Economics II: Micro — Fall 2009
Exercise session 5 — VŠE

1 Review

Optimal production: Independent of the level of market concentration, optimal level of production is where $MR = MC$.

Monopoly: Market with a sole supplier is Monopolistic.

Oligopoly: Market with small number of firms producing one good. Typically characterised by the absence of a natural monopoly situation.

Antoine Augustin Cournot Model of Oligopoly This is a model where each firm competes by setting a quantity which is a best response to the quantity decisions of other firms in the market.

Cournot Equilibrium: This is the equilibrium in the Cournot model and is obtained as the intersection of the reaction functions of oligopolistic firms in the market.

Heinrich von Stackelberg Model of Oligopoly: Firms still choose quantities, however, as opposed to the Cournot model, there is a leader and followers in this model: The leader chooses and action first, and the followers best-respond to it.

Joseph Bertrand Model of Oligopoly: This is a model where firms compete via prices.

Francis Ysidro Edgeworth Model of Oligopoly: This is a model where firms compete via prices while being capacity constrained (are not able to supply entire market).

Collusion or Cartels: Firms behave as if they are a monopoly firm in the market, in order to maximise total profits. The firms then share among themselves the maximised profit.

Perfectly Competitive Market: A market consisting of 'price taking firms' only.

(Short-run) Supply curve of a firm in a perfectly competitive market is that portion of the marginal cost which lies above the minimum of the average variable cost curve.

Competitive equilibrium is determined by the intersection of the market supply curve and the market demand curve.

2 Problems

Problem 1 *Suppose a monopolist has* $TC = 100 + 10Q + 2Q^2$*, and the demand curve it faces is* $p = 90 - 2Q$*. What will be the price, quantity, and profit for this firm?*

Solution: First, determine $MR = 90 - 4Q$. Second, $MC = 10 + 4Q$. Setting $MR = MC$ yields $90 - 4Q = 10 + 4Q$. Rearranging yields $80 = 8Q$ or $Q = 10$. Price equals $p = 90 - 2(10) = 70$. $TR = 70 \cdot 10 = 700$. Total cost equals $100 + 10(10) + 2(102) = 400$. Profit equals $700 - 400 = 300$.

Problem 2 *It is a conventional practice among apparel retailers to set the retail price of clothing at twice the cost paid to the manufacturer. For example, if the retailer pays $7 for a pair of jeans, the jeans will retail for $14. What must the price elasticity of demand be for this practice to be profit maximizing?*

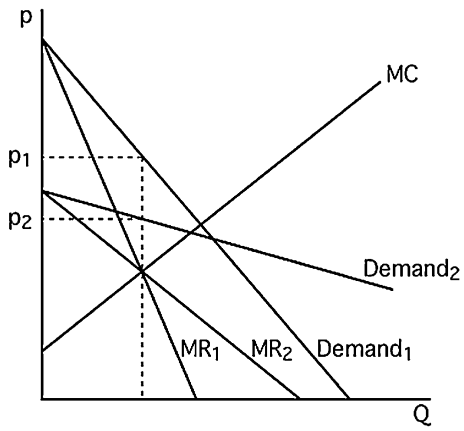
Solution: Since price is twice marginal cost, the *Lerner Index* is 1/2. This practice is profit maximizing if the price elasticity of demand is -2.

Problem 3 *For profit-maximizing monopolies, explain why the boundaries on the Lerner Index are 0 and 1.*

Solution: The Lerner Index equals $\frac{p-MC}{p}$. Because marginal cost is greater than or equal to zero and the optimal price is greater than or equal to the marginal cost, then $0 \leqslant p - MC \leqslant p$. So, the Lerner Index ranges from 0 to 1 for a profit-maximizing firm. As price gets higher, the Lerner Index approaches 1. As price gets lower, the index approaches zero.

Problem 4 *Draw a graph that shows a shift in the demand curve that causes the optimal monopoly price to change, while the quantity remains the same.*

Solution:



Problem 5 *Assume that there are two firms, each producing with a constant* $MC = 2$. *Assume that the demand function for the product is defined as follows:* $p = 100 - 0.5\,(q_1 + q_2)$. *If firm 2 sets a quantity of* $q_2 = 100$*, what's the best response quantity for firm 1.*

Solution: Residual demand curve facing firm 1 is $p = 100 - 0.5\,(q_1 + 100) = 50 - 0.5q_1$. If the marginal cost of production is constant at 2, then the firm should set a quantity at which the marginal revenue associated with the residual demand curve is equal to the MC. The marginal revenue curve associated with this demand curve has the form $MR = 50 - q_1$, since it has a slope twice the steepness of the linear demand curve. Setting $MR = MC$ means setting $50 - q_1 = 2 \Longrightarrow q_1 = 48$ is the best resopnse.

Problem 6 *Assume the market demand function defined as* $p = 100 - 0.5\,(q_1 + q_2)$*, and assume further a constant* $MC = 0$. *Find the best response functions.*

Solution: The residual demand function firm 2 is $p = (100 - 0.5q_1) - 0.5q_2$. $MR = (100 - 0.5q_1) - q_2$. Using condition $MC = 0$ we get $q_2 = 100 - 0.5q_1$. Case of firm one is absolutely symmetric, thus: $q_1 = 100 - 0.5q_2$.

Problem 7 *Assume two firms competing in a market and that these firms have the following reaction functions:*

$$\begin{aligned} q_1 &= 50 - \frac{q_2}{2} \\ q_2 &= 50 - \frac{q_1}{2} \end{aligned}$$

1. *For what quantity produced by firm 2 would firm 1 prefer to shut down and produce nothing?*

2. *What quantity would firm 1 produce if firm 2 never existed?*

3. *Verify that* $q_1 = 33,(3)$ *and* $q_2 = 33,(3)$ *is a Nash equilibrium.*

Solution:

1. Solving for $q_1 = 0 : 50 - \frac{1}{2}q_2 = 0 \Longrightarrow q_2 = 100$ and firm 1 would shut down.

2. If firm 2 never existed, then $q_2 = 0$ and the best output would be $q_1 = 50$.

3. Nash equilibrium $q_1 = q_2 = 33,(3)$ can be verified by seeing if those quantities constitute a best response for each firm:

$$33,(3) = 50 - \frac{1}{2} \cdot 33,(3)$$

If firm 1 is choosing 33,(3), firm 2 will also want to choose 33,(3), and vice versa. That is the definition of an equilibrium.

Problem 8 *Consider two oligopolists producing an identical product with identical cost functions* $C = q^2$ *who face a demand curve* $p = 1 - (q_1 + q_2)$

1. *What is the Cournot equilibrium in this market?*

2. *If firm 1 can choose its output first, what will the outcome be?*

3. *Suppose the two firms choose price instead of quantity. What will the market outcome be?*

Solution:

1. Denote firm 1's output by q_1 and firm 2's output by q_2. Firm 1's total revenue function is $R = \ (1 - (q_1 + q_2))\, q_1 = q_1 - q_1^2 + q_2 q_1$. The associated marginal revenue is $MR_1 = 1 - q_2 - 2q_1$ and the merginal cost is $MC = 2q_1$. Using $MR = MC$

$$q_1 = \frac{1 - q_2}{4}$$

The reaction function of firm 2 is symmetric to this one. Substituting one reaction function into the other we see that

$$4q_2 = 1 - q_1 = \frac{3 + q_2}{4}$$

that is: $15q_2 = 3$. Therefore $q_1 = q_2 = \frac{1}{5}, p = \frac{3}{5}$.

2. If firm 1 produces first, it will take firm 2's reaction to its own output into consideration. Therefore, we can rewrite the demand curve faced by firm 1 as:
$$p = 1 - q_1 - q_2 = 1 - q_1 - \frac{1-q_1}{4} = \frac{3-3q_1}{4}$$
and thus
$$MR = \frac{3}{4} - \frac{3q_1}{2}$$
Using $MR = MC$:
$$2q_1 = \frac{3}{4} - \frac{3q_1}{2}$$
And finally, $q_1 = \frac{3}{14}, q_2 = \frac{11}{56}, p = \frac{33}{56}$.

3. As they engage in price wars the equilibrium is when both firms set a price equal to marginal cost. Since the consumers do not discriminate between the two firms, they'll split the market: $q_1 = q_2 = q$. Thus the demand curve is $p = 1 - 2q$. Using $p = MC : q = \frac{1}{4}, p = \frac{1}{2}$.

Problem 9 *There are 3 perfectly competitive firms in an industry. The firms have no fixed costs. Their marginal costcurves are given by:*
$$\begin{aligned} MC_A &= 21 + 3q \\ MC_B &= 30 + 2q \\ MC_C &= 40 + q \end{aligned}$$
Determine the market supply as a function of price.

Solution: From the optimising condition $p = MC$ we can conclude that when $p < 21$, no firm will produce (otherwise q should be negative), in case $p \in [21, 30)$ only firm A will produce, supplying according to its marginal cost curve: $p = 21 + 3q$ and thus $q = \frac{p-21}{3}$. On the price range $p \in [30, 40)$ firms A and B will produce, each according to their marginal cost curve, and the market supply would be the horizontal summation of the individual supplies: Thus $q_A = \frac{p-21}{3}$ and $q_B = \frac{p-30}{2} : Q = q_A + q_B = \frac{5}{6}p - 22$. And when the price is above 40 all three firms produce. Thus the market supply curve is:
$$Q = \begin{cases} 0 & p < 21 \\ \frac{p-21}{3} & p \in [21, 30) \\ \frac{5}{6}p - 22 & p \in [30, 40) \\ \frac{11}{6}p - 62 & p \geqslant 40 \end{cases}$$

Problem 10 *Coconut Unlimited produces cocnuts using only one variable input, labor. It's a perfectly competitve firm. Suppose that the fixed cost associated with production is $F = 50$. Let y denote the total number of coconuts produced. The total variable costs and marginal cost associated with the production of y units of output is*

$$\begin{aligned} MC &= 3y^2 - 16y + 21 \\ TVC &= y^3 - 8y^2 + 21y \end{aligned}$$

(For your info, the firm has U-shaoed average cost curves.)

1. *Determine how much the firm will supply (in the short run) as a function of the price level (i.e. determine the supply function of this firm).*
2. *What is the price below which the firm does not supply any output in the short-run?*
3. *Now suppose that the current market price is $p = 21$.*
   (a) *How much will this firm choose to produce?*
   (b) *How much profit it will make?*

Solution: THe supply curve is the portion of the MC curve which lies above the AVC curve. The lowest point on the AVC curve occurs where the AVC intersects the MC curve. The $AVC = \frac{TVC}{y} = y^2 - 8y + 21$.Intersection is where $AVC = MC : y^2 - 8y + 21 = 3y^2 - 16y + 21 \iff y(2y - 8) = 0$ so $y = 4$. At $y = 4, MC = AVC = 5$.

1. The inverse supply function is: $p = 3y^2 - 16y + 21$ when $p > 5$; and $y = 0$ if $p \leq 5$
2. Already answered above, $p = 5$.
   (a) From inverse supply function: $3y^2 - 16y = 0 : y = \frac{16}{3}$
   (b) $\pi = 21y - TC = 21y - 50 - \left(y^3 - 8y^2 + 21y\right) = -50 - \left(\left(\frac{16}{3}\right)^3 - 8\left(\frac{16}{3}\right)^2\right) = \frac{698}{27}$