Intermediate Microeconomics

Lecture 5: Production and Costs

Agribusiness Teaching Center

Easter Term 2015

Equilibrium

Definition

A condition in which all acting influences are canceled by others, resulting in a stable, balanced, or unchanging system.

Definition (Economics)

A state of the economy in which for every good the excess demand is zero (total supply and demand are exactly equal).

Definition (Game theory)

A condition which no actor has an incentive to deviate from (given the payoffs and available strategies).

Formal Microeconomics

- Consumer theory
  - People
  - Applications
- Producer theory
  - *Internal organisation*
  - Industrial organisation
- Equilibrium
  - Existence
  - Efficiency

Formal Theory of Production

- Translate Neoclassical Consumer Theory into Theory of Production
- Revise graphs from introductory Microeconomics
- Introduce a few new concepts

Firms and technologies

Definition

A **firm** is a unit that organises production of a good (or service) for sale in order to maximise its profit.

Definition

Technology is the sum total of society's pool of knowledge concerning the art of production.

Axiomatic Base for Production Theory

- No Land of Cockaigne
  - (no free lunch) zero inputs result in zero output
- Free disposal
  - (monotonicity) more inputs produce at least as much as less inputs
- Convexity
  - (no-increasing return to scale) eighted average produces at least as much output as the original inputs
- Other technical

Production function

Definition

Production function is the relationship between the quantities of inputs used and the maximum quantity of output that can be produced.

Example

Two factors of production: Capital, K, and Labour, L:

$$q = f(L, K)$$

Inputs and 'time'-ing

- Immediate Run $q = f(\bar{L}, \bar{K})$
  - All the factors are almost fixed (Basically choice is between inactivity and fixed production)
- Short Run $q = f(L, \bar{K})$
  - One or more of the inputs (factors) are on fixed level.
- Long Run $q = f(L, K)$
  - All inputs can be varied.
- Very Long Run
  - We are all dead.

Average and Marginal Products

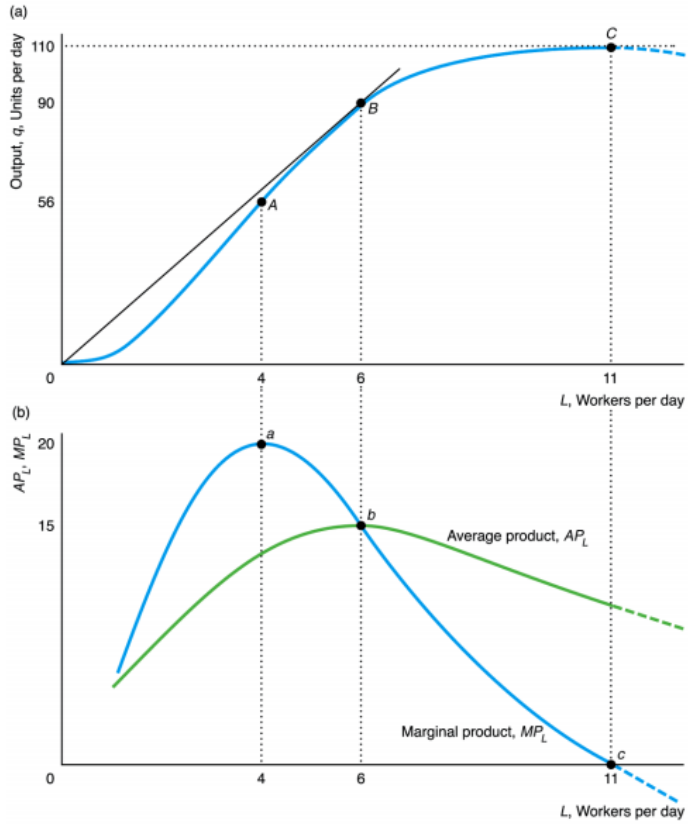
Definitions

Average product is the ratio of output to input used for production

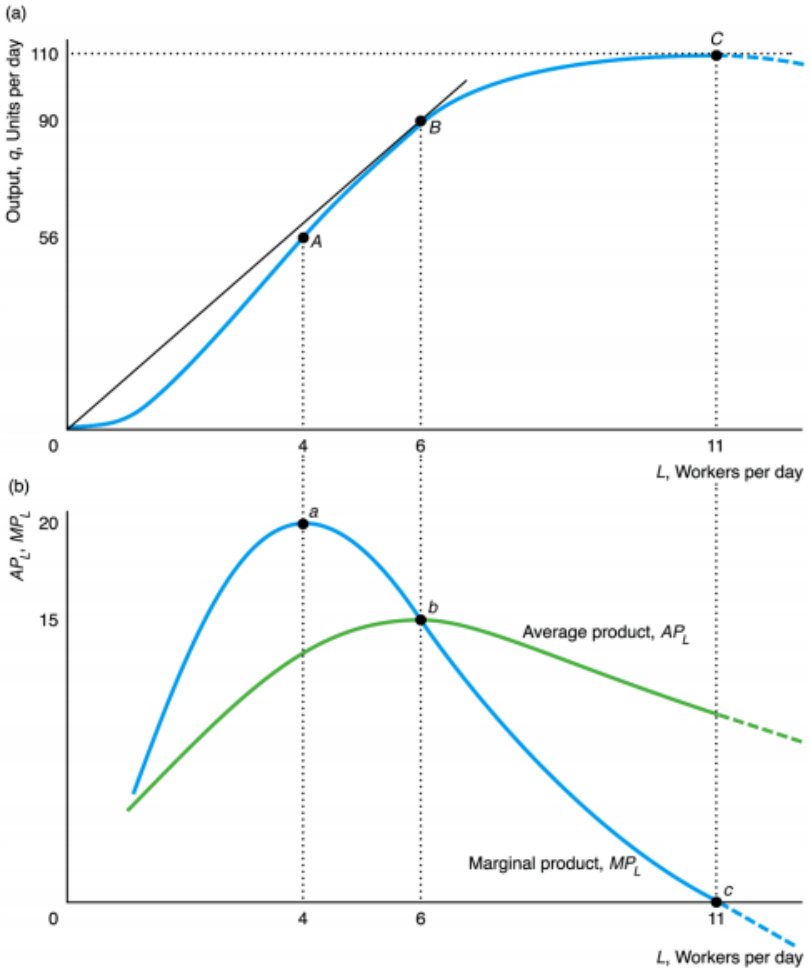
$$AP_L = \frac{q}{L}$$

Marginal product is the change in total output resulting from a marginal change in input (holding other factors constant):

$$MP = \frac{\partial f\left(L, \bar{K}\right)}{\partial L}$$



Average and Marginal Products



Fact

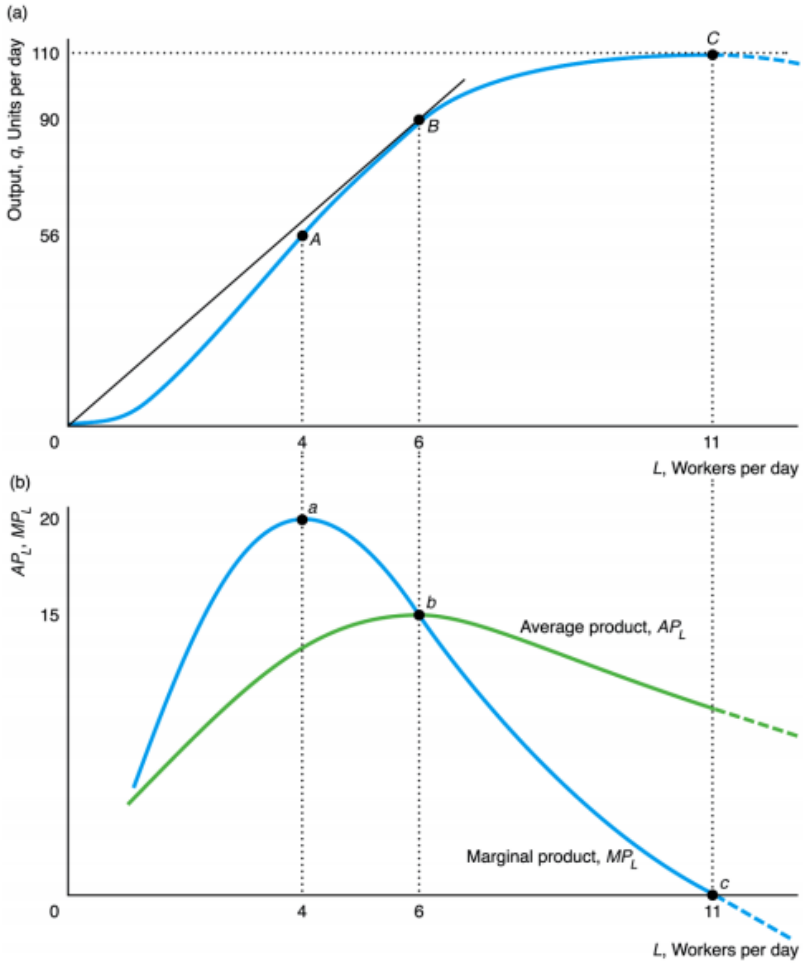
Marginal product equals to the average product when the average product reaches its highest level:

$$\frac{\partial}{\partial L}\left(\frac{q}{L}\right) = 0$$

$$\frac{\partial q}{\partial L} \cdot L - \frac{\partial L}{\partial L} \cdot q = 0$$

$$\frac{\partial q}{\partial L} = \frac{q}{L}$$

Average and Marginal Products



Fact

The **law of diminishing marginal returns** *(or product) holds that, if a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will become smaller eventually.*

- Diminishing returns *vs.* diminishing marginal returns

Law of Diminishing Marginal Returns

Was the Revd Thomas R. Malthus wrong?

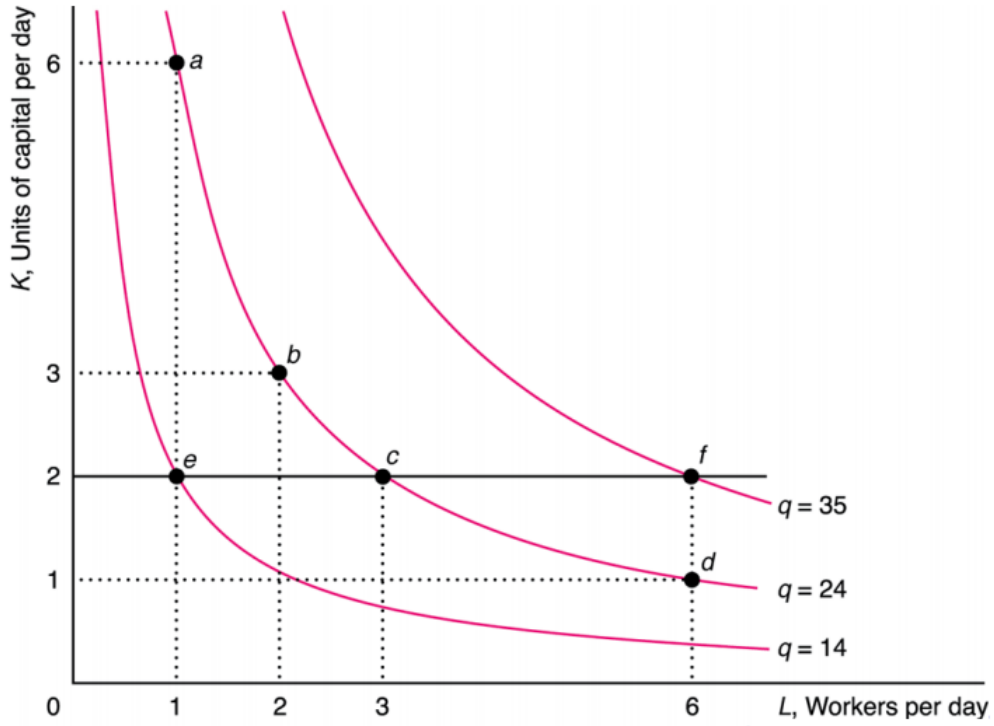
Fact

The law of diminishing marginal returns (or product) holds that, if a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will become smaller eventually.

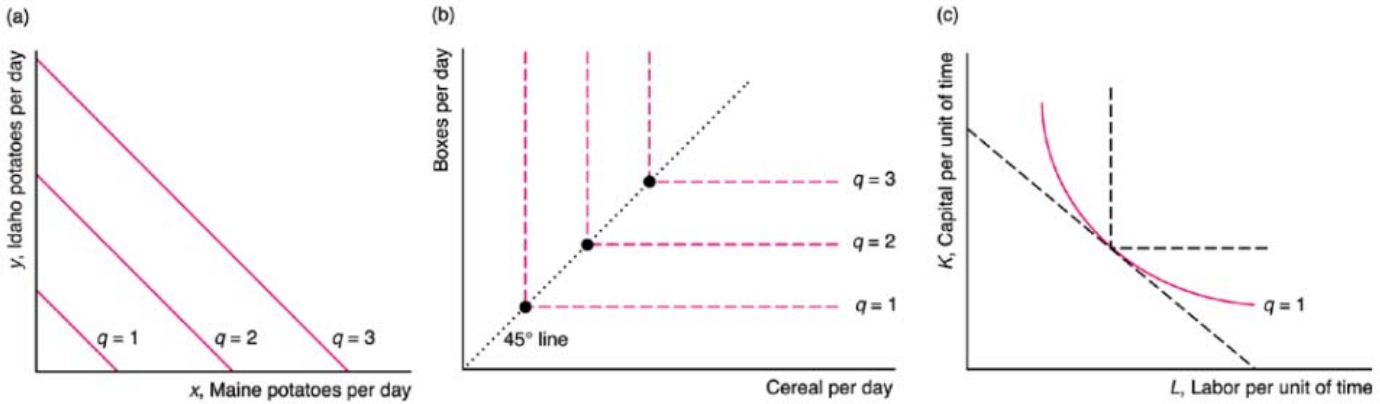
Isoquants

Definition

Isoquant is a curve that shows the efficient combinations of inputs that can produce single (iso-) level of output (*quant*-ity).



Isoquants: Substitutes and Compliments



Definition

Marginal rate of technichal substitution is the number of extra units of one input needed to replace one unit of another input while keeping the amount of output constant:

$$MRTS = -\frac{MP_L}{MP_K} = \frac{dK}{dL}$$

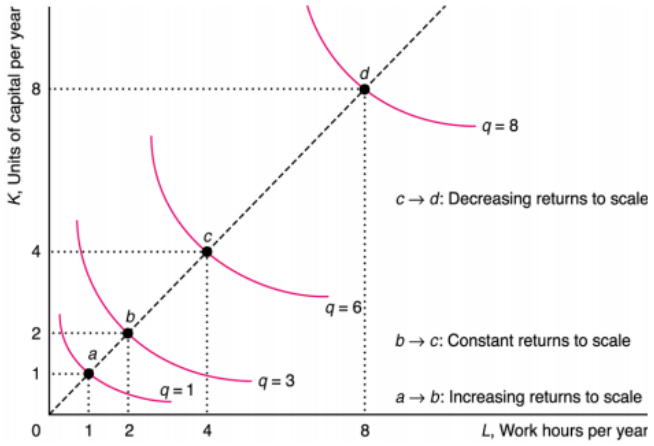
Returns to Scale

Definition

Increasing returns to scale is a property of a production function whereby output rises more than in proportion to an equal increase in all inputs.

Definition

Decreasing returns to scale is a property of a production function whereby output rises less than in proportion to an equal increase in all inputs.



Definition

Constant returns to scale is a property of a production function whereby when all inputs are increased by certain percentage, output increases by that same percentage.

COSTS

Iso-cost line

Definition

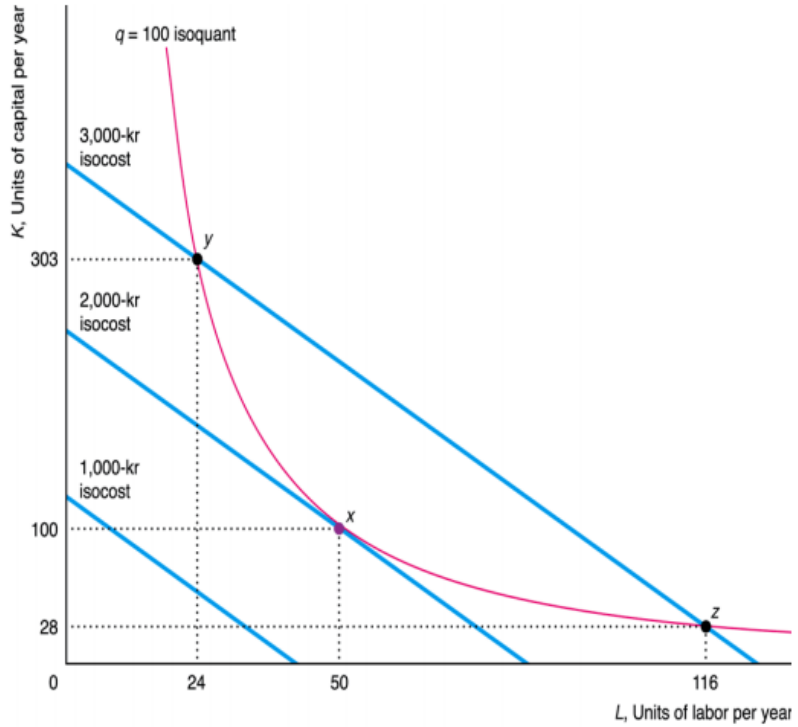
Production costs:

$$w_1 x_1 + w_2 x_2 = \bar{C}$$

rearrange:

$$x_2 = \frac{\bar{C}}{w_2} - \frac{w_1}{w_2} x_1$$

All the combinations of inputs that require the same (iso-) total expenditure (-cost) is called isocost line.



Optimisation Problem

Problem

Production costs:

$$\min w_1 x_1 + w_2 x_2$$

$$s.t. \qquad f(x_1, x_2) = \bar{y}$$

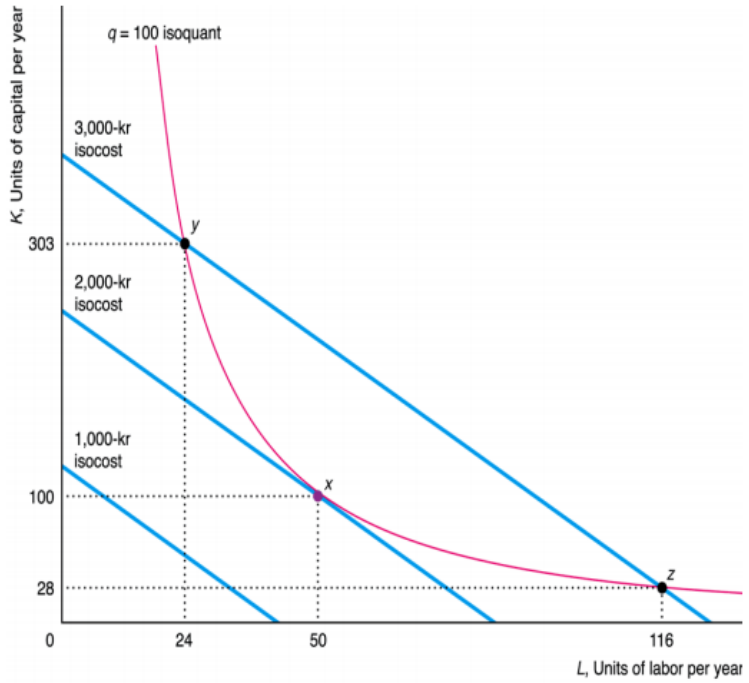
Solution

Cost function

$$C = c(w_1, w_2, y)$$

Condition

$$\frac{MP_1}{MP_2} = [-MRTS =] \frac{w_1}{w_2}$$



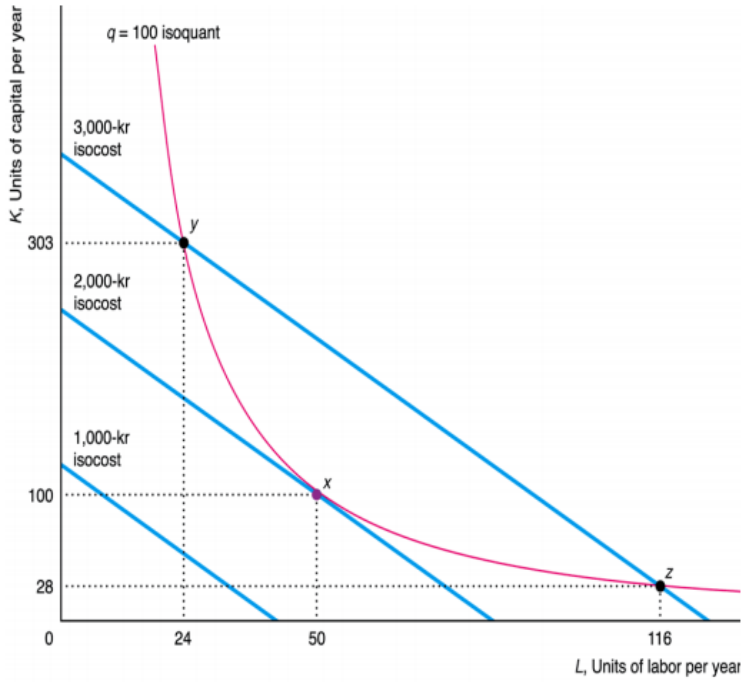
Optimisation

Fact

Condition

$$\frac{MP_1}{MP_2} = [-MRTS =] \frac{w_1}{w_2}$$

- *Lowest isocost rule!*
- *Tangency rule!*
- *Last dollar rule (pick the bundle of inputs where the last dollar spent on one input gives as much extra output as the last dollar spent on any other input).*



Short-run Costs

Definitions

Fixed cost (F) is a production expense that does not vary with output.
Variable cost (VC) is a production expense that changes with the quantity of output produced.
Cost (total cost, C) is the sum of a firm's variable and fixed costs:

$$C = VC + F$$

Definition

Marginal cost (MC) the amount by which a firm's cost changes of the firm produces one more unit of output (units being infinitesimally small):

$$MC = \frac{\partial C}{\partial q} \left[= \frac{\partial VC}{\partial q} \right]$$

Average Costs

Definitions

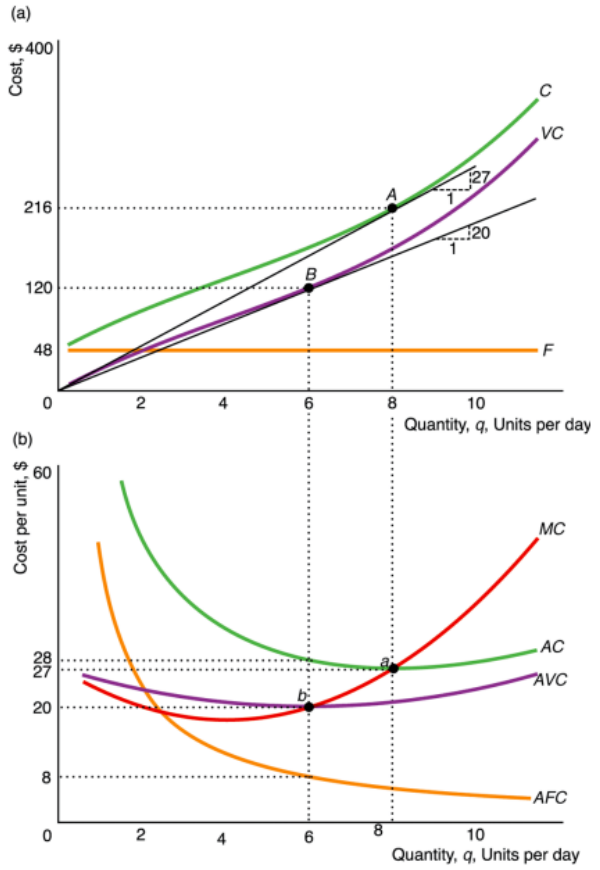
Average fixed cost (AFC) is the fixed cost divided by the units of output produced:

$$AFC = F/q$$

Average variable cost (AVC) is the variable cost divided by the units of output produced:

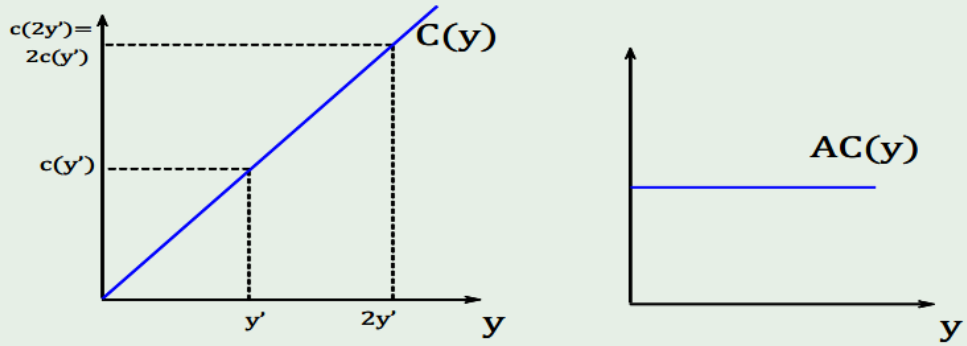
$$AVC = VC/q$$

Average cost (AC) is the sum of the two: $AC = AVC + AFC$

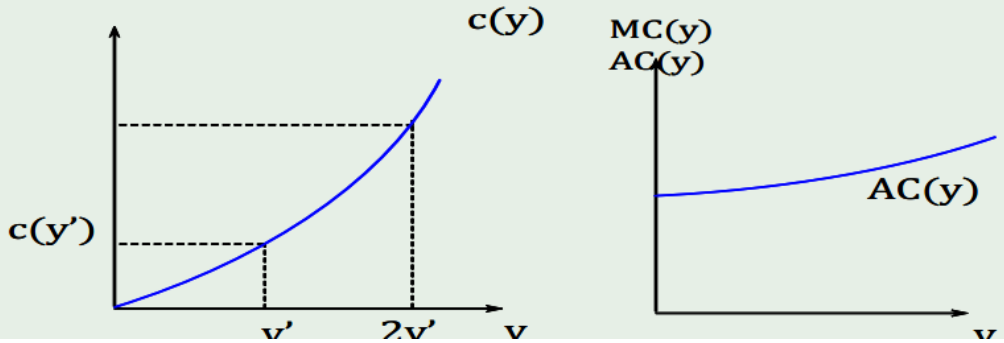


Returns to Scale

Example (CRS)

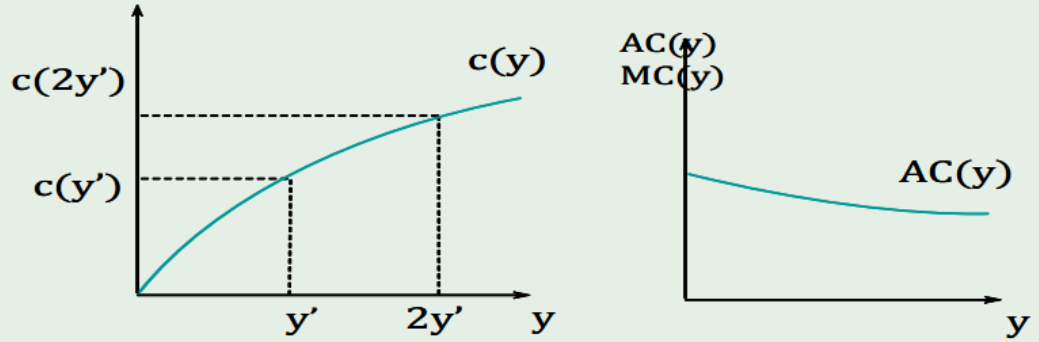


Example (DRS)



Returns to Scale

Example (IRS)



Fact

$$MC = \left[\frac{\partial VC}{\partial q} = w\frac{\partial L}{\partial q} =\right] \frac{w}{MP_l}$$

$$AVC = \left[\frac{VC}{q} = w\frac{L}{q} =\right] \frac{w}{AP_L}$$

Long-run *v* Short-run

Problem

Long run cost minimisation:

$$\min_{x_1,x_2} w_1 x_1 + w_2 x_2$$

$$s.t. \quad f(x_1, x_2) = \bar{y}$$

Problem

Short run cost minimisation:

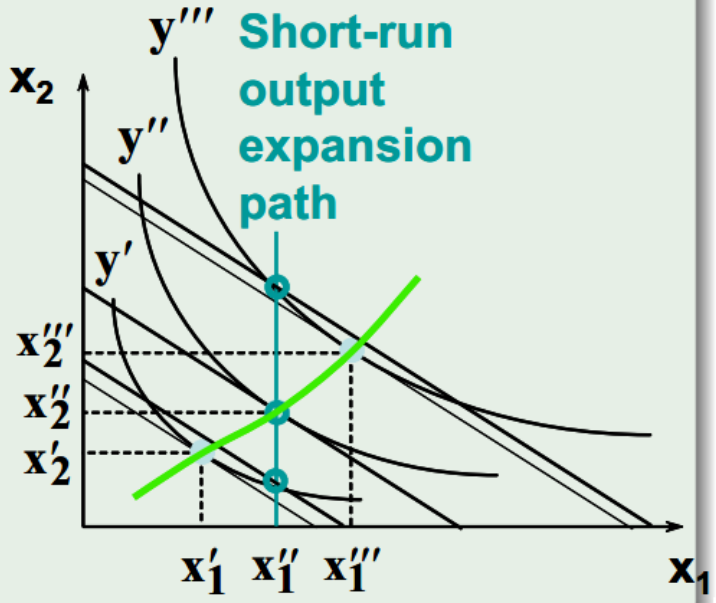
$$\min_{x_1} w_1 x_1 + w_2 \bar{x}_2$$

$$s.t. \quad f(x_1, \bar{x}_2) = \bar{y}$$

Fact

SR problem is LR problem with constraint $x_2 = \bar{x}_2$

Example



Exercises

- Show that on its minimum the average cost is equal to the marginal cost. Explain.
- To dig a trench, each worker needs a shovel. Workers can use only one shovel at a time. Workers without shovels do nothing, and shovels cannot operate on their own. Graphically determine the number of shovels and workers used by a firm to dig 2 trenches when:
  - w = 10 and r = 10
  - w = 10 and r = 5

Exercise

For the production function

$$Q = F(K,L) = (K \bullet L)^{1/2}$$

with $P_K = 4$ and $P_L = 2$, find the values of K and L that minimise the cost of producing 2 units of output.