

- 1) At low levels of production, marginal productivity of labor increases as labor increases. At high levels of production, marginal productivity of labor decreases as labor increases. Are these two statements contradictory? Explain.

Answer: No. At low levels of production, marginal productivity of labor can increase due to specialization. Eventually, however, diminishing marginal productivity will commence.

- 2) Explain the difference between diminishing returns to labor and diminishing marginal returns to labor.

Answer: Diminishing returns to labor means that an increase in the number of labor units will decrease the amount of output. Diminishing marginal returns means that additional units of labor increase output at a decreasing rate.

- 3) Consider the following short-run production function: $q = 5L^2 - 1/3 L^3$. At what level of L do diminishing marginal returns begin? At what level of L do diminishing returns begin?

Answer: $MP = 10L - L^2$. Marginal product peaks when $L = 5$ and equals zero when $L = 10$. Thus, diminishing marginal returns begin when $L = 5$, and diminishing returns begin when $L = 10$.

- 4) Suppose the production function for T-shirts can be represented as $q = L^{0.25} K^{0.75}$. Show that the marginal productivity of labor diminishes in the short run.

Answer: In the short run, $MPL = 0.25 * (q/L)$. The change in MP with respect to L equals $d(MP)/dL = -0.25 * q/L^2$. Thus, for all levels of labor hired, MP falls as L increases.

- 5) Suppose the production function for T-shirts can be represented as $q = L^{0.25} K^{0.75}$. When $K = 1$ and $q = 2$, what is the slope of the isoquant? If there is insufficient information to answer the question, describe what information is needed.

Answer: Since the slope of the isoquant is $-K/3L$, one needs to know the level of L. Since $K = 1$ and $q = 2$, L must equal 16. Thus, the slope of the isoquant at this point is $-(1/48)$.

- 6) Suppose the production function for T-shirts can be represented as $q = L^{0.25} K^{0.75}$. Show that the production function has constant returns to scale.

Answer: When $L = L_1$ and $K = K_1$, $q = q_1$. Let $L_2 = \mu L_1$ and $K_2 = \mu K_1$ so that both L and K have increased by the same proportion. Then, $q_2 = (\mu L_1)^{0.25} (\mu K_1)^{0.75} = \mu^{0.25+0.75} L_1^{0.25} K_1^{0.75} = \mu q_1$. Thus, for any μ , output changes by the same proportion that the inputs changed.

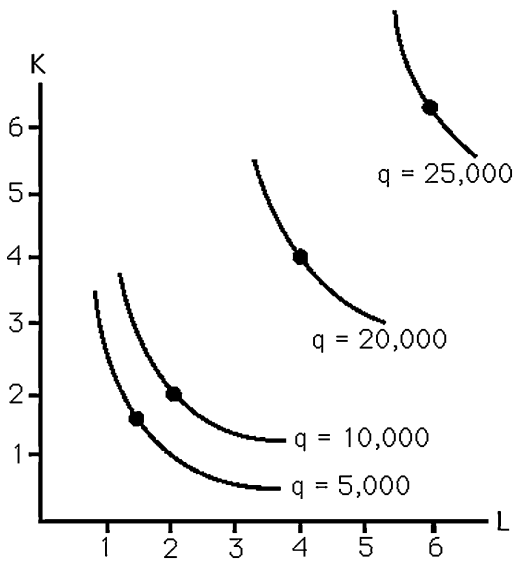


Figure 1

- 7) Figure 1 shows the isoquants for the production of steel. In which regions of production are there increasing, decreasing, and constant returns to scale?

Answer: When output is less than 10,000 tons, there are increasing returns to scale. Between 10,000 and 20,000 tons, there are constant returns to scale. For output greater than 20,000 tons, there are decreasing returns to scale.

- 8) Suppose firms A and B each make T-shirts. Firm A's production function is $q = L^{0.5} K^{0.5}$. Firm B's production function is $q = 1.2 * L^{0.5} K^{0.5}$. If the two firms each hire the same amounts of capital and labor, compare the two firms in terms of AP_L and MP_L .

Answer: Firm B's AP_L and MP_L will equal 1.2 times the AP_L and MP_L of firm A.

- 9) You have two career options. You can work for someone else for \$50,000 a year. You can run your own business, your annual revenue is \$100,000, and explicit costs are \$40,000 annually. Explain which career option a profit-maximizer would select and why.

Answer: A profit-maximizer will run their own business. The business owner will receive $\$100,000 - \$40,000 = \$60,000$. The opportunity cost is only \$50,000.

L	Q
2	10
5	20
9	30
14	40
20	50

Figure 2

- 10) A firm's production function for pretzels is shown in Figure 2. If the firm's fixed cost equals \$100 per time period and the wage rate equals \$1 per unit of labor per time period, calculate the firm's MC, AVC, and AC schedules. Do these cost functions follow the general rules concerning the relationships between MC, AVC and AC?

Answer:

q	AFC	AVC	AC	MC
10	10.00	0.20	10.20	—
20	5.00	0.25	5.25	0.30
30	3.30	0.30	3.60	0.40
40	2.50	0.35	2.85	0.50
50	2.00	0.40	2.40	0.60

MC > AVC, AVC is rising.

MC < AC, AC is falling.

- 11) Explain why the marginal cost curve intersects a U-shaped average cost curve at its minimum point.

Answer: At low quantities, the average cost curve declines as the quantity increases. The marginal cost is below the average cost. The marginal cost represents the cost of an additional unit of production. Thus, as the marginal cost curve declines, this pulls the average cost down from its previous level. Then, the marginal cost curve will begin to rise. However, the marginal cost is still below the average cost, which will continue to lower the average cost. When the two costs are equal the marginal cost will leave the average cost unchanged. Then, the marginal cost will be above the average cost so it will start to pull up the average cost. Thus, the marginal cost curve will intersect the average cost curve at its minimum point.

- 12) What are the functions for MC and AC if $TC = 100q + 100q^2$? Are the returns to scale increasing, decreasing, or constant?

Answer: $MC = 100 + 200q$

$AC = 100 + 100q$

Since AC increases with an increase in output, there are decreasing returns to scale.

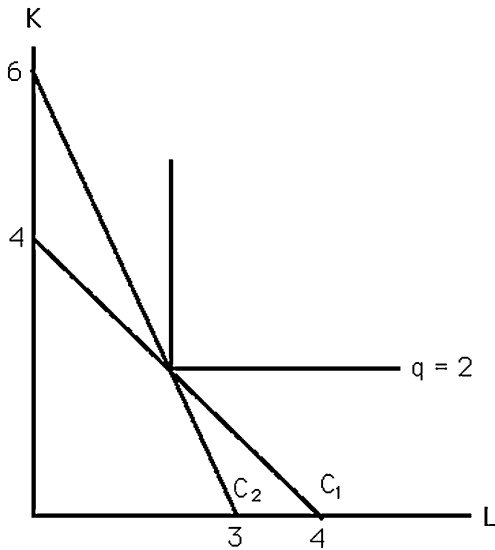


Figure 7.5

- 13) To dig a trench, each worker needs a shovel. Workers can use only one shovel at a time. Workers without shovels do nothing, and shovels cannot operate on their own. Graphically determine the number of shovels and workers used by a firm to dig 2 trenches when:

- (a) $w = 10$ and $r = 10$
 (b) $w = 10$ and $r = 5$

Answer: See Figure 7.5. Because the production process requires fixed proportions of K and L, the firm cannot change input mix when the relative factor costs change.

- 14) “If the wage rate paid to one form of labor is twice the cost of another form of labor, the first type of labor must be twice as productive.” Comment.

Answer: This is true. Firms minimize cost by setting the ratio of marginal productivity per unit cost equally across all inputs. If one form of labor is twice as expensive as another, the firm will want the MP of the first type of labor to be twice that of the second.

- 15) Suppose capital and labor are perfect substitutes resulting in a production function of $q = K + L$. That is, the isoquants are straight lines with a slope of -1 . Derive the long-run total cost function $TC = C(q)$ when the wage rate is w and the rental rate on capital is r .

Answer: The firm will always hire the input that is cheapest. So, the function is

If $w < r$, $TC = wq$.

If $w > r$, $TC = rq$.

If $w = r$, either equation works.

- 16) Explain: Were it not for the law of diminishing marginal returns, we could grow the world’s food supply from a flowerpot.

Answer: Given the current state of technology, the law of diminishing marginal returns limits the amount of additional output yielded from additional labor working the flowerpot. Revoking the “law” would allow output to increase at a non-decreasing rate when more labor is applied to the flowerpot.

- 17) Explain: If marginal productivity is decreasing as more labor is hired, then average productivity must be decreasing as well.

Answer: The statement is false. The change in average productivity is not determined by the change in marginal productivity. Average productivity can be increasing even when marginal productivity is decreasing. Average productivity can only be decreasing when marginal productivity is below average productivity.

- 18) Explain: Cobb-Douglas production functions can never possess varying returns to scale.

Answer: The Cobb-Douglas function takes the form $q = L^\alpha K^\beta$, where the exponents are constant parameters. The returns to scale equals $\alpha + \beta$ which is constant for all q .