OVS452 Intermediate Economics II
VSE NF, Fall 2009
Lecture Notes #10
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10 Economic Growth II - Solow model

- 3 main sources of growth - capital, people, technolog. progress
- SOLOW: how these three interact and affect national output; build up in steps

10.1 STEP 1: Accumulation of capital

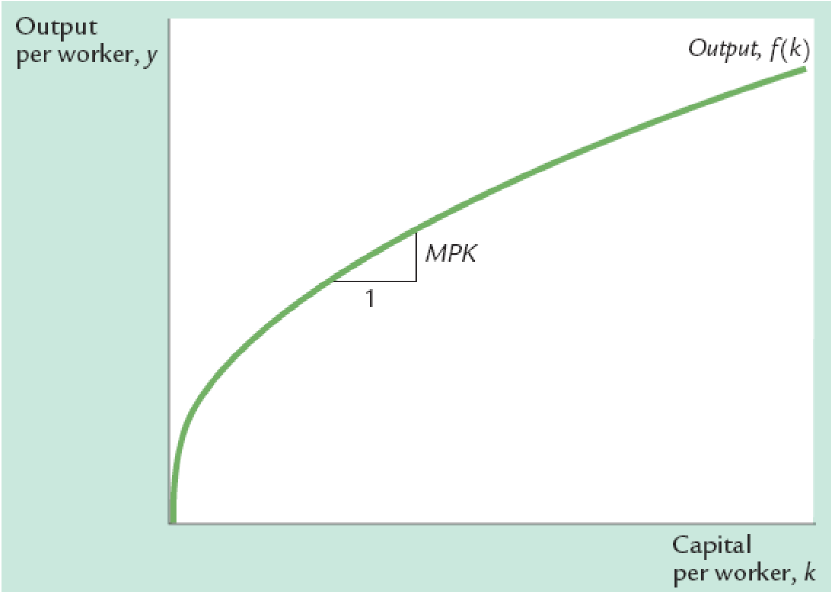
10.1.1 Assumptions of the model:

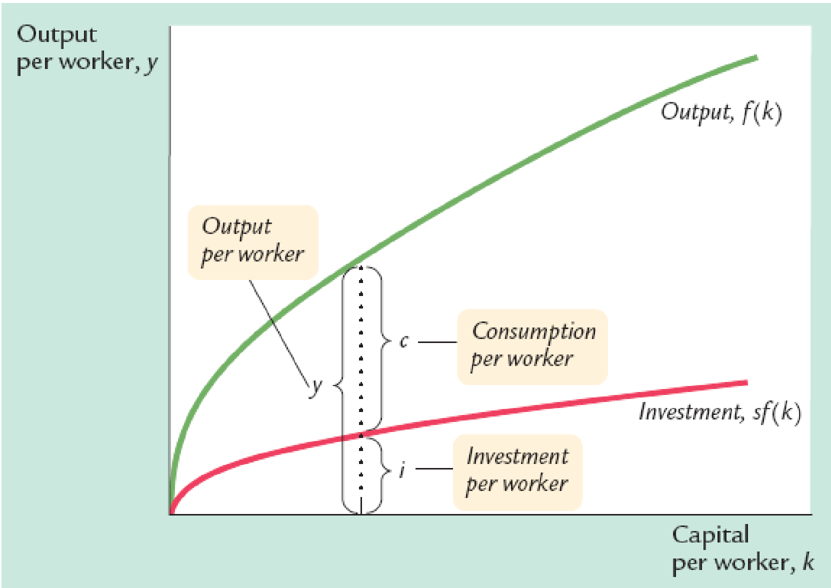
- **Neoclassical production function:** $Y = F(K, L)$
  - Ass.1: Constant returns to scale - i.e. $zY = F(zK, zL)$
    - allows us to analyze the per capita quantities: output per worker $y = Y/L$ and capital per worker $k = K/L$
    - take $z = 1/L \Rightarrow Y/L (=: y) = F(K/L, 1)(=: f(k)$
  - Ass.2: Marginal product is positive and diminishing
    - applies also for transformed function - $f'(k) > 0, f''(k) < 0$
  - Ass.3: Inada conditions + essentiality
- output is divided between consumption and investment: $y = c + i$
- HHs save a constant fraction of their income $s \in (0, 1)$: $i = sy,\ c = (1 - s)y$

10.1.2 Basic analysis:

- capital stock of economy changes over time
  - increases due to investment - new plants and equipment
  - decreased due to depreciation - wearing out of capital
- Investment: $i = sf(k)$
- Depreciation: fraction δ of capital stock "disappears" δk
- **change of capital stock** = investment - depreciation

$$\Delta k = sf(k) - \delta k$$



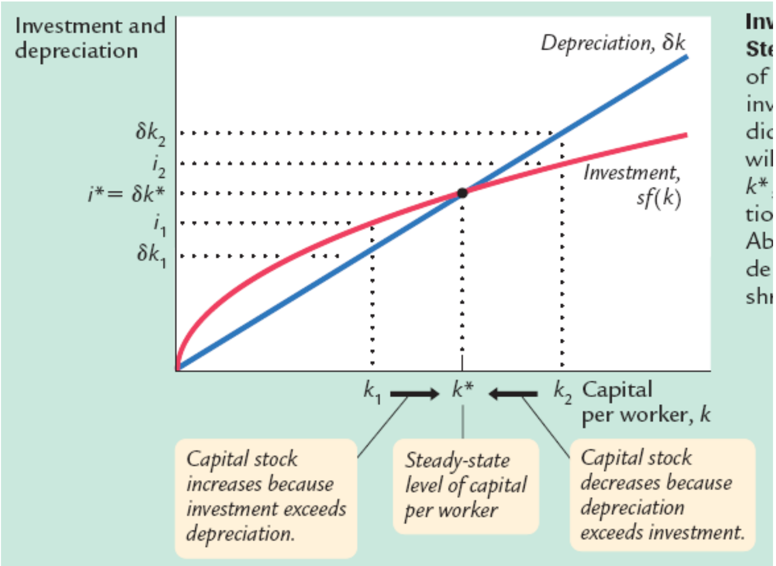


- **STEADY STATE:** there exist single capital stock k^* for which amount of depreciation equals the invested amount

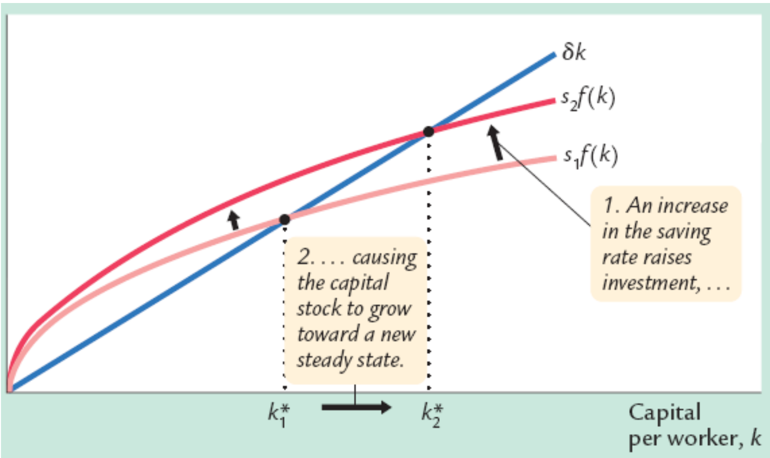
  $$\exists! k^* : \Delta k = 0 \text{ or } sf(k) = \delta k$$

  - if economy is in steady-state, it will stay there
  - if economy starts with any other level of capital, it will converge to steady state (stable equilibrium)

- **Prediction of model:**
  - in the long run, all economies will converge to their respective steady state
  - if country starts from relatively lower level of capita per person, it will grow faster (Japan, Germany after WWII)

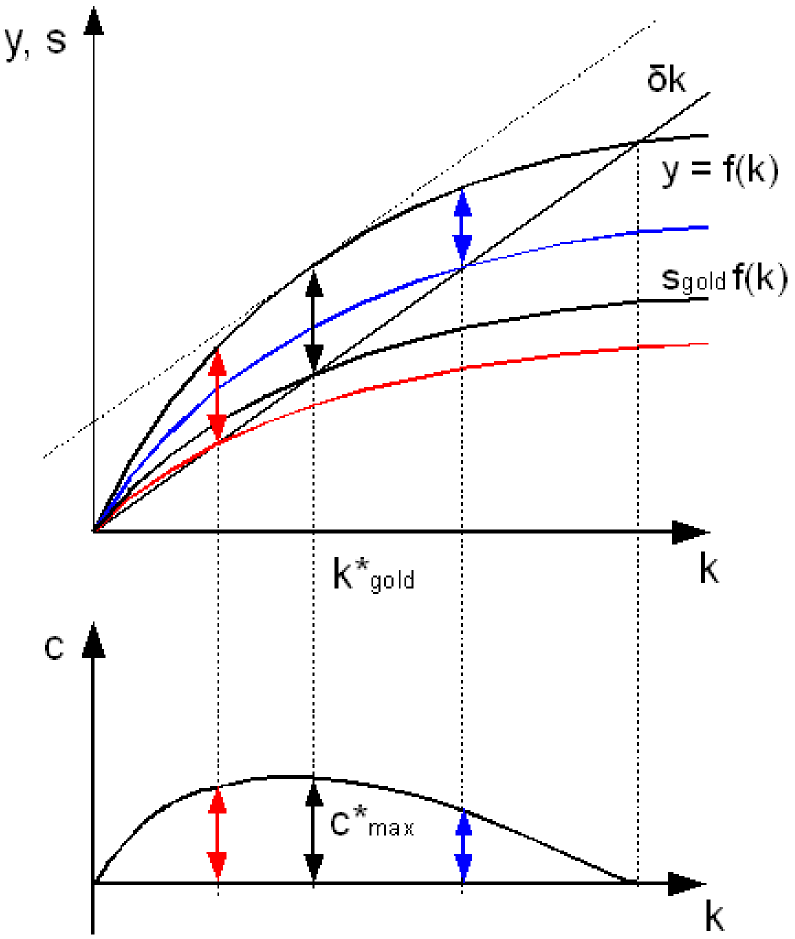


- **Effect of savings:** as key determinant of capital stock
  - higher saving rate => higher steady state level of capital and output per capita
  - increase in saving rate => temporary increase in growth rate of economy



10.1.3 Golden Rule level of capital:

- different saving rates lead to different steady states - with corresponding steady state level of capital, output and consumption.
- Questions: How do we compare these different steady states? What is optimal from HH's point of view?
- Answer: Choose saving rate (and corresponding capital level) that **maximizes the consumption**.

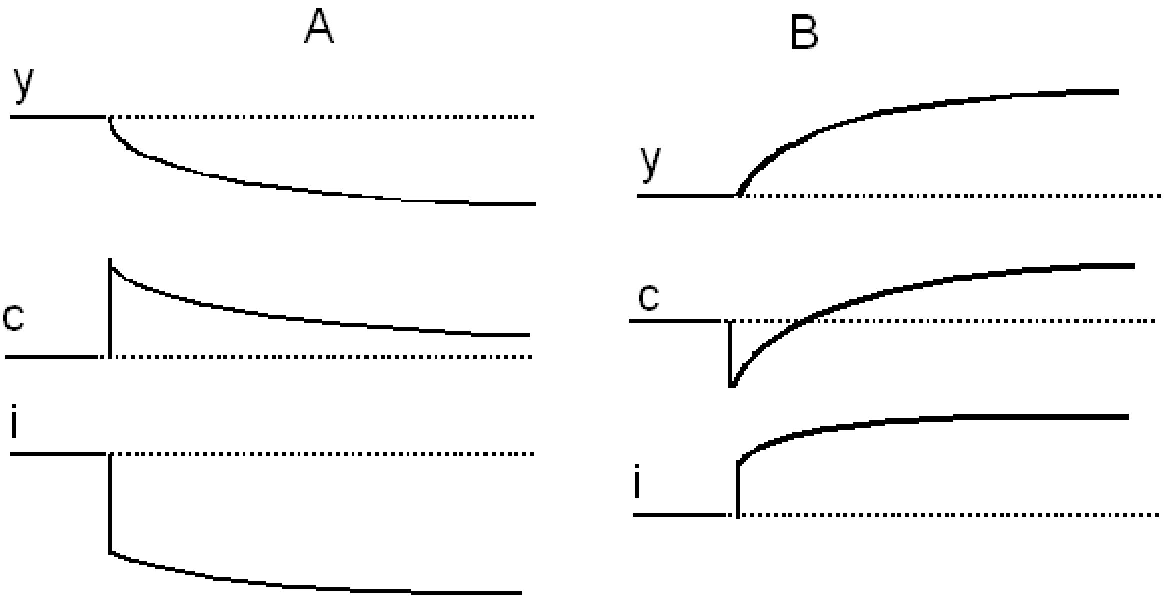


- Computation:
  - $y = c + i$
  - in the steady state: $i = sf(k^*) = \delta k^*$
  - therefore, consumption can be expressed as $c^* = f(k^*) - \delta k^*$

– Maximum:

$$\frac{\partial c^*}{\partial k^*} = 0\Big|_{k^*=k^*_{gold}} \quad \rightarrow \quad f'(k^*_{gold}) - \delta = 0$$

- Intuition:
  – small s => small k^* => small y^* => small consumption
  – high s => high k^* => high y^* and high depreciation δk^* => high investment needed to cover for depreciation => small consumption



- **Transition:** what are the costs of transition to optimal steady state?
  – A. starting with **too much capital**=> POLICY = reduce saving rate
    * investment drops immediately
    * consumption jumps up - keeps over the initial level over all transition time
  – B. starting with t**oo low capital** => POLICY = increase saving rate
    * investment partially jumps up
    * consumption jumps down - first lower than initial level => then increases
  – tradeoff among welfare of different generations - decision depends on the weight that policy makers put on different generations

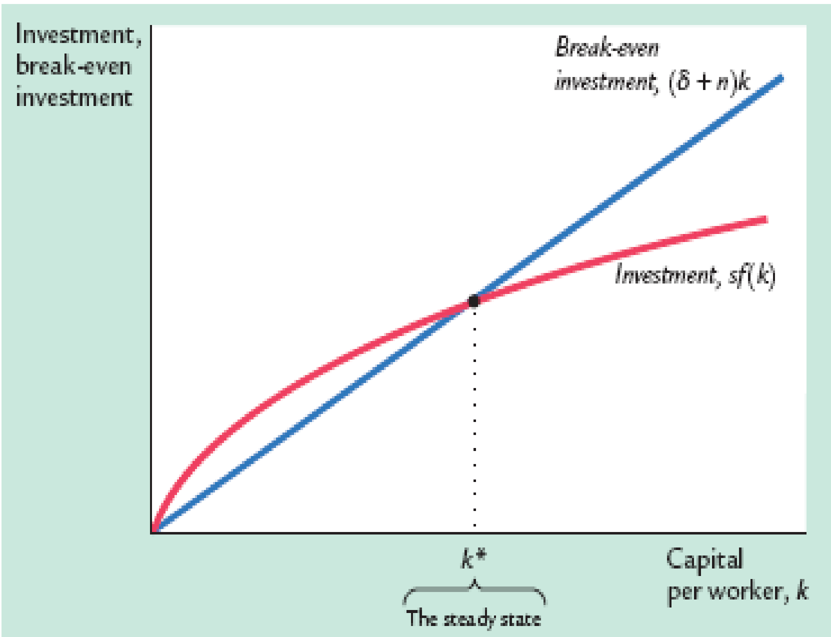
10.2 STEP 2: Population growth

- capital accumulation alone cannot explain sustained growth
  - eventually converge to steady state => capital and output p.c. are constant
  - assuming constant labor force => constant total output and capital
- assumption: population grows at constant rate n

10.2.1 Effect of population growth:

- with increasing population, capital per worker is decreasing
  - ∗ capital is distributed among larger population of workers
  - ∗ similar effect as depreciation
- new condition for steady state: investment = replacement of depreciation + capital for new workers

$$\Delta k = sf(k) - (\delta + n)k$$
$$\Delta k = 0 \quad \Leftrightarrow sf(k^*) = (\delta + n)k^*$$

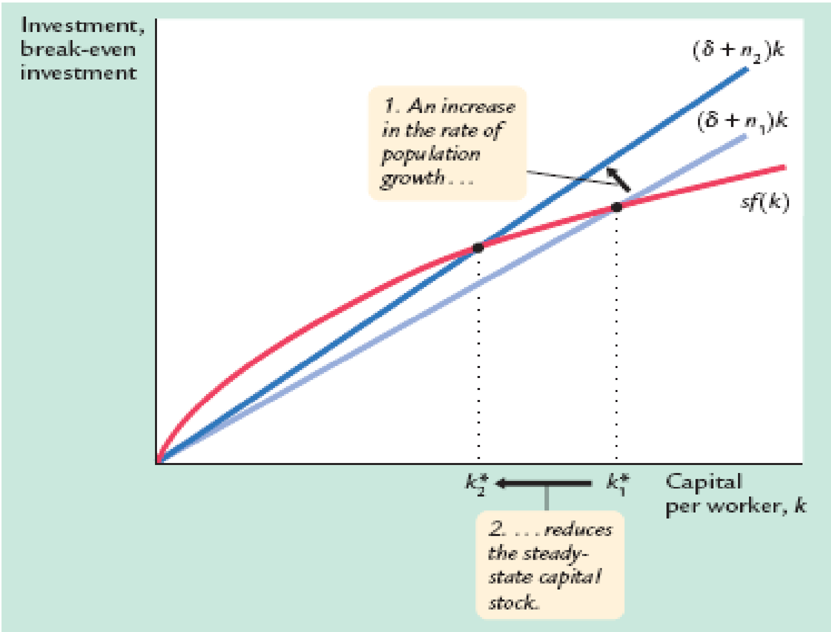


1. partial explanation of sustained economic growth
   - output and capital per worker is constant
   - total output and capital grow at rate n
2. higher population growth => lower level of GDP per capita

- consistent with empirical data
- possible reverse causation

3. new equation for Golden Rule level of capital

$$MPK = f'(k) = \delta + n$$



10.3 STEP 3: Technological growth

10.3.1 Efficiency of labor

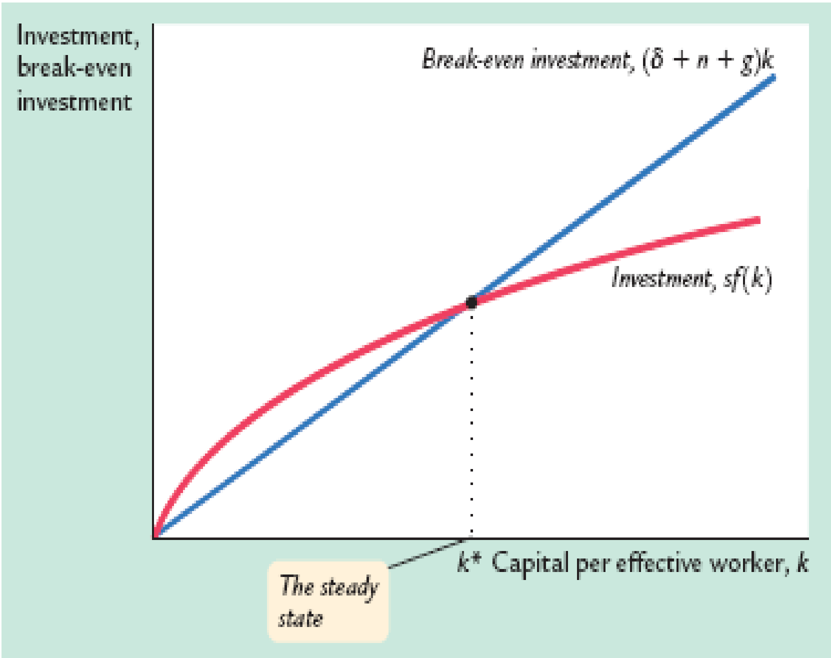
- we rewrite production function: $Y = F(K, L \times E)$
- E - efficiency of labor
- $L \times E$ - number of productive workers
- Assumption: technological growth causes the efficiency E to grow **at constant rate g**
  - labor augmenting technological progress
  - number of effective workers grows approx. at rate $n + g$

10.3.2 Effects of technological growth

- new variables definition: $\hat{y} = \frac{Y}{LE}$ and $\hat{k} = \frac{K}{LE}$

- analogous to population growth

$$\begin{aligned}\Delta\hat{k} &= sf(\hat{k}) - (\delta + n + g)\hat{k} \\ \Delta\hat{k} &= 0 \quad \Leftrightarrow sf(\hat{k}^*) = (\delta + n + \mathbf{g})\hat{k}^*\end{aligned}$$



1. explanation of sustained economic growth
   - total output and capital grow at rate n + g (like effective workers)
   - output and capital per worker is growing at rate g
2. new equation for Golden Rule level of capital

$$MPK = f'(\hat{k}) = \delta + n + g$$

10.4 Endogenous growth models

- Solow-Swan model: s.s. growth rate of $Y/L = \frac{\Delta Y/L}{Y/L} = g$
- g - rate of technological progress, **EXO**GENOUSLY given + assumed to be positive and constant
- want to have growth **ENDO**GENOUS, i.e. we are able to explain it as the outcome of the decisions of agents within the model

Possible solutions:

- **AK models** - abandon diminishing returns to capital (DRC)
  - broad definition of capital (physical + **human**)
    - 1 sector: production of goods - basic model
    - 2 sectors: production of both goods and (human) capital - education
  - **learning-by-doing** + **spillovers** of knowledge
    - individual firms - DRC, aggregate level - CRC/IRC
- **R&D models** Advances in technology level determined by purposeful activity (explicitly model determinants of g)
  - expanding **variety** of products
  - **quality** improvements of existing products

10.4.1 Basic AK model

- production function: $Y = AK \quad => MPK = A > 0$
- abolished diminishing product of capital - A is positive constant => constant return to capital (CRC)
- accumulation of capital:

$$\begin{aligned} \Delta K &= sY - \delta K \\ \frac{\Delta K}{K} &= \frac{\Delta Y}{Y} = sA - \delta \end{aligned}$$

- as long as $sA > \delta$, economy grows forever
- saving decision alone leads to permanent growth

  Is CRC reasonable assumption?

  - NO, if we assume classical definition of capital - stock of plants and equipment.
  - YES, if we consider broad definition of capital - including knowledge (know-how).

10.4.2 2 sector model of Human capital

- 2 sectors: production of output (firms) and production of education or human capital (universities)
- production function - CRS

$$Y = F(K, (1-\mu)EL)$$

- accumulation of capital (physical + human):

$$\begin{aligned} \Delta E &= g(\mu)E \\ \Delta K &= sY - \delta K \end{aligned}$$

  - μ - fraction of labor force in universities
  - g - production of new knowledge, dependent on share of labor force in that sector
- persistent growth attained endogenously - production of knowledge on universities will explain g - otherwise similar to Solow
- two societal decision variables - s and μ