OVS452 Intermediate Economics II
VSE NF, Spring 2008
Lecture Notes #2
Eva Hromádková

3 General equilibrium model of national income

3.1 Overview

4 basic questions about GDP:

1. What are the factors of production?
2. How is the income from production distributed in compensation for labor and capital?
3. Who buys the output of the economy? How do we determine how much output is demanded?
4. How are supply of output (production) and demand for output put together in equilibrium?

For graphical illustration of interaction of all parts of economy see diagram of circulation in macroeconomics.

3.2 Factors of production

The level of GDP depends on

1. quality and quantity of resource (inputs) = factors of production
2. ability to turn inputs into goods and services = production function

3.2.1 Factors of production

- **labor** L: time that people spend working
- **capital** K: everything that is replaceable and used in production process (machinery, tools, offices, etc.)

Simplifications we are making:

- do not consider quality differences or specialization

- do not consider quality other potential factors of production (e.g. land)
- we take supply of factors as given ($K = \bar{K}, L = \bar{L}$) - do not analyze how it is created and how it evolves over time
- assume full utilization, BUT in reality unemployment

3.2.2 Production function

$Y = F(K, L)$ - F represents technology which turns inputs into output

Assumptions about production function (neoclassical):

1. **constant returns to scale** (CRS):

   $$F(cK, cL) = cF(K, L) \quad \text{for all } c \geq 0$$

   - Ex.: 4kg of oranges = 1l of juice, 8kg of oranges = 2l of juice

2. **positive and diminishing marginal product** of inputs:
   - what happens if we add one more unit of just one of the factors?

   $$\begin{aligned} F(K, L) &< F(K+1, L) < F(K+2, L) \\ F(K+1, L) - F(K, L) &> F(K+2, L) - F(K+1, L) \end{aligned}$$

   - mathematically expressed:

   $$\frac{\partial F}{\partial K} > 0, \ \frac{\partial F}{\partial L} > 0; \quad \frac{\partial^2 F}{\partial K^2} < 0, \ \frac{\partial^2 F}{\partial L^2} < 0$$

   - Ex.: tractors on the same field

3. **Essentiality** and **Inada conditions**:
   - both inputs are essential for production - i.e. $F(K, 0) = F(0, L) = 0$
   - Inada conditions describe the extremes of marginal productivity

   $$\begin{aligned} \lim_{K \to 0} \frac{\partial F}{\partial K} &= \lim_{L \to 0} \frac{\partial F}{\partial L} = \infty \\ \lim_{K \to \infty} \frac{\partial F}{\partial K} &= \lim_{L \to \infty} \frac{\partial F}{\partial L} = 0 \end{aligned}$$

Example 1: **Cobb-Douglas production function:** defined as

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}, \quad \alpha \in [0, 1]$$

where $A > 0$ is a parameter that captured effectivity of production.

Let us check the propertied of production function:

ad 1. $F(zK, zL) = A(zK)^{\alpha}(zL)^{a-\alpha} = zAK^{\alpha}L^{1-\alpha} = zF(K, L)$

ad 2. $MPK = \dfrac{\partial F(K, L)}{\partial K} = A\alpha K^{\alpha-1}L^{1-\alpha} > 0$

$$MPL = \frac{\partial F(K, L)}{\partial L} = A(1-\alpha)K^{\alpha}L^{-\alpha} > 0$$

$$\frac{\partial^2 F(K, L)}{\partial K^2} = A\alpha \underbrace{(\alpha - 1)}_{<0} K^{\alpha-2}L^{1-\alpha} < 0$$

$$\frac{\partial^2 F(K, L)}{\partial L^2} = A(1-\alpha) \underbrace{(-\alpha)}_{<0} K^{\alpha}L^{-\alpha} < 0$$

3.3 Distribution of national income

- labor is provided by HHs - traded on labor market
- capital is provided primarily from savings of HHs (in reality capital is owned by firms) - traded on financial market
- government transfers back all money from taxes (redistribution)
- firms are owned by HHs

=> national income is divided between labor and capital

3.3.1 Factor prices

- **wage** that workers earn + **rent** paid to the owners of capital
- we assume supply of factor is fixed
- demand for factors => **firm's decision problem:**
  - competitive firm => small => take prices of goods & services P as well as labor W and capital R as given
  - **profit maximization:**

    $$\max_{L,K} \pi = P \times Y - W \times L - R \times K$$

  - solve using **marginal product of labor**: product of one more unit of labor, diminishing

    $$\Delta\pi = P \times MPL - W \quad \Rightarrow \quad \text{hire} \Leftrightarrow W \leq MPL$$

  - hire new people until $P \times MPL = W$ or $MPL = \frac{W}{P}$ (real wage)
  - same logic with **marginal product of capital**
  - rent new capital until $P \times MPK = R$ or $MPL = \frac{R}{P}$ (real rent)

3.3.2 Division of national income

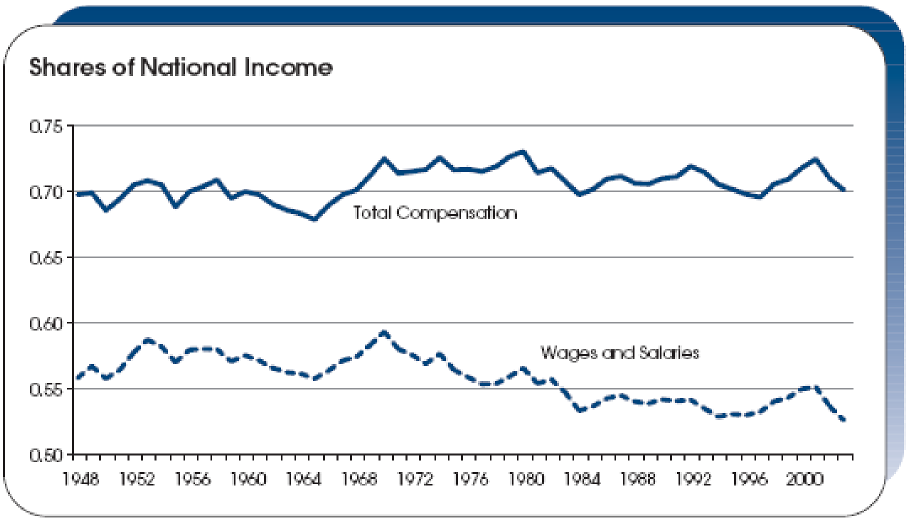
- we assume that all firms in the economy are homogenous ("the same") and profit maximizing => $W = MPL$, $R = MPK$

$$\begin{aligned} \pi &= Y - (MPL \times L) - (MPK \times K) \\ Y &= \pi + (MPL \times L) + (MPK \times K) \end{aligned}$$

- if production function is CRS, then profit $\pi = 0$;

Proof:
$$\begin{aligned} cY &= F(cK, cL)/\ \partial c \\ Y &= \frac{\partial F}{\partial cK}\frac{dcK}{dc} + \frac{\partial F}{\partial cL}\frac{dcL}{dc} \\ &= \frac{\partial F}{\partial cK}K + \frac{\partial F}{\partial cL}L \\ Y &= MPK \times K + MPL \times L \end{aligned}$$

- in real life "profit" = return on capital
- assumption for MPL - "given the supply/level of other factor is fixed"



Example 1 (cont.): $Y = AK^{\alpha}L^{1-\alpha}$

$$\begin{aligned} MPK &= A\alpha K^{\alpha-1}L^{1-\alpha} = \alpha\frac{AK^{\alpha}L^{1-\alpha}}{K} = \alpha\frac{Y}{K} \\ MPL &= A(1-\alpha)K^{\alpha}L^{-\alpha} = (1-\alpha)\frac{AK^{\alpha}L^{1-\alpha}}{L} = (1-\alpha)\frac{Y}{L} \end{aligned}$$

- income of capital = $MPK \times K = \alpha Y$; income of labor = $MPL \times L = (1-\alpha)Y$

3.4 Demand for goods and services

- 4 components: consumption (C), investment (I), government purchases (G) and net export (NX)
- ass. closed economy => $NX = 0$

3.4.1 Consumption

- financed from **disposable income** = $Y - T$, where T = taxes - transfers
- consumption function (positive relationship): $C = C(Y - T)$
- **marginal propensity to consume** (MPC): how C changes if income increases by one unit (e.g. CZK, \$); $MPC \in [0, 1]$, usually $MPC < 1$

3.4.2 Investment

- firms add to their stock of capital / replace the one that has worn out; HHs buy new houses
- decision to invest based on comparison if return to investment and interest rate (payment for borrowed funds, opportunity cost)
  - nominal interest rate = reported and payed cost
  - real interest rate = adjusted for inflation => true cost of borrowing
- investment function (negative ralationship): $I = I(r)$

3.4.3 Government purchases

- expenditures on army, education, health care, infrastructure
- transfer payments to HHs and firms: welfare, social security
- government budget
  - $G = T$ - balanced budget
  - $G > T$ - budget deficit
  - $G < T$ - budget surplus
- ass.: fiscal policy is already determined $G = \bar{G}, T = \bar{T}$

3.5 Equilibrium mechanism

How is assured that planned expenditures ($C + G + I$) are equal to produced output Y?

3.5.1 Market for goods and services

$$\begin{aligned} Y &= C + I + G = F(\bar{K}, \bar{L}) = \bar{Y} \\ C &= C(\bar{Y}) - \bar{T} \\ I &= I(r) \\ G &= \bar{G}, T = \bar{T} \\ \bar{Y} &= C(\bar{Y} - \bar{T}) + I(r) + \bar{G} \end{aligned}$$

In this market, it is only interest rate that can affect the level of I and thus balance expenditures with output.

3.5.2 Financial market

How is interest rate determined? By equilibrium of disposable loanable funds (savings of HHs or gvt) and investments desired by firms.

$$\begin{aligned} \underbrace{(Y - T - C)}_{\text{private savings}} + \underbrace{(T - G)}_{\text{public savings}} &= I \\ \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} &= I(r) \\ \underbrace{\bar{S}}_{\text{national savings = loanable funds}} &= \underbrace{I(r)}_{\text{desired investments}} \end{aligned}$$

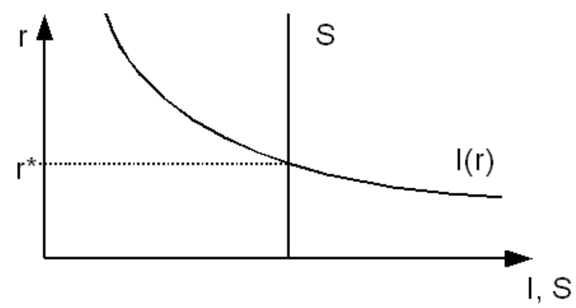


Figure 2: Equilibrium on financial market

3.5.3 Changes in saving - fiscal policy:

Increase in government spending: What if government spending increases by ΔG?

- $\bar{C}, \bar{Y}$ remains unchanged => $I(r)$ has to fall => interest rate r increases
- **crowding - out effect:** gvt expenditures crowd out private investment

Decrease in taxes What if taxes are cut by ΔT?

- lower T => higher disposable income $\bar{Y} - T$ => higher consumption => no change in $\bar{y}$ => lower $I(r)$ => higher interest rate r
- same effect as increased gvt expenditures, different mechanism

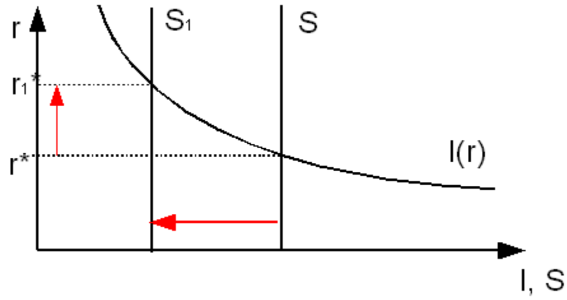


Figure 3: Effect of fiscal policies on equilibrium interest rate

3.5.4 Changes in investment demand:

- reasons: technological innovation, encouragement from government (e.g. tax law)
- 2 scenarios: constant $\bar{C}$ or $C(Y - T, r)$ and consequently $S(r)$

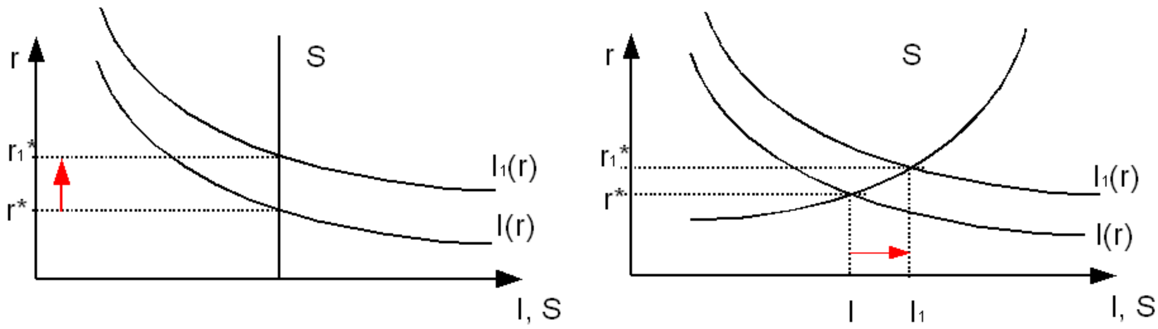


Figure 4: Effect of change in investment demand on equilibrium interest rate