Intermediate Microeconomics

Lecture 3: Demand and elasticities

Agribusiness Teaching Center

Easter Term 2015

Consumer Optimisation Problem

Calculus Approach

Problem

$$\max_{x_1, x_2} U(x_1, x_2),$$

$$s.t. \qquad p_1 x_1 + p_2 x_2 = m$$

Problem (Lagrange function)

$$\max_{x_1, x_2, \lambda} \mathcal{L} = U(x_1, x_2) + \lambda (m - p_1 x_1 - p_2 x_2)$$

Consumer Optimisation Problem

Calculus Approach

Problem

$$\max_{x_1,x_2,\lambda} \mathcal{L} = U(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2)$$

Solution

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0 : \frac{\partial U(x_1, x_2)}{\partial x_1} - \lambda p_1 = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0 : \frac{\partial U(x_1, x_2)}{\partial x_2} - \lambda p_2 = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 : m - p_1x_1 - p_2x_2 = 0 \quad (3)$$

Demand Function

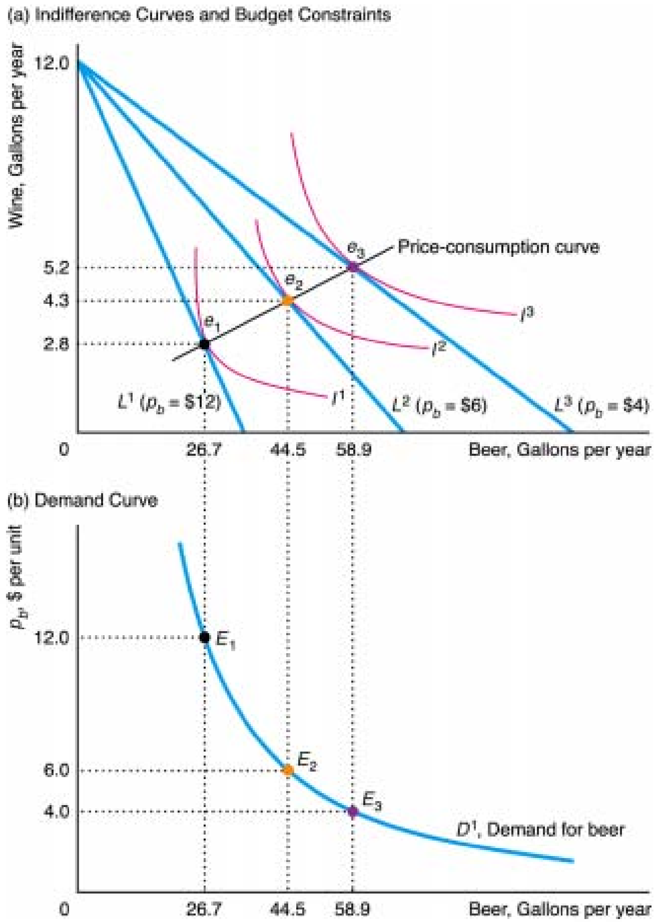
Definition

Demand:
The quantity demanded at each possible price.

- Demand function:

$$x_1 = x_1\left(p_1, \bar{p}_2, \bar{m}\right)$$

- Comparative statics
  - Shifts in m.
  - Shifts in p_1.
  - Shifts in p_2.



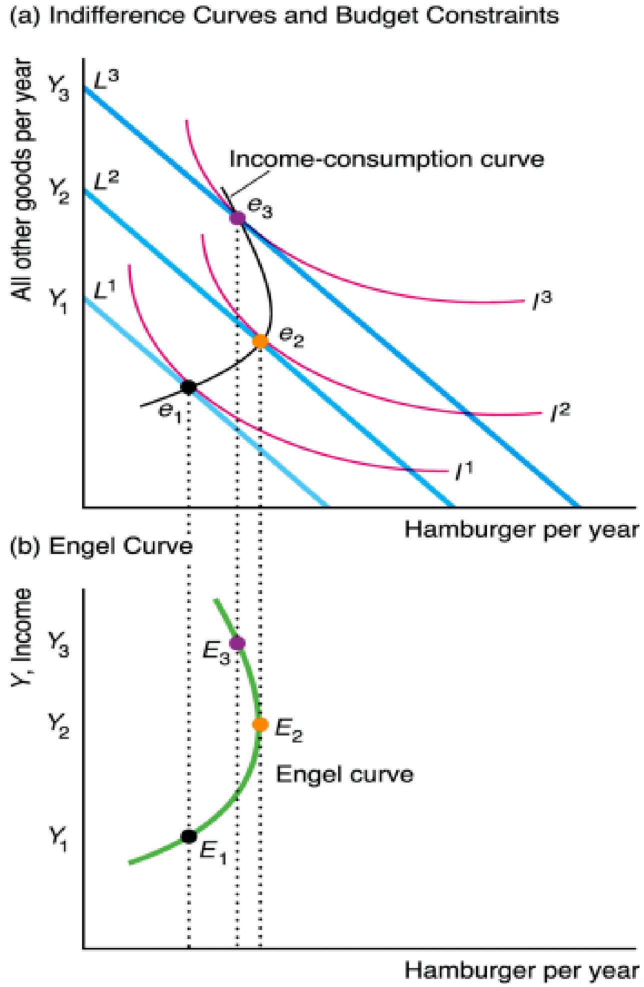
Income and Engel Curve

- Income expansion path
  - Bundles demanded at different income levels
  - Income-consumption curve

- Engel curve

$$x_1 = x_1(\bar{p}_1, \bar{p}_2, m)$$

  - Demand as a function of income only!



Engel Curve and Income Elasticity

Definition

The income elasticity of demand (or income elasticity) is the precentage change in the quantity demanded in response to a given percentage change in income.

$$\varepsilon_M = \frac{\%\Delta x_1}{\%\Delta m} = \frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta m}{m}} = \frac{\frac{x_1' - x_1}{x_1}}{\frac{m' - m}{m}} = \frac{\partial x_1}{\partial m} \cdot \frac{m}{x_1}$$

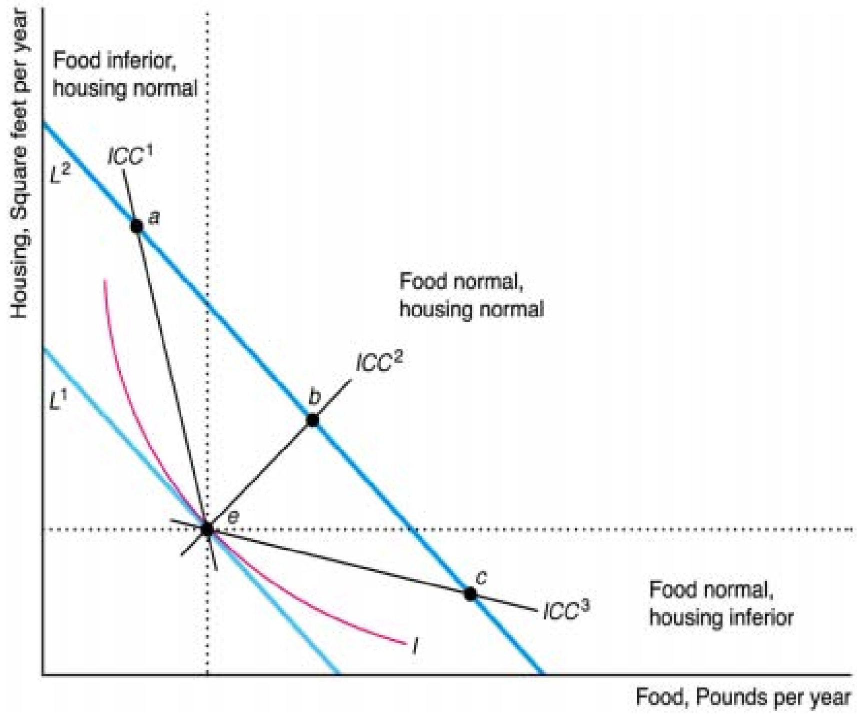
- Note:

$$\varepsilon_M \neq \frac{\Delta x_1}{\Delta m}$$

Engel Curve and Income Elasticity

- Normal good : $\varepsilon > 0$
- Inferior good: $\varepsilon < 0$
- Quasilinear: $\varepsilon = 0$

- Luxury goods: $\varepsilon > 1$
- Necessities: $\varepsilon < 1$
- Homothetic: $\varepsilon = 1$



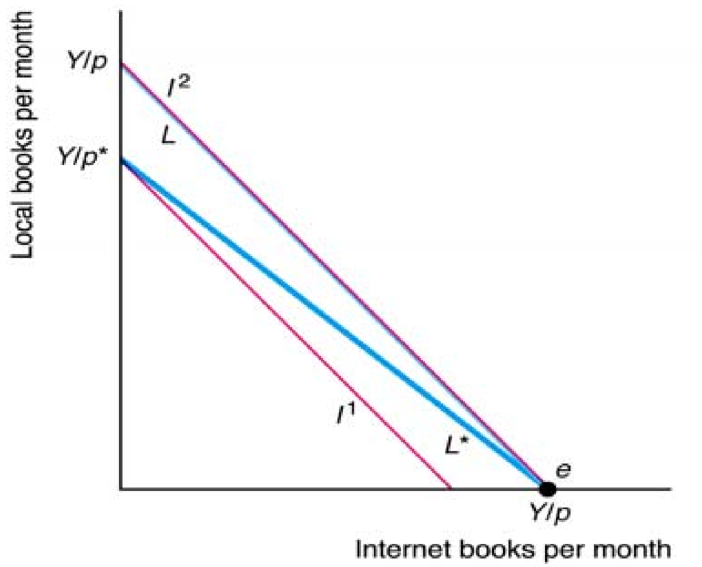
Indifference curves

Suppose Absurdistani people have 2 left feet and 1 right foot. We want to derive a utility function for a Absurdistani person who has L left shoes and R right shoes.

1. Draw the indifference curves.
2. Is it:
    1. rational
    2. monotonic
    3. convex
        1. strictly convex
        2. weakly convex

Demand Function

- Derive demand function (graphically) for a good with perfect substitute available.



Homothetic goods

- Assume Cobb-Douglas utility function

$$U = x^{\alpha} y^{\beta}$$

- Write the *Lagrangian function* for consumer's optimisation problem

- Derive the FOCs and the *demand function*

- Show that this is an example of *homothetic preferences*

Quasi-linear preferences

- Assume utility function of the following form $U(x, y) = x + \sqrt{y}$
  - It would work with any function of the form $u(x_1, x_2) = x_1 + v(x_2)$
- Derive the demand functions and show that the demand for *y* does not depend on income

Income and Substitution Effects

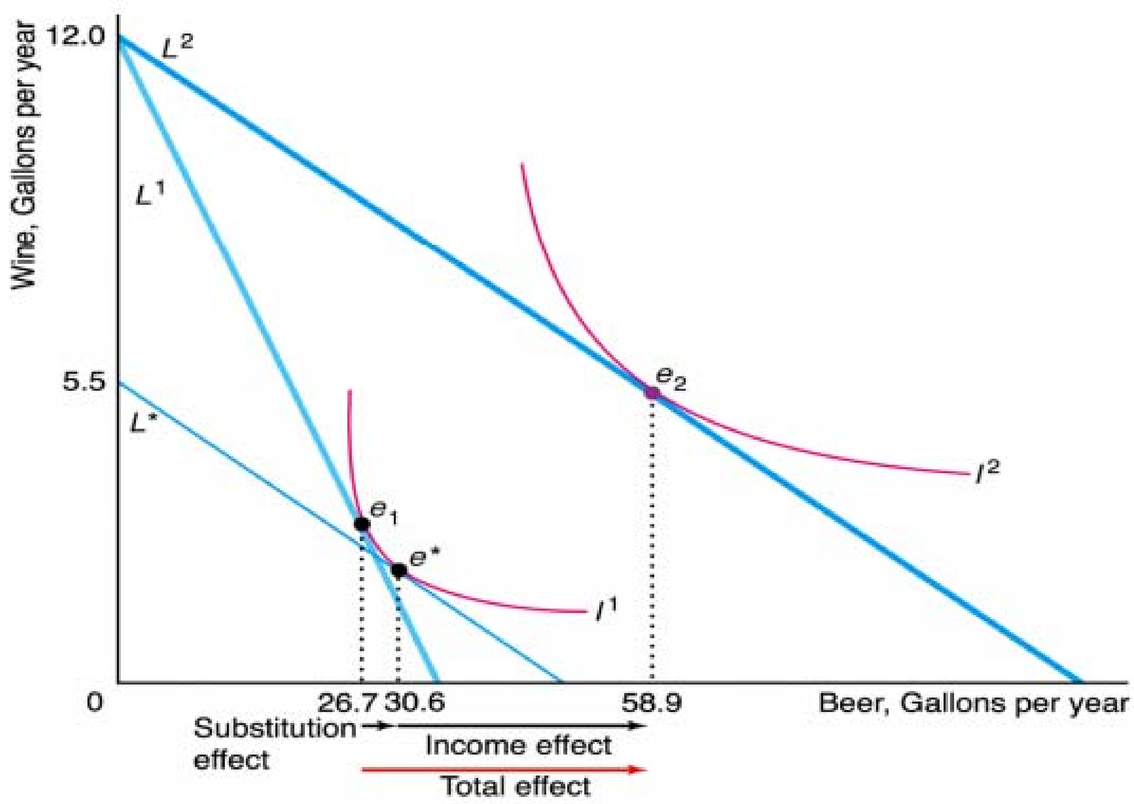
Definition

The change in demand due to the change in the rate of exchange between the two goods is called **substitution effect.** (changed own price, other prices and utility constant).

Definition

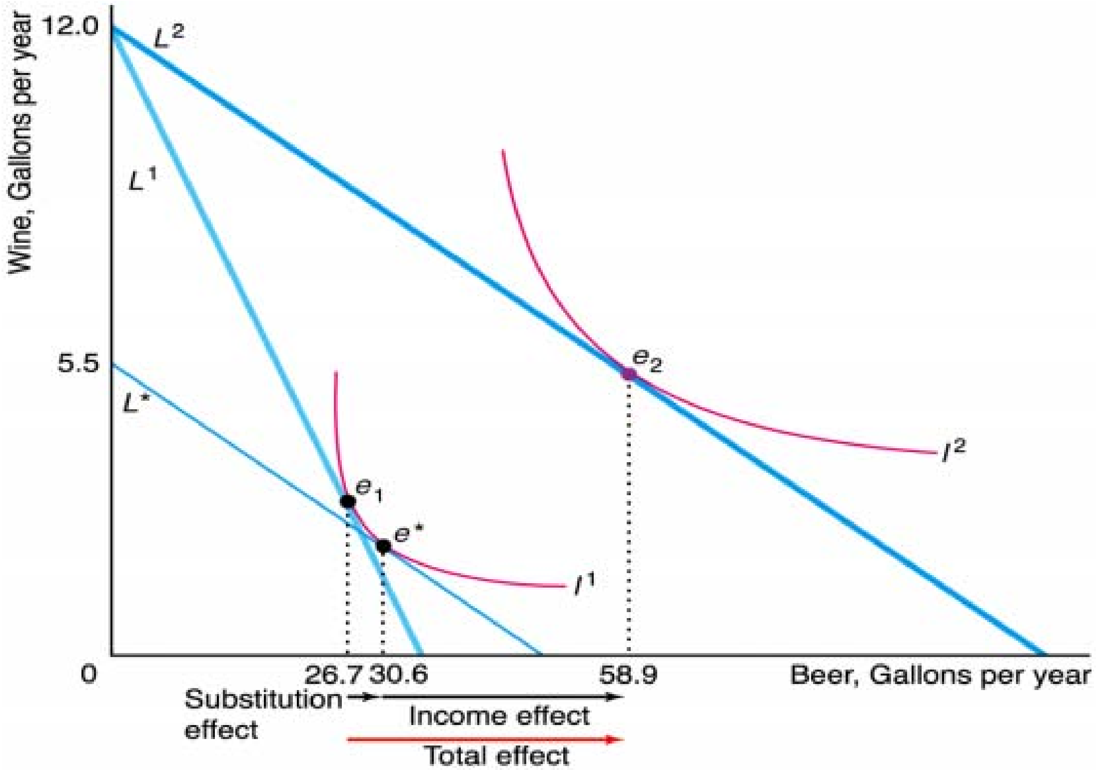
The change in demand due to the change in purchasing power is called **income effect.** (prices are hold constant)

Income and Substitution Effects



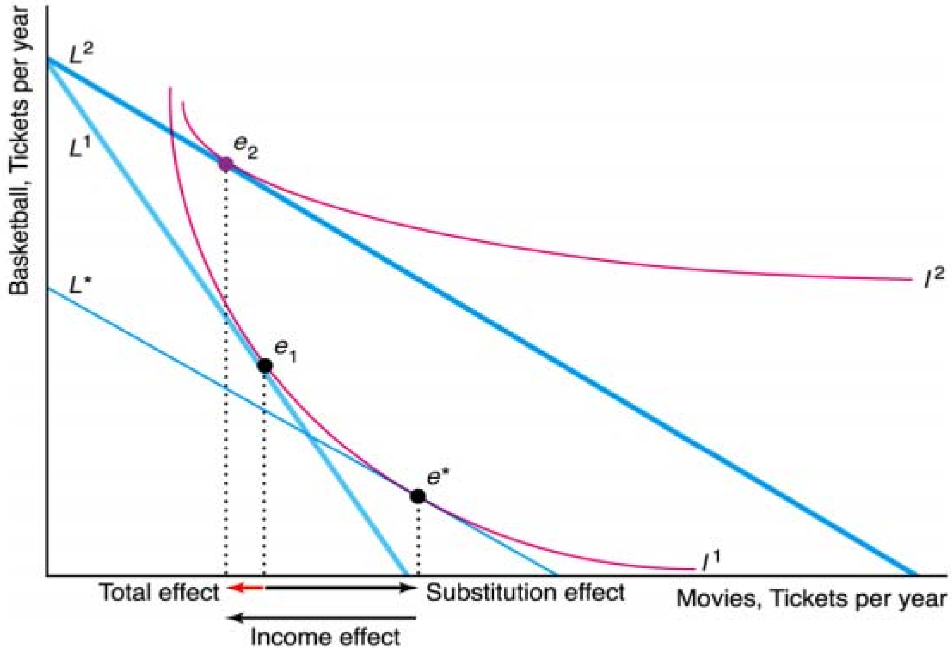
- Substitution effect is always negative due to the 'well-behaved' indifference curves!

Income and Substitution Effects



- Substitution effect is always negative due to the 'well-behaved' indifference curves!
- What about the direction of the income effect?

Income, Substitution Effects and Giffen Goods



Giffen goods (Inferior)

- What about perfect compliments?

Price and cross-price changes

- Own price change
  - Ordinary goods:

$$\frac{\partial x_1}{\partial p_1} < 0$$

  - Giffen goods:

$$\frac{\partial x_1}{\partial p_1} > 0$$

- Cross price change
  - substitute (not perfect)

$$\frac{\partial x_1}{\partial p_2} > 0$$

  - complement (not perfect)

$$\frac{\partial x_1}{\partial p_2} < 0$$

Some problems (budget constraint)

Draw a budget line to illustrate each of the following:

1. $p_1 = 1, p_2 = 1, m = 10$
2. $p_1 = 2, p_2 = 1, m = 20$
3. $p_1 = 1, p_2 = 0, m = 10$
4. $p_1 = p_2, m = 15p_1$

Exercise

Milan's utility function is $U(X,Y) = X \cdot Y$

1. *Suppose he originally consumed 4 units of* X *and 12 units of* Y*. If his consumption of* Y *is reduced to 8, how much* X *must he have to be as well as he was to begin with.*

2. *which bundle would Milan like better:*

   (a) *3 units of* X *and 10 units of* Y*, or*

   (b) *4 units of* X *and 8 units of* Y*.*

Question

Does an increase in the price of *Giffen good*

make the consumers better off?