Intermediate Microeconomics Strategic Choice and Game Theory

> Agribusiness Teaching Center Easter Term 2015

## Cuban Missile Crisis

**Problem 1 (Cuban Missile Crisis)** There are two players: USA and USSR. The USA must decide whether to Blockade the ports of Cuba or have an Airstrike to eliminate the missiles. The USSR must decide whether to Withdraw the missiles or to Maintain them. The payoffs are as in the matrix below:

		USSR		
		Withdraw	Maintain	
	Blockade	3, 3	2, 4	
USA				
	Airstrike	4, 2	0,0	

1. How many Nash equilibria are there?

Best Response (reaction given the other player's action)

## Prisoner's Dilemma

Problem 2 (Prisoner's Dilemma) Payoffs as follows:

		$Player \ B$		
		Silence	Confess	
	Silence	3, 3	1, 4	
Player A				
	Confess	4, 1	2, 2	

- 1. Is (Silence, Silence) a Nash Equilibrium?
- 2. Is Confess dominant strategy for player B?
- 3. Is Silence dominant strategy for Player A?

# Sharing Gold

**Problem 3** Players A and B must make bids for the division of a pot of gold equal to £1000. The rules are as follows: Each must bid an amount between zero and £1000. If the sum of their bids is less than or equal to £1000 then they both get their bids (e.g., if A bids £400 and B bids £100 then A gets £400 and B gets £100). If the sum of their bids exceeds £1000 then they both get nothing. Which pairs of bids are Nash equilibria? (For simplicity you can assume that players can make bids only in hundreds of pounds, i.e. £200 or £500 but not £350.)

#### Battle of Sexes

**Problem 4 (Battle of the Sexes)** There are two players, Husband and Wife, who must decide whether to go to watch Ballet or Football. The payoffs are as in the matrix below:

		W i f e		
		Ballet	Football	
	Ballet	3, 4	1, 1	
Husband				
	Football	2, 2	4, 3	

- 1. is (Football, Ballet) a Nash equilibrium?
- 2. is (Football, Football) a Nash equilibrium?
- 3. does player Wife have a dominant strategy?
- 4. Now consider the extensive form where Husband moves first, and then Wife, having observed Husband's move, then plays second. What's the (sub-game perfect) Nash equilibrium to this extensive form game?

#### Modified Battle of Sexes

**Problem 5** Suppose a man and a woman each choose whether to go to a Football match or a Ballet performance. The man would rather go to Football and the woman to Ballet. What is more important to them however is that the man wants to show up to the same event as the woman (he adores her) but the woman wants to avoid him (she cannot stand his... whatever... something... cologne). Construct a game matrix to illustrate this game, choosing numbers to fit the preferences described verbally.

#### Many actions

Problem 6 (2x5 game) Consider the normal form game presented below. Find Nash equilibria for the 2x5 game? (Yes, each player has 5 actions!)

		Mrs. Column				
		$\beta(1)$	$\beta$ (2)	$\beta$ (3)	$\beta$ (4)	$\beta$ (5)
	$\alpha(1)$	0, 4	18, 1	8,9	7, 5	11,7
Mr. Row	$\alpha(2)$	11, 5	8, 8	0, 8	13, 12	2, 0
	$lpha\left(3 ight)$	11, 3	10, 5	9, 4	9, 2	10, 1
	$\alpha$ (4)	11, 5	19, 3	1,9	9, 8	2,10
	$lpha\left(5 ight)$	12, 4	15, 0	5, 1	5, 1	4,3

## Nash equilibrium

**Problem 7** Provide an example of a simple two-person game, with each player having two actions, to illustrate that the following statement is NOT always true:

'If your opponent does NOT play her Nash equilibrium strategy you should, however, still always play yours since it is always in your best interest to do so.' **Problem 8 (Centipede Game)** Consider the following extensive form game: There are two players, A and B. Players take turns in making decisions begining with Player A. At each decision node each player must choose either the action  $\langle C \rangle$  meaning 'continue' in which case the other player has a chance to make a decision; or to choose  $\langle S \rangle$  meaning 'stop,' in which case the game ends and the payoffs indicated in the extensive form (the game tree) are received. As indicated bellow, the game will automatically end if  $\langle C \rangle$  is played by each player 4 times and the payoff will then be (1000,700) for A and B, respectively. What is the sub-game perfect Nash equilibrium to this extensive form game?



# Three players

**Problem 9 (3x2 game)** Consider the following three-person game in which player 1 chooses the row, player 2 chooses the column, and player 3 chooses the matrix that will be played.



The first number i each cell is the payoff to player 1, the second number is the payoff to player 2, and the third number is the payoff to player 3. Find the Nash equilibrium for this game.

## Backward induction



Problem 10 (Backward induction) Use backward induction to find subgame perfect equilibrium given following game in extensive form : Ching-chang-chong

# Problem 11 (Rock, scissors, paper) Find mixed strategy Nash equilibrium

