Economics II: Micro — Fall 2009
Exercise session 2 — VŠE

1 Deriving demand function

Assume that consumer's utility function is of Cobb-Douglass form:

$$U(x,y) = x^{\alpha} y^{\beta} \tag{1}$$

To solve the consumer's optimisation problem it is necessary to maximise (1) subject to her budget constraint:

$$p_x \cdot x + p_y \cdot y \leq m \tag{2}$$

To solve the problem Lagrange Theorem will be used to rewrite the constrained optimisation problem into a non-constrained form:

$$\max \mathcal{L}(x,y,\lambda) = x^{\alpha} y^{\beta} + \lambda (m - p_x x - p_y y) \tag{3}$$

The first order (necessary) conditions will result in:

$$\alpha x^{\alpha-1} y^{\beta} = \lambda p_x \tag{4}$$

$$\beta x^{\alpha} y^{\beta-1} = \lambda p_y \tag{5}$$

$$m = p_x \cdot x + p_y \cdot y \tag{6}$$

Combining (4) and (5) will result in:

$$\alpha p_y y = \beta p_x x \tag{7}$$

which, combined with (6) will give:

$$\alpha (m - p_x x) = \beta p_x x \tag{8}$$

and finally, after some rearrangements becomes:

$$x = \frac{\alpha}{\alpha+\beta} \frac{m}{p_x} \tag{9}$$

This is the demand function for the good x. When the price of the good x, p_x, is fixed then (9) is the Engel curve for the good x. It is easy to see that this was an example of homothetic preferences: It is enough to check the income elasticity to be equal to unity:

$$\varepsilon_m^x = \frac{m}{x}\frac{\partial x}{\partial m} = \frac{\cancel{m}}{\frac{\alpha \cancel{m}}{(\alpha+\beta)p}} \frac{\partial}{\partial m}\left(\frac{\alpha m}{(\alpha+\beta)p}\right) = \frac{\frac{\alpha}{(\alpha+\beta)p}}{\frac{\alpha}{(\alpha+\beta)p}} = 1$$

Re-writing (9) as:

$$p_x = \frac{m}{x}\frac{\alpha}{\alpha+\beta} \tag{10}$$

gives the *Inverse Demand* function!

1.1 Quasi-linear preferences

Remark 1 *Quasi-linear utilities have the form* $u(x_1, x_2) = x_1 + v(x_2)$*!*

Suppose the agent is maximising the following utility function:

$$U(x, y) = x + \sqrt{y} \tag{11}$$

subject to standard budget constraint (2). Assuming that a rational agent will spend all her money on purchasing the goods (more rigorous alternative is to set up Lagrangian function), the optimisation problem will beocome:

$$\max_y \left[\frac{m}{p_x} - \frac{p_y}{p_x}y + \sqrt{y}\right] \tag{12}$$

The first order (necessary) condition after rearranagements reads:

$$y = \left(\frac{p_x}{2p_y}\right)^2 \tag{13}$$

This is the demand function for the good y. It is *independent on the income level*, *i.e.* the agent is going to consume exactly the same amount of the good y as long as the prices remain constant. On the other hand the agent is spending all her 'leftover' money on purchasing good x. From (2) and (13) the demand function is:

$$x = \frac{m}{p_x} - \frac{p_x}{4p_y} \tag{14}$$

which is of the usual form: $x = x(p_x, p_y, m)$.

Q: Are x and y substitutes or compliments?

2 Exercises

2.1 True/False

Claim 1 *If the Engel curve for a good is upward sloping, the demand curve for that good must be downward sloping.*

TRUE: Upward sloping Engel curve $\rightsquigarrow$ Normal good $\rightsquigarrow$ (negative income effect $\overset{Slutsky}{\rightsquigarrow}$) downward sloping demand curve

Claim 2 *If the demand function is $q = \frac{3m}{p}$ (m is the income, p is the price), then the absolute value of the price elasticity of demand decreases as price increases.*

FALSE: The elasticity is: $\frac{p}{d}\frac{\partial q}{\partial p} = \frac{p}{q}\left(-\frac{3m}{p^2}\right) = -\frac{1}{q}\frac{3m}{p} = -\frac{q}{q} = -1$. Thus has constant elasticity equal to unity. *Note: Any utility function of the form $q = Ap^{\varepsilon}$ has constant elasticity equal to ε.*

Claim 3 *An increase in the price of Giffen good makes the consumers better off.*

FALSE: Increase in price of any good makes the consumer poorer and thus worse off. (A graphical representation may be helpful!)

Claim 4 *The demand function $q = 1000 - 10p$. If the price goes from 10 to 20, the absolute value of the elasticity of demand increases.*

TRUE: The elasticity of demand is: $\varepsilon = -10\frac{p}{q}$. $\varepsilon_{p=10} = -10\frac{10}{1000-100} = -\frac{1}{9}$; $\varepsilon_{p=20} = -10\frac{20}{1000-200} = -\frac{1}{4}$. $\left\|-\frac{1}{4}\right\| > \left\|-\frac{1}{9}\right\|$

Claim 5 *In case of perfect complements, decrease in price will result in negative total effect equal to the substitution effect.*

FALSE: In case of perfect compliments there is no substitution effect, and the total effect is equal to the income effect.

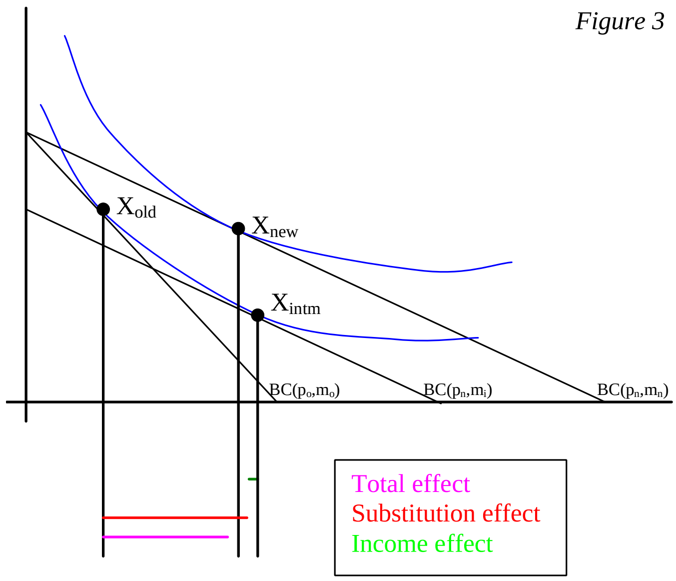
Claim 6 *When all other determinants are held fixed, the demand for a Giffen good always falls when income is increased.*

TRUE: To prove the claim we need to show that Giffen good is always an inferior good.

We are going to use the version of Slutsky equation that we had in class and ilustrated in Figure 3 (Note: the figure is illustrative and does not explain Giffen good). Thus:

$$\Delta x = \Delta x^s + \Delta x^m \qquad \textit{Slutsky}$$

$$\begin{aligned} \Delta x &= X_{old} - X_{new} && \text{total effect} \\ \Delta x^s &= X_{old} - X_{intm} && \text{substitution e.} \\ \Delta x^m &= X_{intm} - X_{new} && \text{income e.} \end{aligned}$$



As we can see, the figure illustrates a case when the price of the good went down, *viz.*

$$\Delta p = p_o - p_n > 0$$

and we can rewrite the Slutsky equation as

$$\frac{\Delta x}{\Delta p} = \frac{\Delta x^s}{\Delta p} + \frac{\Delta x^m}{\Delta p}$$

and check for the signs. We know that substition effect is always negative. We also know that for the Giffen good the total effect is positive. Thus the income effect should be positive:

$$sgn\left[\frac{\Delta x^m}{\Delta p}\right] = 1 \tag{15}$$

In order to prove the claim we need to show that

$$\begin{aligned} sgn\left[\frac{\Delta x^m}{\Delta m}\right] &= -1 \\ & \quad or\ (same\ as) \\ \frac{\Delta x^m}{\Delta m} &< 0 \end{aligned} \tag{16}$$

that is *the demand falls when income increases.* Thus we need to see that

$$sgn\left[\Delta m\right] = -1 \cdot sgn\left[\Delta p\right]$$

From the budget constraint we know that when the price goes down, the agent 'gets' richer, *i.e.* $\Delta p = p_o - p_n > 0 \Longrightarrow m_n - m_o > 0$ as the shift of the budget constraint is paralel to right. Thus we have

$$\Delta m = m_o - m_n < 0 \tag{17}$$

Again from (15) and (17) directly follows (16). *Q.E.D.*

Claim 7 *If the goods are substitutes, then an increase in the price of one of them will reduce the demand for the other.*

False: According to the definition!

2.2 Problems

Problem 1 *Demand functions for beer is given:*

$$q_b = m - 30p_b + 20p_c$$

where m is the income; p_b and p_c are the prices of beer and cake, respectively; q_b is the demanded quantity.

1. *is beer a substitute or compliment for cake? (A: $\frac{\partial q_b}{\partial p_c} = 20 > 0 \Longrightarrow$ substitute)*
2. *assume income is 100, and cake costs 1, what is the demand function? (A: $q_b = 120 - 30p_b$)*
3. *write the inverse demand function. (A: $p_b = 4 - \frac{1}{30}q_b$)*
4. *at what price would 30 beers be bought? (A: $p_b = 4 - \frac{1}{30}30 = 3$)*
5. *Draw the inverse demand. (Hint: It's a linear function)*
6. *Draw the inverse demand when $p_c = 2$. (Hint: It's parallel to the above, but higher.)*

Problem 2 *Suppose the demand function is $q = (p+a)^{\beta}, a > 0, \beta < -1$.*

1. *Find the price elasticity of demand.*
$\left(A : \frac{p}{q}\frac{\partial q}{\partial p} = \frac{p}{q}\beta\,(p+a)^{\beta-1} = \frac{p}{(p+a)^{\beta}}\beta\,(p+a)^{\beta-1} = \frac{\beta p}{p+a}\right)$
2. *Find the price level for which the elasticity is equal to -1?*
$\left(A : \frac{\beta p}{p+a} = -1 : p = -\frac{a}{\beta+1}\right)$