Intermediate Microeconomics

Strategic Choice and Game Theory

Agribusiness Teaching Center

Easter Term 2015

Games and strategic choice

Some Strategic Interactions

Game of strategy

Definition

A person is engaged in a game of strategy with someone else (or with several other people) when her utility or payoff is affected not only by the actions that she takes but also by the actions that her opponents take.

- Chess
- Free riding, bargaining
- Arms race, political campaigns
- Wedlock

Fact

Attested as early as 2600 BC, games are a universal part of human experience and present in all cultures.
– Wikipedia (Game)

Game of strategy

Definition

A **game of strategy** is an abstract **set of rules** that constrains the behaviour of players and defines outcomes on the basis of the actions taken by the players.

The essentials:

- The players (all the particiants, including Nature)
- The strategies (all the options that the player can use in action)
- Payoffs (for every possible profile of strategy choices of all players.

Fact

When we buy a board game like Monopoly, the accompanying instructions give us this type of information.

Representation of Games

A game tree or an extensive form of game is a diagram that provides a detailed description of the rules of the game.

Example

Biblical story (Genesis 22): Abraham is told to bring his son Isaac for sacrifice.

HW: Own examples and representations from the Bible or some other major work of literature.

Representation of Games

Example

Abraham contemplates disobeying God's command. Hence, Abraham has two choices:

- to bring Isaac, or
- to refuse to bring him.

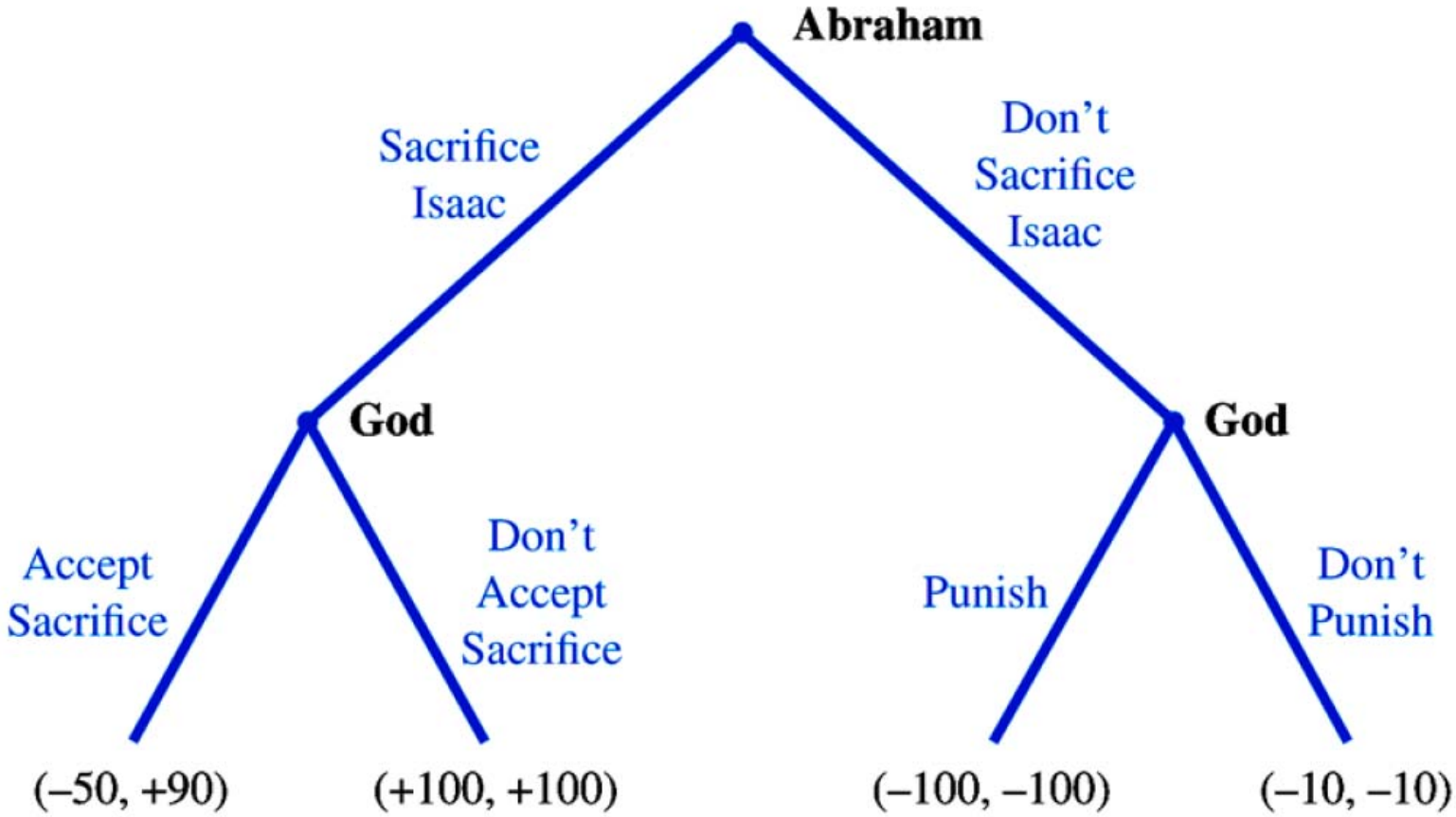
Since God is omniscient, He will see in advance Abraham's move; after He will decide on his move. His moves, therefore, will be conditional on what Abraham does. If Abraham brings Isaac

- he can accept the sacrifice or
- substitute a sheep in his place.

If Abraham refuses, then God can

- excuse Abraham or punish him.

Representation of Games



Games of Perfect and Imperfect Information

Definitions

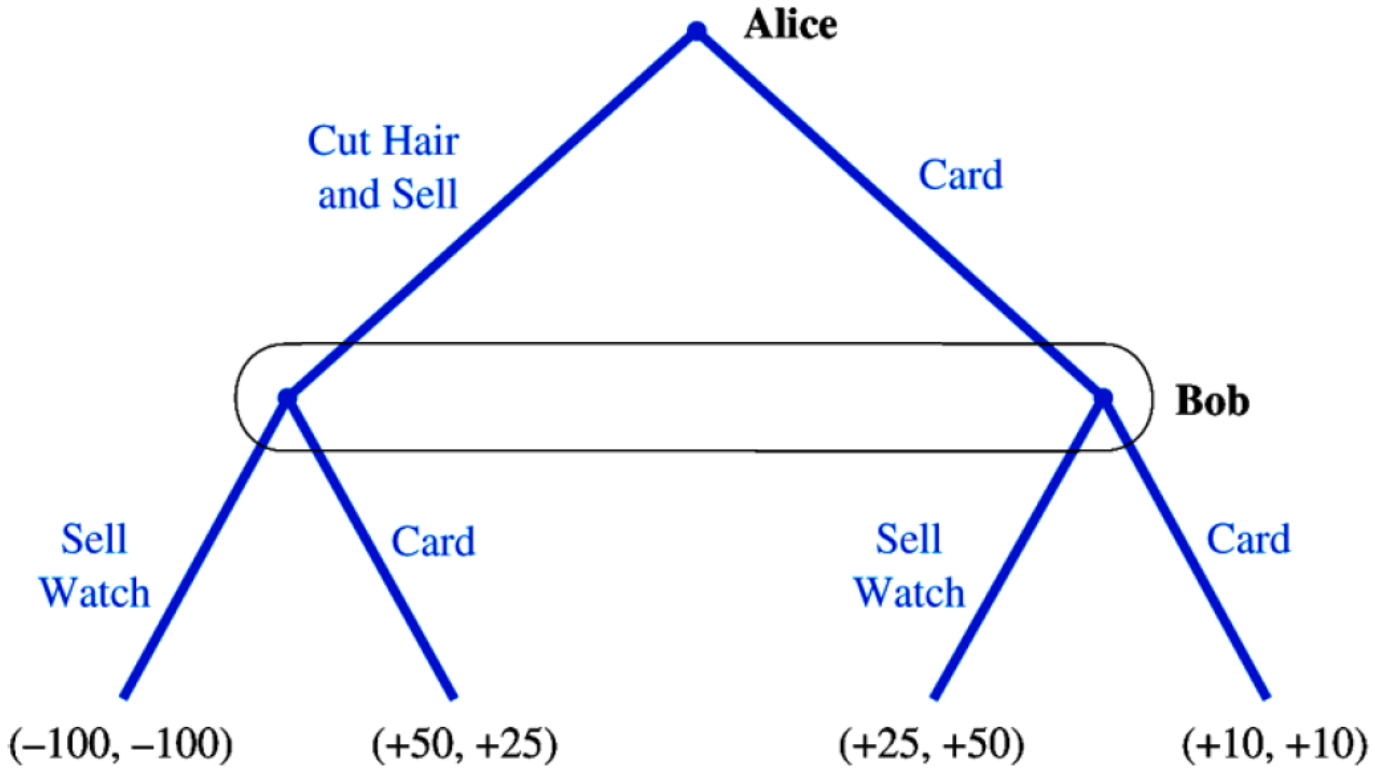
In a game of perfect information any player makes a move knowing all the prior choices made by the other players.

Definitions

In games of imperfect information player does not know all the choices of the other players who preceded her.

Games of
Perfect and Imperfect Information

O'Henry: Gift of the Magi



Representation of Games

Definition

The normal form of a game is a matrix that represents the strategic situation of the players and tells what payoff each of them will receive depending on the strategies chosen.

		Bob	
		Sell watch	Buy card
Alice	Sell hair	-100 -100	25 50
	Buy card	50 25	10 10

Solution

Definition

The **equilibria** for games are those states in which no player will want to change his or her behavior given the behavior of the other players in the game.

Once an equilibrium is achieved in a game, no matter how it is achieved, it will continue without change.

No individual players will have any incentive to change their actions if they assume that their opponents also will not change their actions.

Nash Equilibrium

- Players: $1, \ldots, n$
- array of strategy choices: $s^* = (s_1^*, \ldots, s_n^*)$
- payoff to player i : $\pi_i(s_1^*, \ldots, s_n^*)$

Definition (Nash Equilibrium)

An array of strategy choices $s^* = (s_1^*, \ldots, s_n^*)$ is a Nash equilibrium if $\pi_i(s_1^*, \ldots, s_i^*, \ldots s_n^*) \geq \pi_i(s_1^*, \ldots, \hat{s}_i, \ldots s_n^*)$ for all strategy choices $\hat{s}_i \in S_i$ (that is, the set of all possible strategies from which player i can choose) and all players i.

Example

- Players: Mr. Column and Ms Row
- array of strategy choices: (Left, Right) and (Up and Down)
- payoffs in the normal form

		Mr. Column	
		Left	Right
Ms. Row	Up	25 20	15 25
	Down	40 15	35 20

Example

- Players: Mr. Column and Ms Row
- array of strategy choices: (Left, Right) and (Up and Down)
- payoffs in the normal form

		Mr. Column	
		Left	Right
Ms. Row	Up	**25** 20	**15** 25
	Down	**40** 15	**35** 20

Example

- Players: Mr. Column and Ms Row
- array of strategy choices: (Left, Right) and (Up and Down)
- payoffs in the normal form

		Mr. Column	
		Left	Right
Ms. Row	*Up*	**25*** 20	**15** 25
	Down	40 15	35 20

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Prisoners' Dilemma

- Players: 2 prisoners, Row and Column, who perpetrated a crime
- Actions: each may confess (and implicate other) or deny
- Payoff matrix (years in prison)

		Mr. Column	
		Confess	Deny
Ms. Row	Confess	-3 -3	-6 0
	Deny	0 -6	-1 -1

Prisoners' Dilemma

		Mr. Column	
		Confess	Deny
Ms. Row	Confess	**-3*** **-3***	-6 0
	Deny	0 -6	-1 -1

Definition

A prisoner's Dilemma is a game in which all players have dominant strategies that result in payoffs that are inferior to what they could achieve if they used cooperative strategies.

Games with many equilibria

- Players: Ms. Row and Mr. Column, who got interupted
- Actions: each may call or wait for the call
- Payoff matrix

		Mr. Column	
		Call	Wait
Ms. Row	Call	0 0	6 3
	Wait	3 6	0 0

Games with many equilibria

- Players: Ms. Row and Mr. Column, who got interupted
- Actions: each may call or wait for the call
- Payoff matrix

		Mr. Column	
		Call	Wait
Ms. Row	Call	0 0	**6*** **3***
	Wait	**3*** **6***	0 0

Need further refinements in the concept of equilibrium!

Games with no (pure) equilibrium

- Players: Two Army Generals
- Actions: each may attack or retreat
- Payoff matrix

		General 2	
		Retreat	Attack
General 1	Retreat	$5, 8^*$	$6^*, 6$
	Attack	$8^*, 0$	$2, 3^*$

Pure *vs.* Mixed Strategies