1 Consumer Theory Applications

1.1 Problems

Problem 1 *Charles' utility function is $u(x,y) = xy$. Anne's utility function is $u(x,y) = 100xy$. Diana's utility function is $u(x,y) = -xy$. Elizabeth's utility function is $u(x,y) = -1/(xy+1)$. Fergie's utility function is $u(x,y) = xy - 10,000$. Margaret's utility function is $u(x,y) = x/y$. Philip's utility function is $u(x,y) = x(y+1)$. Mike's utility function is $u(x,y) = lnx + lny$. Which of these persons have the same preferences as Charles? Justify your reasoning.*

Answer: Two utility functions represent the same preferences, if there exist a positive monotonic transformation between them. In this case, Anne has the same preferences as Charles, because multiplication by positive constant is a positive monotonic transformation. Diana does not, because the constant used is negative. The transformation $-1/(t+1)$ is positive monotonic, so Elisabeth also has the same preferences as Charles. Subtracting or adding any number is positive monotonic transformation, so Fergie has the same preferences, too. On the other hand, neither Margaret nor Philip has the same preference. Finally, Mike has the same preferences, since the transformation is log, which is positive monotonic.

Problem 2 *William earns 5 pounds per hour. He has 100 hours per week which he can use for either labour or leisure. The government institutes a plan in which each worker receives a £100 grant from the government, but has to pay 50 per cent of his or her labour income in taxes. If his utility function is $U(c,r) = cr$ where c is pounds worth of consumption of goods and r is hours of leisure per week, how many hours per week will William choose to work?*

Answer: William's problem is: $\max cr$, $s.t.$ $100 - (r+l) = 0$ (time constraint); and $c - [m + (1-\alpha)wl] = 0$ (budget constraint), where l is labour in hours, m is the non-labour income, w is the wage rate, and α is the tax rate. The Lagrange function will be: $\mathcal{L}(c,r,\lambda) = cr + \lambda(c - m - (1-\alpha)(100-r)w)$, and the solution is:

$$\frac{m}{2w(1-\alpha)} + \frac{100}{2} = r$$

Using the values for m, w, α and the time constraint we'll have $l = 30$.

Problem 3 *Aristotle earns 5 euros per hour. He has 110 hours per week available for either labour or leisure. In the old days he paid no taxes and received nothing from the government. Now he gets a €200 payment per week from the government and he must pay half of his labour income in taxes. (His before-tax wages are the same as they were before; and he has no other source of income than wages and payments from the government.) He notices that with the government payment and his taxes, he can exactly afford the combination of leisure and consumption goods that he used to choose. How many hours per week did he work in the old days?*

Answer: The budget constraint is $c = m + (1 - \alpha)wl$. $m_1 = 200, \alpha_1 = 0.5$ while $m_0 = 0, \alpha_0 = 0$. As the consumption and leisure are the same, the problem can be rewritten as:

$$m_0 + (1 - \alpha_0)wl = m_1 + (1 - \alpha_1)wl$$

which gives: $l = \frac{200}{5 \cdot 0.5} = 80$.

Problem 4 *Schoschana receives a lump sum child-support payment of €150 per week. She has 80 hours a week to divide between labour and leisure. She earns €5 an hour. The first €150 per week of her labour income is untaxed, but all labour income that she earns above €150 is taxed at the rate 30 per cent. Graph her budget line with leisure on the horizontal axis and consumption on the vertical axis.*

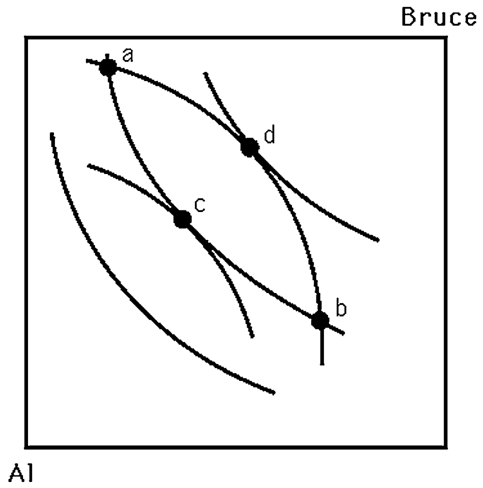
Answer: It has a kink where her income is 300 and her leisure is 50.

2 Exchange and Edgeworth Box

2.1 Problems

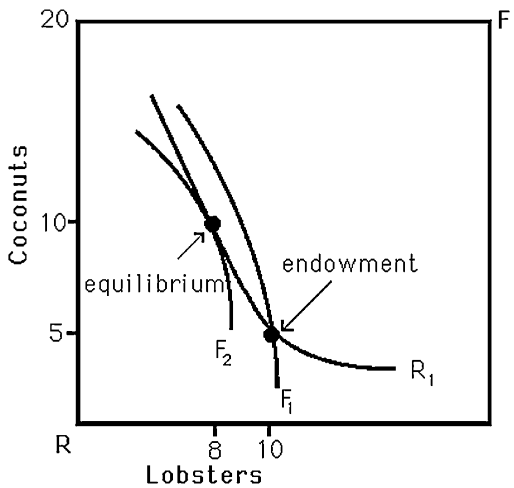
Problem 5 *Figure 1 depicts the Edgeworth box for two consumers, Al and Bruce. Explain why point "a" cannot be a competitive equilibrium.*

Answer: Point "a" cannot be a competitive equilibrium because at point "a", Al and Bruce each have a different marginal rate of substitution. Since they both face the same prices in a competitive market, at least one of them is not in equilibrium.



Problem 6 *Robinson starts out with 10 lobsters and 5 coconuts. Friday starts out with 10 lobsters and 5 coconuts. After trading, Robinson ends up with 8 lobsters and 10 coconuts. Robinson feels neither better nor worse off than when he started but cannot get Friday to agree to any more trades. Friday feels better off than when he started. Draw the Edgeworth box consistent with this story.*

Answer: Figure 2



Problem 7 *Consider a society consisting of just a farmer and a tailor. The farmer has 10 units of food but no clothing. The tailor has 20 units of clothing but no food. Suppose each has the utility function $U = F \cdot C$. Derive the contract curve.*

Answer: Let F and C denote the farmer's final allotment of food and clothing. Setting MRS's equal yields $\frac{F}{C} = \frac{10-F}{20-C}$ or $\frac{F}{C} = \frac{1}{2}$. The contract curve is a ray from the origin with a slope of 1/2 (assuming food is on the vertical axis).

Problem 8 *Continue with the Problem 7. The price of clothing is always \$1. If the price of food is \$3, does a competitive equilibrium exist? If not, what will happen to the price of food?*

Answer: The farmer will have \$30. His budget line is $30 = 3F + C$. His budget line is tangent to an indifference curve when $C/F = 3$. Substituting yields $30 = 6F$ or $F = 5$ and $C = 15$. The tailor's budget line is $20 = 3F + C$. His budget line is tangent to an indifference curve when $C/F = 3$. Substituting yields $20 = 6F$ or $F = 3.33$ and $C = 10$. In total 8.33 units of food and 25 units of clothing are demanded. There is excess demand for clothing and excess supply of food. The price of food will fall. When the price of food is \$2, both markets clear.

2.2 True or False

Claim 1 *Any point on the contract curve is Pareto efficient regardless of the initial endowment.*

TRUE: Efficiency requires that no gains from trade are possible. This true along the contract curve.

Claim 2 *If two school children willingly trade their lunches with one another, then at least one of them preferred the other's lunch to his own.*

TRUE: Either one or both preferred the other's. If only one preferred the other's, then the other must have been indifferent between the two lunches.

Claim 3 *When two people trade their initial endowments to a point on the contract curve, only the level of the endowments will determine the new allocation.*

FALSE: The respective bargaining abilities will also play a role in determining the final allocation.

Claim 4 *Having different marginal rates of substitution is necessary for trade to occur.*

TRUE: The marginal rate of substitution is the rate at which a person is willing to trade one good for another. If these rates are not equal for all people, trade can occur. With different marginal rates of substitution at least one person gains by trading. When the marginal rates of substitution are the same for everyone, everyone is willing to trade goods at the same rate, so no one can gain by trading.

Claim 5 *Robin Hood's practice of stealing from the rich to give to the poor is Pareto efficient.*

FALSE: When Robin steals from the rich, those people are made worse off. This cannot be Pareto efficient.