The Likelihood Function

In the proposed model buy and sell orders follow one of three Poisson processes on each day. Whether new information occurs is not directly observable. It is, however, reflected in the data, so more buy orders are expected on good-signal days and more sell orders on bad-signal days. On no-event days, there are no informed traders arriving at the market so fewer trades can be expected. The probabilities of these cases are determined by the probability and type of new information occurring.

In constructing the likelihood function for the whole model, the likelihood of order arrivals on a day of known type is derived first. Consider a good-signal day. The buy orders arrive at rate $\mu + \varepsilon$ as both uninformed and informed traders submit these orders. The sell orders arrive at rate ε as only uninformed traders sell. The distributions of the total number of shares supplied/demanded are independent Poisson distributions. Then, the likelihood of observing the total of A^D buy orders and A^S sell orders submitted on a good-event day, conditional on nonzero trade, is

$$e^{-(\mu+\varepsilon)} \frac{(\mu+\varepsilon)^{A^D}}{A^D\,!} e^{-\varepsilon} \frac{\varepsilon^{A^S}}{A^S\,!}. \tag{4}$$

Similarly, on a bad-event day, this conditional likelihood is

$$e^{-\varepsilon} \frac{\varepsilon^{A^D}}{A^D\,!} e^{-(\mu+\varepsilon)} \frac{(\mu+\varepsilon)^{A^S}}{A^S\,!}, \tag{5}$$

and for a no-event day, it is

$$e^{-\varepsilon} \frac{\varepsilon^{A^D}}{A^D\,!} e^{-\varepsilon} \frac{\varepsilon^{A^S}}{A^S\,!}. \tag{6}$$

The probabilities of a good-event day, bad-event day, and a no-event day are $\alpha(1\text{-}\delta)$, $\alpha\delta$, and $1\text{-}\alpha$, respectively. The overall likelihood function for a given day, therefore, is

$$L((A^D, A^S)\,|\,\theta) = \alpha(1-\delta)e^{-(\mu+\varepsilon)} \frac{(\mu+\varepsilon)^{A^D}}{A^D!} e^{-\varepsilon} \frac{\varepsilon^{A^S}}{A^S!} + \\ + \alpha\delta e^{-\varepsilon} \frac{\varepsilon^{A^D}}{A^D!} e^{-(\mu+\varepsilon)} \frac{(\mu+\varepsilon)^{A^S}}{A^S!} + (1-\alpha)e^{-\varepsilon} \frac{\varepsilon^{A^D}}{A^D!} e^{-\varepsilon} \frac{\varepsilon^{A^S}}{A^S!} \tag{7}$$

The values of A^D and A^S are not directly observable and are modeled here by (1) and (2) (see the article), from which we can write

$$A^D = \lfloor B[1+e^{\beta p}]\rfloor, \tag{8}$$

$$A^S = \lfloor S[1+e^{-\beta p}]\rfloor, \tag{9}$$

where B and S are the number of shares demanded/supplied at the new market price, and $\lfloor x \rfloor$ denotes the nearest integer to x; this rounding is employed as A^D and A^S are assumed to follow a (discrete) Poisson distribution. Substituting from (8) and (9) into (7), we get the likelihood $L((B,S)|\theta)$ in terms of observable variables. As days are independent, the likelihood of observing the data $(B_i, S_i)^I_{i=1}$ over I days is the product of daily likelihood,

$$L = \prod_{i=1}^{I} L(\theta|(B_i, S_i)). \tag{10}$$

This function is then maximized to estimate the parameter vector θ.