ R = MPK$

$$\begin{aligned} \pi &= Y - (MPL \times L) - (MPK \times K) \\ Y &= \pi + (MPL \times L) + (MPK \times K) \end{aligned}$$

- if production function is CRS, then profit $\pi = 0$;

$$\begin{aligned} \textbf{Proof:} \quad cY &= F(cK, cL) /\ \partial c \\ Y &= \frac{\partial F}{\partial cK}\frac{dcK}{dc} + \frac{\partial F}{\partial cL}\frac{dcL}{dc} \\ &= \frac{\partial F}{\partial cK}K + \frac{\partial F}{\partial cL}L\Big|_{c=1} \\ Y &= MPK \times K + MPL \times L \end{aligned}$$

- in real life "profit" = return on capital
- assumption for MPL - "given the supply/level of other factor is fixed"



Example 1 (cont.): $Y = AK^{\alpha}L^{1-\alpha}$

$$MPK = A\alpha K^{\alpha-1}L^{1-\alpha} = \alpha\frac{AK^{\alpha}L^{1-\alpha}}{K} = \alpha\frac{Y}{K}$$
$$MPL = A(1-\alpha)K^{\alpha}L^{-\alpha} = (1-\alpha)\frac{AK^{\alpha}L^{1-\alpha}}{L} = (1-\alpha)\frac{Y}{L}$$

- income of capital = $MPK \times K = \alpha Y$; income of labor = $MPL \times L = (1-\alpha)Y$

3.4 Aggregate demand for goods and services

- 4 components: consumption (C), investment (I), government purchases (G) and net export (NX)
- ass. closed economy => $NX = 0$

3.4.1 Consumption

- financed from **disposable income** = $Y - T$, where T = taxes - transfers
- consumption function (positive relationship): $C = C(Y - T)$
- **marginal propensity to consume** (MPC): how C changes if income increases by one unit (e.g. CZK, $); $MPC \in [0, 1]$, usually $MPC < 1$

3.4.2 Investment

- firms add to their stock of capital / replace the one that has worn out; HHs buy new houses
- decision to invest based on comparison if return to investment and interest rate (payment for borrowed funds, opportunity cost)
  - nominal interest rate = reported and payed cost
  - real interest rate r = adjusted for inflation => true cost of borrowing
- investment function (negative relationship): $I = I(r)$

3.4.3 Government purchases

- expenditures on army, education, health care, infrastructure
- transfer payments to HHs and firms: welfare, social security
- government budget
  - $G = T$ - balanced budget

- $G > T$ - budget deficit
- $G < T$ - budget surplus

- ass.: fiscal policy is already determined $G = \bar{G}, T = \bar{T}$

3.5 Equilibrium mechanism

How is assured that planned expenditures $(C+G+I)$ are equal to produced output Y?

3.5.1 Market for goods and services

$$\begin{aligned}
Y &= C + I + G = F(\bar{K}, \bar{L}) = \bar{Y} \\
C &= C(\bar{Y}) - \bar{T} \\
I &= I(r) \\
G &= \bar{G}, T = \bar{T} \\
\bar{Y} &= C(\bar{Y} - \bar{T}) + I(r) + \bar{G}
\end{aligned}$$

In this market, it is only interest rate that can affect the level of I and thus balance expenditures with output.

3.5.2 Financial market

How is interest rate determined? By equilibrium of disposable loanable funds (savings of HHs or gvt) and investments desired by firms.

$$\begin{aligned}
\underbrace{(Y - T - C)}_{\text{private savings}} + \underbrace{(T - G)}_{\text{public savings}} &= I \\
\bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} &= I(r) \\
\underbrace{\bar{S}}_{\text{national savings = loanable funds}} &= \underbrace{I(r)}_{\text{desired investments}}
\end{aligned}$$

3.5.3 Changes in saving - fiscal policy:

Increase in government spending: What if government spending increases by ΔG?

- $\bar{C}, \bar{Y}$ remains unchanged => $I(r)$ has to fall => interest rate r increases
- **crowding - out effect:** gvt expenditures crowd out private investment

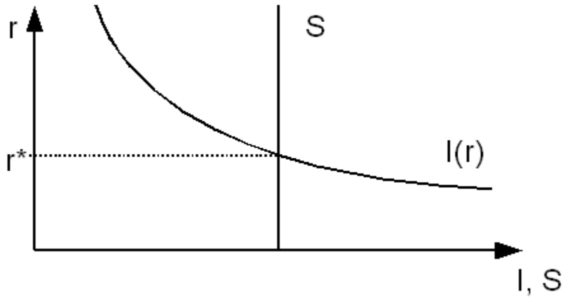


Figure 2: Equilibrium on financial market

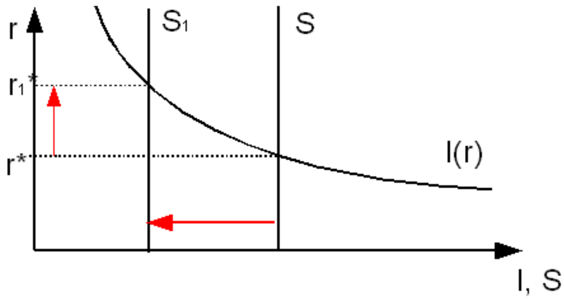


Figure 3: Effect of fiscal policies on equilibrium interest rate

Decrease in taxes What if taxes are cut by ΔT?

- lower T => higher disposable income $\bar{Y} - T$ => higher consumption => no change in $\bar{Y}$ => lower $I(r)$ => higher interest rate r
- same effect as increased gvt expenditures, different mechanism

3.5.4 Changes in investment demand:

- reasons: technological innovation, encouragement from government (e.g. tax law)
- 2 scenarios: constant $\bar{C}$ or $C(Y - T, r)$ and consequently $S(r)$

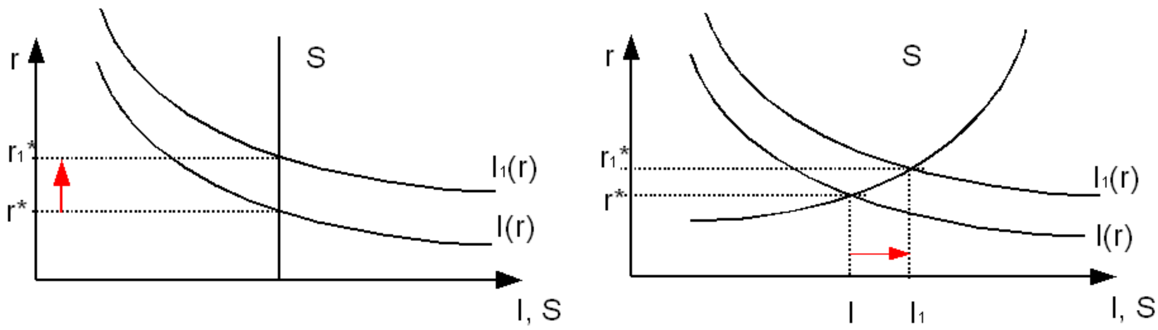


Figure 4: Effect of change in investment demand on equilibrium interest rate