Estimates of the Displacement Deadweight Loss from Tax Evasion: A Firm Survey Approach using Data from the Czech Republic

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June, 2006

In the presence of the underground economy taxes give rise to a deadweight loss from displacement of efficient producers by inefficient producers. We consider an economy in which a producer faces two types of costs: the cost of production, and taxes. If the ability to evade taxes is inversely proportional to the ability to keep production costs down, high tax rates may cause inefficient producers to crowd out efficient producers. We estimate this deadweight loss from a survey of one hundred and seven Czech firms taken in 2004. We find that the deadweight loss due to this crowding out can be several times as large as the triangle deadweight losses from discouraged consumption. Our paper provides the first estimates ever of the displacement loss from tax evasion as elaborated by Palda (1998, 2001, 2002).

Keywords: Underground economy; social cost of public funds; taxation.
JEL Classification: H26, H43, K42, O17

Shell Brasil, the Brazilian subsidiary of the Anglo-Dutch oil group, is to sell 285 service stations and six fuel deposits to Agip do Brasil, the local subsidiary of Eni, the Italian group. Shell said the move was part of efforts to concentrate on the most profitable parts of its business in Brazil, but it is understood to have sold the stations, in remote central and western regions of the country, after failing to compete with smaller distributors undercutting bigger companies by evading taxes.

Financial Times of London, February 25, 2000, page 18

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1. Introduction

The present paper seeks to estimate the deadweight loss from the displacement of efficient producers by efficient tax evaders by using surveys of 426 Czech firms taken in 2004 and 2005. Uneven enforcement of taxes creates an uneven playing field on which inefficient producers with a willingness and ability to evade taxes can oust honest, efficient producers from the market. The difference between the costs of the surviving evaders and what costs would have been without evasion is the "displacement" loss from tax evasion. We put the term displacement in quotation marks because it is a term new to economics. Public finance theorists have ignored displacement loss, or have hurried past it on sprinkling a few words of warning. Vito Tanzi (1982, p.88) is one of the few economists to have noticed that "untaxed underground activities will compete with taxed, legal ones and will succeed in attracting resources even though these activities may be less productive...There will of course be significant welfare losses associated with this transfer." Jonathan Kesselman (1997, p.300) made a related point: "If pure tax evasion is concentrated in particular industries or sectors it will raise net returns from activities in those sectors, and this will in turn tend to expand those sectors and their products as against the efficient pattern arising with uniform compliance." To the best of our knowledge, only Usher (1981) has attempted to model displacement loss in his graphical treatment of government subsidies to business.

In a series of works Palda (1998, 1999, 2000, 2001) examined the circumstances under which a displacement loss from uneven enforcement of taxes arises. The amount of loss depends on how closely tied are a firm's productive efficiency and evasive ability. If efficient producers are honest taxpayers and inefficient producers are dishonest, then a rise in taxes creates a climate that favors the survival of tax evaders above the survival of firms with low production costs. Using a simple model of profit maximizing firms he showed how displacement losses from the tax tend to rise as the correlation between honesty and efficiency rises.

Palda relied exclusively on simulations to get an estimate of the deadweight loss of tax evasion. His conclusions did not follow from data. The present paper takes as its basis Palda’s framework and uses a survey of firms to calculate the displacement loss from evasion. We refine Palda’s earlier work to show that displacement loss depends on two fundamental variables: one variable
spans a range of firm efficiency in production and another variable spans a range of firm efficiency in tax evasion. If we know how firms are distributed along these two axes we can venture an educated guess of the displacement loss from tax evasion by weighting this distribution with the costs of surviving firms and subtracting this cost from the hypothetical cost of firms if there were no evasion.

To get an idea of the joint distribution of evasive and productive abilities we asked firms their opinions. We presented each firm with a five-by-five matrix with evasive ability on one axis and productive ability on the other axis. We asked each firm to state what percentage of firms in their industry they believed fell into each of the twenty-five cells of the evasion-productivity matrix. We then gathered the answers of all firms and used these answers to estimate a Lebesgue-type weighting scheme applied to the costs of firms. We then compared these costs to what costs would be in the absence of tax evasion. The difference in the two costs is the displacement loss from tax evasion.

Our work lies somewhere between testing and simulation. We present no hypothesis, and wish to emphasize this point in order to avoid confusion about our goals. We do not suggest that productive and evasive talents are negatively correlated, nor positively correlated, or without correlation. Our goal is to use our survey data as the parameters of a theoretical model and then to use this model to measure displacement loss, much as work in computational general equilibrium models proceed by using real-world information and demand and supply to calibrate models that simulate deadweight losses and changes in output due to changes in government policies.

The plan of the present paper is first to present a theory of displacement deadweight loss from tax evasion and to show how this theory guides the measurement displacement deadweight loss. We follow the theoretical exposition with survey results that allow us to measure displacement loss.

2. Simulation of Costs under Fixed Firm Output

The strategy of the present paper is to estimate what are the costs of firm production under tax evasion and then to compare this cost to what costs would be if firms did not evade taxes. We do
not estimate firms’ cost functions; rather we assume these functions take a specific functional form. Our displacement loss measure will appear at first sight to be heavily dependent upon the form of the cost function we use. We will show that this dependence is not as great as it might seem at first glance.

Displacement loss arises because by virtue of their ability or willingness to evade taxes some firms with high production costs oust from the market firms with low production costs. To model displacement loss we need a model of firm survival in the presence of two characteristics that “code” for survival. The two characteristics we consider are efficiency in production and efficiency or willingness to evade taxes. Two parameter survival models in economics are rare and to the best of our knowledge a two parameter survival model where one parameter is socially productive while the other may be destructive does not exist, so before considering this two-parameter firm survival world it is perhaps useful to go over the case of firm survival when only productive efficiency determines who produces.

2.1 Firms endowed only with productive ability

We start with a model of an industry in which firms are atomistic and produce each an identical and fixed quantity of output. We follow Telser (1978) in working with the extensive margin. In the present paper we do not consider, for reasons of space, more complex cost structures in which firms can vary their outputs in response to changes in market parameters, though we refer readers to Palda (2001) for a theoretical discussion of this issue.

In the extensive margin model, potential producers are infinite in number, and indexed by $A$. $A$ is a productivity parameter that differs from firm to firm. Nature grants each firm its $A$ by drawing from a truncated normal distribution $f(A)$ along the interval $[0,1]$ with mean $\mu_A$ and standard deviation $\sigma_A$. Implicitly we assign the set of producers a measure of one, though we could have assigned them an explicitly weight of say $N$. We choose the normal distribution because of the ease with which it can be used to account for correlations between variables. We need to account for such correlations because the purpose of this paper is to understand how differently correlated levels of evasive and productive ability influence displacements loss. In the empirical section of
the present paper we depart from the normal assumption and gather real-world information on the
distribution of productive talents in order to measure displacement loss.

To keep notation simple we avoid making the measure explicit and assume that firms that find it
profitable to produce are constrained to producing the identical infinitesimal output \( dq \) and that the
sum of these outputs cannot exceed one. The costs a firm perceives depend on its particular
efficiency index \( A \) and taxes \( T \) in the following manner: \( (1+T)dq/A \). Costs, as perceived by the
firm, fall as productivity rises and rise with taxes.

The above cost function may be arrived at by assuming a Cobb-Douglas technology which makes
a firm's output \( q \) a function of its labor \( L \) and capital \( K \) inputs as well as productivity parameters \( A, \alpha, \beta \), and that government levies a tax of \( T \) on each unit of labor or capital hired \( T \) may be thought of as unemployment insurance and a capital tax).

\[
q = AL^\alpha K^\beta 
\]

\( q = AL^\alpha K^\beta \)  \hspace{1cm} (1)

Assume firms share the same \( \alpha \) and \( \beta \) but differ in the parameter \( A \). Firms pay \( w \) for a unit of labor
and \( r \) for a unit of capital. The costs the firm perceives of hiring labor are \( w(1+T) \) and its costs of
capital are \( r(1+T) \). If we set \( \alpha=\beta=.5 \) (i.e. constant returns to scale) and \( w=r=.5 \) then the firm's cost
function can be shown to be

\[
c = \frac{1+T}{A} 
\]

\( c = \frac{1+T}{A} \)  \hspace{1cm} (2)

If we assume each firm which decides to produce is constrained to producing an infinitesimal
quantity \( dq \) then the above becomes the cost function mentioned earlier. We assume that firms
either produce or do not produce depending on whether their costs are lower than price, and that all
are restricted to producing the same amount so that we may be certain that the results on industry
cost falling out of simulations emerge, as taxes change, from some firms taking the place of other
firms. The cost structure we examine in the present section can be considered a special case a Cobb-Douglas world without capacity constraints, where everyone produces, and displacement takes place at the intensive rather than the extensive margin.

A firm's decision to produce depends on whether its costs \((1+T)/A\) are less than the market price \(P\) which is paid in terms a numeraire (produced in some other industry without distortions). Supply \(Q^s\) is the proportion of firms who satisfy this condition:

\[
Q^s = \Pr \left( \frac{1 + T}{A} \leq P \right) \\
= \int_{1+T/P}^{\infty} f(A) dA 
\]

Equation (3) traces a logistic shaped supply function with a maximum output of 1 attained as \(P\) tends to infinity and sweeps even the highest cost firms (those with very low \(A\)) into the market. Figure 1 illustrates the simple logic of the supply curve. We assume a linear demand curve of the form \(Q^d = a + bP\) where \(a > 0\) and \(b < 0\).

Despite the simple layout, there is no analytical solution to equilibrium owing to the indeterminacy of the integral of the normal distribution and due to the fact that price \(P\) appears in the limits of
integration. Finding an equilibrium in this case calls for numerical methods which search over a broad range of possible prices to see which price equates supply and demand. To narrow the range of possible equilibrium prices searched we have used the bisection method described by Judd (1998). Details of the algorithm are available in Maple V format from the authors upon email request. To ensure the area under the normal curve comes to unity we have renormalized the truncated normal between zero and one using the procedure Madalla (1983) prescribes for truncated normal distributions. Figure 2 shows one particular combination of supply and demand curves (for the case where $T=2$).

As taxes rise, average firm production costs should fall. This is the standard public finance result. In a high tax environment only firms with the lowest costs will remain standing. To see this more formally recall that a firm's production costs depend on its productivity parameter $A$ as follows: $1/A$. This differs from total costs $(1+T)/A$ as perceived by the firm. Production costs of the industry are

$$\int_{(1+T)/P}^{1} f(A) \frac{1}{A} dA \quad (4)$$

Simulations (not shown here but available on request and which assume demand parameters ranging from completely inelastic to completely elastic demand, and $\mu_A=.5, \sigma_A=.1$) show the expected result that as taxes rise, the average industry costs (total production costs divided by total output, equation 4 divided by equation 3) fall.
2.2 Firms endowed with productive and evasive abilities

Tax evasion can be introduced into the above model by endowing each firm with a particular propensity to pay a proportion $i$ of its taxes. In this case, a firm perceives its costs to be $(1+iT)/A$. A high $i$ signifies the firm pays most of its taxes. We assume that each firm draws its productivity $A$ and its evasive ability $i$ from a truncated joint normal distribution. As mentioned earlier, the assumption of a bivariate normal distribution is made for the convenience this distribution affords in specifying a correlation between evasive and productive abilities. In the present section we want to give the reader an idea of how displacement loss can be calculated given a particular assumption about the joint distribution of evasive and productive abilities. Armed with the techniques we describe here we can then proceed to measure displacement loss given a joint distribution we cull from a survey of firms.

Both $A$ and $i$ range between zero and one. Their correlation coefficient is $\rho_{A,i}$. The marginal distributions of $A$ and $i$ have means and standard deviations of $\mu_A$, $\mu_i$, $\sigma_A$, $\sigma_i$. Supply falls out of an infinite number of firms, each asking itself whether the sum of its tax and production cost $(1+iT)/A$ is below the market price $P$. The sum of answers to these many questions is summarized by the supply function:

$$Q^*(P) = \int \int f((A,i))dAdi \quad \text{(5)}$$

The term in curly brackets beneath the double integrals is the area of integration that captures the criterion each firm uses in deciding whether to produce. The precise bounds of integration can be gleaned from Figure 3.
If \((1 + T)/P > 1\) then firms with the combinations of productivity parameter \(A\) and evasion parameter \(i\) in the shaded area 1, are those firms who produce. If \((1 + T)/P < 1\) firms in the area 1 and 2 produce. Integrating the density function \(f(A, i)\) over the appropriate bound gives the weight of firms producing. This density function is a truncated bivariate joint distribution of \(A\) and \(i\). The distribution is truncated by the fact that \(A\) and \(i\) are both limited in range between zero and one.

How supply is determined can also be seen in Figure 4, the three dimensional representation of Figure 3. Those firms north of the "wall" described by \((1 + iT)/A\) decide to produce. How many of these firms there are depends on how much of the density function falls on this north side of the wall. The density function We have graphed has \(\rho_{A,i} = 0.9\), which means that there is a strong tendency for firms with high productivity to pay most of their taxes. We have set the means of both \(A\) and \(i\) to be 0.5 and their standard deviations to be 0.1.
Figure 3 indicates that the supply function comes in two parts. If \((1+T)/P<1\), then

\[
Q' = \int_0^{1+T/P} \int_0^1 f(A,i) dAdi \tag{6}
\]

and when \((1+T)/P>1\)

\[
Q' = \int_0^{P-1/T} \int_0^{1+T/P} f(A,i) dAdi \tag{7}
\]
Figure 5 shows the industry supply function for a range of prices and taxes in the cases where $\rho_{A,i}=(-0.9,0.0,0.9)$. The supply curves have the familiar logistic shape, and tend to one as price rises and zero as tax rises, holding all else constant. It is only at higher tax levels that a difference emerges between the three supply curves. At around $T=5$ the curve with the highest supply is the one generated by $\rho_{A,i}=0.9$ the lowest supply curve is the one with $\rho_{A,i}=-0.9$.

Figure 5. Industry supply function. $\rho_{A,i}=(-0.9, 0.0, \text{ and } 0.9)$

How do average (also the same as unit), industry production costs vary with the tax? Recall that in the case of no evasion, unit costs unambiguously fell with the tax. The same might not hold true in the presence of tax evasion if rising taxes allow firms skilled at evasion but unskilled at production to oust from the market firms with great productive skills but poor evasive skills. Intuition suggests that when $A$ and $i$ are positively correlated (high productivity accompanies high tax-paying) average cost may rise with the tax level. Figure 3 indicates that total industry costs in the case of tax evasion again come in two parts. If $(1+T)/P<1$, then
\[
C = \int \int f(A,i) \frac{1}{A} dAdi \quad (8)
\]

and when \((1+T)/P > 1\)

\[
C = \frac{P-1}{T} \int \int f(A,i) \frac{1}{A} dAdi \quad (9)
\]

Unit costs are the above industry production costs divided by total industry output as given by equations 6 and 7.

We now have all the structure we need to see whether average costs rise with the tax and whether the rise is larger for higher levels of \(\rho_{A,i}\). Figure 6 shows equilibrium average industry costs for a range of taxes and correlations. To generate this graph our computer program took a value of \(T\) and \(\rho_{A,i}\). This nailed down some of the parameters in the supply function (see equations 6 and 7). The program then searched for that price which would equalize supply and demand. We then took this equilibrium price and plugged it into the cost equations 9 and 10. This gave us industry costs. To get average industry costs at a particular \((T, \rho_{A,i})\) combination we plugged equilibrium price into the supply function (equations 6 and 7) and divided total industry costs by total industry output. One repeats this process for a wide range of taxes and correlations between productive and evasive abilities.

To isolate the production side we want to minimize interaction between the demand and supply curve. To do so we make demand infinitely inelastic so that changes in taxes have no effect on output but only on the identity of producers. Equilibrium output is fixed at \(Q_{\text{fixed}}^*\). As supply parameters vary, equilibrium price varies. Output stays the same, but the identity of the producers’ changes. If our notions about displacement are correct then as taxes rise, higher cost producers displace lower cost producers when good evaders tend to be inefficient producers \((\rho_{A,i} > 0)\). In
Figure 6 we have set fixed demand at $Q_{\text{fixed}}^* = .5$ (recall that maximum supply is one). Figure 6 shows that for positive levels of correlation between A and i (inefficient producers are good evaders) average industry costs rise with taxes.

Figure 6: Average industry cost under evasion and honesty

Figure 6 shows that when efficiency and evasive ability are uncorrelated, costs can still rise with taxes. This result may strike one as odd. If production costs bear no systematic relation to evasive ability, should not evasive ability simply be a "noise" through which emerges the well-accustomed opposite flux between taxes and average industry costs? Palda uncovered that average costs may rise with the tax in an earlier work (1998) which assumed that evasive ability and productive efficiency were independently and uniformly distributed. He explained there that "even in such a world, some firms with poor productive ability but a verve for tax evasion will manage to survive and displace more productive, but less wily rivals (p.1136)." In that paper he was unable to explore whether the standard result that taxes drive down average industry costs arose the instant the correlation between $A$ and $i$ passed from zero, to less than zero because the assumption of a uniform distribution does not lend itself easily to the construction of different correlations between $A$ and $i$. 


The structure of the present paper allows us to vary this correlation with ease. Remarkably, even as we move into an industry where $\rho_{A,i} < 0$, average industry costs continue to rise with the tax. It seems that even when good producers tend to be good tax evaders, some poor producers will be even better tax evaders and will displace from the market some more efficient competitors. What accords nicely with intuition in Figure 6 is that as $\rho_{A,i}$ falls, so does the average industry cost, except at the highest range of correlations. Average costs fall with correlation for high levels of correlation because as correlation rises, two effects are at work. Inefficient firms (low $A$) are becoming better tax evaders (even lower $i$), but also some very good tax evaders are becoming less efficient firms, and these are falling out of the market and so bringing down unit costs. This second effect may be what is coming to push unit costs down at the high end of $\rho_{A,i}$.

Displacement loss is the difference between actual production costs and minimum possible production costs. We calculate the minimum possible costs of producing an output $Q^*_\text{fixed}$ by solving the following equation for price:

$$Q^*_\text{fixed} = \int_{1/P^*_\text{min}}^{1} f(A) dA$$  \hspace{1cm} (10)

The right hand side of this equation is the supply curve when no one evades the tax and when the tax level is zero. The price $P^*_\text{min}$ we have solved out for here is the price that when plugged into the industry cost equation will sweep under the cost integral the most efficient firms who could produce the amount $Q^*_\text{fixed}$. The price $P^*_\text{min}$ can be plugged into the cost equation when there is no tax evasion to give the minimum cost of producing the level of output $Q^*_\text{fixed}$. This minimum cost is

$$\int_{1/P^*_\text{min}}^{1} f(A) \frac{1}{A} dA$$  \hspace{1cm} (11)
We can set the tax level to zero in the above calculations for convenience because when everyone pays their full tax, there is no displacement loss, so that the cost to the industry of producing a certain quantity with a tax is the same as producing it without a tax.

The minimum average industry cost of producing an output $Q_{fixed}^* = .5$ is 1.7421 units of the numeraire and does not vary either with tax or correlation between productive and evasive talents. We have represented this minimum cost as the flat plane in Figure 6. At all tax and correlation levels the evasion unit cost is above the no evasion unit cost of producing the same quantity. Displacement loss is the difference between the two planes in Figure 6. Figure 6 shows displacement loss as a percentage of the value of industry output.

In the above exposition we kept demand insensitive to price. We did this to isolate the change of the identity of suppliers due to the displacement effects of a tax. In simulations not shown here we varied demand elasticity and found displacement losses to be present. We do not present these results because the purpose of the present paper is, not to carry out simulations of displacement loss under widely varying assumptions about supply and demand parameters, but to set us on the path to measuring displacement loss with real-world data. The data we glean will provide us with information on the distribution of evasive and productive talents. We admit that our measurement of displacement loss will not be free of assumptions because we will have to posit a cost function and a system of taxation in order to come up with displacement loss estimates.

3. Measuring Displacement Loss Using Firm Surveys

The exercise of the present paper is to compare the costs of firms that survive when evasion is possible to the costs of firms that survive when evasion is excluded, by using real-world data. To perform this exercise we must first calculate the costs of existing firms and compare these costs to what they would be if no one evaded. Two sorts of data are needed to measure displacement loss. First we need to know the cost functions of firms. Then we need to know how firms are distributed in efficiency-evasion space. Of these two necessary sets of data, cost functions are perhaps the most difficult to know and we tackle the problem simply by assuming a Cobb-Douglas function
with varying values of its parameters. Economists have at best a fuzzy notion of cost-functions. Their lack of knowledge is a challenge to the profession but not one we propose to take on here.

Our main empirical quest in the present paper is to estimate how firms are distributed in efficiency-evasion space. We represented this space in two dimensions in Figure 3 and in three dimensions in Figure 4. We can get an idea of this distribution by asking firms how they believe other firms are distributed in efficiency-evasion space. Our data on firms come from a combined 2004 and 2005 surveys of 107 and 319 Czech firms, respectively. These firms were drawn from retail (220 firms) and construction (202 firms).

3.1 The Survey

Respondents were of three kinds as summarized in Table 1. The large percentage of respondents who were company owners gives us confidence that our survey will pick up the best available knowledge about the underground economy facing the firms interviewed.

<table>
<thead>
<tr>
<th>Working position</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company owner</td>
<td>305</td>
<td>71.6%</td>
<td>71.6%</td>
</tr>
<tr>
<td>Director of the company/division</td>
<td>17</td>
<td>4.0%</td>
<td>75.6%</td>
</tr>
<tr>
<td>Manager with subordinate departments</td>
<td>104</td>
<td>24.4%</td>
<td>100.0%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>426</td>
<td>100.0%</td>
<td></td>
</tr>
</tbody>
</table>

We presented each firm with a 5X5 matrix. On one axis was a parameter scaled from one to zero indicating a firm’s evasive ability. On the other axis was a parameter scaled from one to zero indicating a firm’s productive efficiency. We asked firms to indicate what percentage of firms in their market fell into each of the twenty-five categories of the matrix. Figure 7 shows the distribution of firm answers weighted to achieve representativeness. The answers of each firm were forced to sum to 100% by a Java™ algorithm. Firms answered the question on the joint distribution of evasive and productive abilities by entering an internet site that forced their answers
to sum to one by not allowing respondents to finish until their answers summed to one. The algorithm designed to achieve such consistency was devised specifically for the purposes of this study by the Czech survey firm Median.

The joint distribution illustrated in Figure 7 can in no way be said to correspond to either the normal or uniform distributions which Palda used in his simulations. Whatever were Palda’s distributional assumptions are of no concern to us. We do not start from any assumption as to the nature of the distribution. We simply seek information on this distribution so that we may use it as a weighting function in our calculations of displacement loss.

A quick glance at Figures 7(b) and 7(c) suggests that firm opinions about the underground economy are similar in the construction and the retail sectors. We carried out a chi-square test of homogeneity of distributions and did not reject (p-value=0.66) at convention levels the hypothesis that the two distributions are the same. The results of the chi-square test led us to pool firm answers from both construction and retail sectors. The polled data can be seen in Figure 7(a) and it is these data we use in calculating displacement loss.
Figure 7: Joint Distribution of Firm Evasive and Productive Abilities (all firms)

A) All firms

Figure 7(a) can be summarized in Table 3. Table 3 is the matrix we use in weighting the cost functions of firms that survive under alternate assumptions about the tax level. Table 3 is similar to Figure 3 which gave a two-dimensional view of the efficiency-evasion space and showed the tax-boundary beyond which some firms survive.

Table 3 differs from Figure 3 in that it presents data, not assumptions, about the distribution of evasive and productive abilities. A potentially confusing aspect of Table 3, but an aspect we preserve in order to maintain consistency with Figure 3, is that on the left of the horizontal axis evasive ability starts out very high and moves to very low going right along the axis. The left-right
The decline of evasive ability in Table 3 can be read from the parameter $i$ in Figure 3. As $i$ rose moving from left to right along the horizontal axis, the ability to evade taxes fell.

**Table 3:** Relationship Between Tax evasion and Production Efficiency

<table>
<thead>
<tr>
<th>PRODUCTIVE EFFICIENCY ($A$)</th>
<th>Very high (0.2)</th>
<th>high (0.4)</th>
<th>medium (0.6)</th>
<th>low (0.8)</th>
<th>Very low (1.0)</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>High efficiency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>very high (1)</td>
<td>6.80</td>
<td>2.37</td>
<td>2.93</td>
<td>1.11</td>
<td>4.48</td>
<td>17.70</td>
</tr>
<tr>
<td>high (0.8)</td>
<td>1.14</td>
<td>3.16</td>
<td>4.38</td>
<td>4.27</td>
<td>5.73</td>
<td>18.68</td>
</tr>
<tr>
<td>Medium (0.6)</td>
<td>1.99</td>
<td>1.40</td>
<td>15.39</td>
<td>4.11</td>
<td>4.82</td>
<td>27.72</td>
</tr>
<tr>
<td>small (0.4)</td>
<td>0.05</td>
<td>2.88</td>
<td>5.04</td>
<td>4.50</td>
<td>2.72</td>
<td>15.20</td>
</tr>
<tr>
<td>very small (0.2)</td>
<td>1.68</td>
<td>3.69</td>
<td>4.57</td>
<td>1.44</td>
<td>9.32</td>
<td>20.70</td>
</tr>
<tr>
<td>Low efficiency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>11.66</td>
<td>13.51</td>
<td>32.32</td>
<td>15.43</td>
<td>27.09</td>
<td>100.00</td>
</tr>
</tbody>
</table>

EVASIVE ABILITY ($i$)

High ability $\Rightarrow$ Low ability

3.2 Measures of Displacement Loss

What exactly must the distribution of talents shown in Table 3 weigh? The answer is that the distribution must weigh some cost function that we postulate. As a first pass we take as our cost function the one given in equation (1). Labor and capital have equal weight as do wages and rents. We note that equation (1) represents the costs a firm perceives. A firm’s true production costs depend only on the productivity parameter and can easily be shown to be $1/A$.

Before we can measure displacement loss we must first calculate market equilibrium. Earlier we calculated equilibrium on the assumption that demand is fixed at some level $Q^*_{\text{fixed}}$. With fixed demand we were forced to calculate thousands of equilibrium prices for each of thousands of different tax levels but the fixity of demand meant that supply was constant, production costs
under no evasion were constant, and that only the identity of producers changed when taxes changed. The smoothness of our assumed normal distribution allowed us to automate calculations of displacement loss. The jagged joint distribution of evasive and productive talents that arose from the firm survey does not lend itself so easily to automation. We discovered that the simplest, yet still tedious, manner in which to calculate displacement loss was to fix a certain tax $T$, and assume demand to be perfectly elastic so that price is fixed at $P$. One then uses Figure 3 as a guide in determining the identity of surviving firms, in other words firms who have productivity parameter $A$ greater than $(1+iT)/P$. For each firm above this line one then calculates its costs $1/A$ and sums these to the costs of all other firms with $A$ above $(1+iT)/P$ to arrive at total industry costs under evasion. We can draw the line $(1+iT)/P$ across Table 3, which, as explained earlier, is the two-dimensional analogue of Figure 3. Suppose that $T=1$ and $P=2$. The line $(1+iT)/P$ intersects the vertical axis of Table 3 at 0.5 and goes precisely through the top right vertex of the table. Drawing on the analogy with Figure 3 we can see that the number of firms who produce is the sum of frequencies in the shaded cells. To be explicit, supply under evasion is:

$$Q'_{\text{evasion}} = 6.80 + 2.37 + 2.93 + 1.11 + 4.48$$
$$+ 1.14 + 3.16 + 4.38$$
$$+ 1.99 = 28.36$$

Costs when firms evade are $(1/A)$ multiplied by the relative frequencies in the colored cells (see equations 8 and 9),

$$C'_{\text{evasion}} = \left(\frac{1}{1}\right)6.8 + \left(\frac{1}{1}\right)2.37 + \left(\frac{1}{1}\right)2.93 + \left(\frac{1}{1}\right)1.11 + \left(\frac{1}{1}\right)4.48$$
$$+ \left(\frac{1}{0.8}\right)1.14 + \left(\frac{1}{0.8}\right)3.16 + \left(\frac{1}{0.8}\right)4.38$$
$$+ \left(\frac{1}{0.6}\right)1.99 = 31.86$$

To measure displacement loss we need to compare the above costs to costs in a world which produces the same quantity but is free of tax evasion. To get these costs we simply add the costs of the most efficient firms that could produce 28.36. This cost comes to 31.03 and can be read off Table 3 by simply taking the most efficient 28.36% of firms and summing their costs. To be sure
the method is clear first we sum the frequencies in the top row of Table 3 and multiply by costs. This gives 17.7*(1/1) because costs are 1/A and A for this row is 1. Then we take the difference between 28.36 and 17.7, which is 10.66, and multiply this by the costs of the firms in the row which is second from the top in Table 3. The costs in the row are 1/A, or 1/0.8=1.25. The total cost of the most efficient way of producing 28.36 are then

\[ C_{\text{no evasion}} = 17.7 \left( \frac{1}{1} \right) + (28.36 - 17.7) \left( \frac{1}{0.8} \right) = 31.03 \]

The displacement loss is the percentage difference between costs under evasion and no evasion which comes to 2.7%.

Table 4 measures the displacement loss for a variety of tax levels. Tax levels are, as before, unit taxes on capital and labour. The table also measures tax revenues and displacement loss per unit of tax dollar raised for each different tax level. This latter measure is analogous to the social opportunity cost of public funds discussed by Usher (1982).

Table 4: Displacement loss and tax revenues for different levels of taxes

<table>
<thead>
<tr>
<th>Tax level</th>
<th>Absolute displacement loss</th>
<th>% Displacement loss</th>
<th>Government tax revenue</th>
<th>Displacement loss per dollar of tax revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.0%</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00</td>
<td>0.0%</td>
<td>11.83</td>
<td>0.00</td>
</tr>
<tr>
<td>1.0</td>
<td>0.83</td>
<td>2.7%</td>
<td>13.95</td>
<td>0.06</td>
</tr>
<tr>
<td>1.5</td>
<td>1.95</td>
<td>9.8%</td>
<td>14.21</td>
<td>0.14</td>
</tr>
<tr>
<td>2.0</td>
<td>0.98</td>
<td>5.3%</td>
<td>11.91</td>
<td>0.08</td>
</tr>
<tr>
<td>2.5</td>
<td>1.61</td>
<td>13.1%</td>
<td>7.34</td>
<td>0.22</td>
</tr>
</tbody>
</table>

That government revenues rise and fall is due to the initially inelastic and then elastic contraction of supply in response to tax increases. This Laffer effect is of peripheral interest to the present paper. The rise, fall, and then rise in percentage displacement loss is due to the irregular disposition of firms on the matrix represented in Table 3 and Figure 7. Displacement loss as a
percentage of the value of output is comparable to Harberger’s triangle calculations for the US. Displacement as a fraction of dollars raised in taxes is comparable to social opportunity cost of government funds presented by Usher (1982).

We admit to an inconsistency between our simulations of displacement loss, and our calculations of displacement loss based on our firm survey. The simulations assumed demand to be fixed. The calculations assumed price to be fixed. In our simulations we wanted to highlight the principle of displacement loss and show that it could arise purely because the identity of producing firms changes in response to tax changes. Once confronted with the task of calculating displacement loss based on our survey we wanted to maintain simplicity in the presentation of the calculations. Our artisanal method of calculating displacement loss relies on drawing a line across our Table 3 matrix and this line shifts in a complicated manner when prices change. To buy simplicity we decided to assume away shifting prices by assuming an infinite elasticity of demand. It is tedious but simple to see how to calculated displacement loss under all assumptions about the elasticity of demand, but we reserve this exercise for future research which seeks to extend, rather than highlight the theory and measurement of displacement loss.

4. Weaknesses and Contributions of the Analysis

We lack the audacity to add a concluding section to research which can be barely said to have begun. After more than a dozen pages of simulation and data analysis we have produced a single table of displacement measures based on one of many possible cost functions. We are the first to admit that much needs to be done to make this figure credible. The theory of displacement loss demands some simple theorizing, but to measure displacement loss demands real world data that are difficult to measure. We estimated the distribution of evasive and productive talents from surveys of firm but we assumed Cobb-Douglas cost functions with parameters chosen for analytical convenience. We also assumed a market price and tax structure that seem simple. Of what use can the results of our paper be to researchers or policy-makers?

The theory and measurement of displacement loss are so new as to be nearly non-existent. This is not a boast, but a lament. We have bent our efforts to developing the methods that, with
refinement, point to how displacement loss may be measured. Future research must build a model of tax evasion based on tax systems which resemble those of the real world. A more complex tax system will produce more complex cost equations of production under evasion than those we presented in equations (8) and (9). Some idea of demand functions in the market studied is needed to allow researchers to calculate production costs without having the arbitrarily fix price by assuming infinitely elastic demand. Using realistic assumptions about tax systems will complicate the form of the cost function and will no doubt generate a decision wall more complicated than that shown in Figure 3 and Table 3.

Future research must also address two further questions:

1) Why not model the firm’s decision to evade? In the present paper we simply assumed a continuum of firms having what we casually referred to as “different evasive abilities.” We summarized these evasive abilities in a single random variable $i$ and by so doing ignored all modeling on the decision to evade that has appeared since Allingham and Sandmo’s (1972) pioneering work. It might appear that a model of a firm’s decision to evade would endogenize the $i$ parameter and lead to a deterministic relation between a firm’s productive ability, its aversion to risk, and its innate talents for hiding from the revenue service and by so doing would do away with the need to estimate distributions of evasive and productive abilities. Such a model would no doubt endogenize the evasive decision of each firm but firms would still differ in their aversion to risk, their raw abilities to evade, and their productive abilities. An equilibrium model based on individual firm decision functions could not avoid the sort of statistical treatment of industry evasion which we have provided. An equilibrium model based on individual firm choices would differ from our simple equilibrium model only in the complexity of its decision wall which would extend into a three dimensional space of random variables (evasive ability, risk aversion, and productive ability). We did not provide such a model at this stage in part for reasons of exposition and in part because the modeling and simulation of such a model is a long-term and collective enterprise which we hope future researchers may take up.

2) The astute reader will notice a quandary in our formulation of the joint distribution function of firms. We asked existing firms to comment on their view of the market as it is. Our theory
postulates a distribution over existing and potential firms. Nothing says that the existing distribution is the same as the potential distribution. Our analysis assumes both distributions to be the same. Such an assumption is questionable and must be seen as casting a shadow over the validity of our results.

3) Some readers will not like the assumption that all firms produce $dq$. One main channel by which the underground economy might lower efficiency might be by inducing firms to remain suboptimally small in a world of increasing returns to scale so as to avoid coming to the attention of the tax authorities. While it would be possible to model this particular deadweight cost of tax evasion by using a non-linear cost of avoidance that does not rely on the ad hoc assumptions about correlations, such modeling would be addressing not displacement loss, but the loss from staying suboptimally small. While we did not model the above case, we did model displacement loss when firms can vary their outputs and evasive and productive talents follow a normal distribution. In calculations not shown here (but available on request) we found the theoretical estimates of displacement loss to be very similar to the case of fixed firm output.

The above shortcomings of our work are more technical than conceptual. We believe it is important to deepen research into this area because tax evasion is a growing phenomenon whose social costs, with the exceptions of Alm (1985) and Usher’s (1986) works, have not been sufficiently documented. The model of evasion we present may also be of greater interest than to public finance. We model survival in an environment where two factors, one productive to society, the other unproductive, determine who survives. The techniques developed here can be used to calculate the social costs of corruption, or the social costs of evading any sort of rule in a world where individuals cannot separate their productive and evasive abilities.
BIBLIOGRAPHY


