

# Are Bismarckian Pensions Immune to Unskilled Immigration?

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**ABSTRACT** The current study assesses the effects of immigration control on the social security system of a host economy. A theoretical model of a small open economy populated with overlapping generations of heterogeneous agents is used to show that if the immigrant population is sufficiently unskilled the native population will prefer to switch from unfunded social security system to debt-funded or fully funded systems even with the fully actuarially fair, Bismarckian, pension system. The result is due to the fact that unskilled immigration causes intergenerational conflict - favouring the future cohorts: Thus pension system privatisation may serve as a redistributive mechanism from future to present cohorts.

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*Keywords:* Non-selective immigration, PAYG, intergenerational conflict

## Introduction

Population ageing - joint decline in fertility and mortality rates - increases social security expenses and burdens public finances. Large number of retired people relative to the shrunk working age population result in more beneficiaries per contributor. With a smaller contribution base and increasing number of beneficiaries governments face problems of financing their pension systems. Those systems have already become a burden for most of the developed countries: The social expenditures have been increasing at least for the last 20 years in tandem with the share of the old-age population. Already in 2007 public pension revenues covered only 88 per cent of the expenditures on average in OECD countries. By 2060 the revenues will constitute only 64 per cent of the expenditures, and in some countries the projected gap is to reach above 10 per cent of GDP (OECD, 2012).

Many possible ways of dealing with this problem are proposed: As the pension system is defined by the tax and benefit rates, parametric reforms (changes in those rates) should be able to restore balance in the system. However, reduction of the social benefits (including reduction in duration of eligibility) contradicts the basic idea of social states of Europe. Also, an increase of the taxes will have harmful consequences in Europe as it may result in 'voting with feet' (Uebelmesser, 2004; Krieger, 2005) when the young working generation prefers to emigrate because of the heavy social policy.

At the same time, pension system can undergo structural reforms - moving from unfunded to funded system. The fully funded system ensures neutrality of the social security system on the government budget. After Peniña reforms in Chile, pension reforms gained popularity. The transition from pay-as-you-go (PAYG) to a fully funded scheme seemingly releases the governments from the problem of financing social security.

Further, Aaron (1966) established the fact that the PAYG system is preferred to fully funded system till the sum of population and the real wage growth rates exceed the real interest rate in the economy. Hence, in the developed world the fully funding is supposedly the preferred scheme as a result of the current and expected negative growth rate of the population: The fully funded system will guarantee higher pensions for the retirees and will eradicate the problem of sustainability for the government.<sup>1</sup>

However, the fully funded system cannot be introduced at once as the PAYG system already exists. When the PAYG system has been introduced the initial generation received benefits without contributing to the system, and from then on all the subsequent generations pay to cover the generated implicit debt. Should there be no terminal-generation the initial-debt-financing problem will never emerge. Alas, the establishment of the funded system terminates the unfunded system, the implicit debt turns into an 'explicit' government debt that requires financing. This makes the political-feasibility of the transition questionable as the taxpayers are still under the burden of the legacy of the PAYG system.<sup>2</sup> Lindbeck and Persson (2003) argue that a cut in benefits and

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<sup>1</sup>Further, Fuster, İmrohoroğlu, and İmrohoroğlu (2007) show welfare gains from elimination of labour-market distortive pension taxes. Kaganovich & Zilcha (2012) claim that the fully funded system produces higher human capital.

<sup>2</sup>At the same time there is a line in literature that advocates for retaining PAYG, *e.g.* Diamond (2004) rationalises it with myopic behaviour, Forni (2005) points at political economy, Krueger & Kubler (2006) base their claim on the intergenerational risk sharing, Nishiyama & Smetters (2007) highlight some missing financial markets, Casarico & Devillanova (2008) note on the redistributive issues. Fehr (2011) provides survey of the literature.

increase in contribution rate suffice to sustain the current system if not the re-distributional concerns. It has been shown in the literature (*e.g.* Breyer, 1989; Conesa & Garriga, 2007) that such a transition per se does not generate an efficiency gain as the burden of the introductory gift (the first benefits distributed to the population who did not contribute) will exist in both cases.

Whilst the traditional economic policies became ineffective to sustain the social security in ageing societies new policies have been proposed: Logical alternative to the parametric and structural reforms of the social security is the demographic reforms. A popular reform discussed in the literature is connected to immigration<sup>3</sup> (Storesletten, 2000; Chen & Fang, 2013), *i.e.* foreign born population may cover for the shortage of the local working population<sup>4</sup>. The literature is mostly favourable towards this theory of ‘replacement migration’ (an excellent survey of the literature is provided by Kerr & Kerr, 2011). That is, migration is seen as an unpaid import of a factor of production that can be taxed to cover the budgetary shortages connected with the social expenses (*e.g.* Razin & Sadka, 2000; Fehr, Jokish & Kotlikoff, 2004).

Meanwhile, the ‘welfare magnet’ literature claims that low skilled immigrants actually are attracted to the redistributive welfare systems of the host economies and thus they are burden on the public finances (Borjas, 1999; Barret & Maître, 2013). Based on that idea, Cohen, Razin & Sadka (2009) and Razin, Sadka & Suwakiri (2011) show that the population will chose to abandon<sup>5</sup> the (redistributive) social welfare programmes in the political equilibrium. However, recent empirical findings do not confirm an existence of those welfare magnet effects. Thus, Giulietti, Guzi, Kahanec & Zimmerman (2013) based on the OECD data claim that migrant skill distribution in the economy cannot be explained by the generosity of the welfare system (at least by the unemployment benefits). Belot & Hatton (2012) show that cultural similarities and colonial legacies are the important factors that determine the direction of migration, especially in Europe.

However, Aslanyan (2012) showed that immigration brings intergenerational welfare conflicts

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<sup>3</sup>Currently the immigration policy formulation does get a high priority among other institution building activities in the EU (Vikhrov, 2013). A solid proof to that is the ‘Green Paper’: It pre-defines an ‘EU approach to managing economic migration.’ The Green Paper aims to design an immigration policy that would fulfill the Lisbon Strategy (of building knowledge-based social economy). Among other policies directly aimed at fulfilling the Lisbon Strategy (*e.g.* attracting students and researchers), the Paper highlights the possibility of using migration policy to secure fiscal sustainability of the social scheme.

<sup>4</sup>While some countries are short in working age population some other countries have excess supply of it. For instance, Fargues (2005) brings the example of Egypt where annually additional half a million workplaces are needed to absorb the new generation entering the labor market.

<sup>5</sup>In their model the social security is cut to absolute minimum, which is still positive, as they have assumed pensions-in-the-utility function.

even in the case of non-redistributive PAYG system.<sup>6</sup> Further, it was shown that the less skilled the immigrants are, the more benefits go to the future population under the restricted PAYG system. Therefore, given that migration control is not always possible,<sup>7</sup> most of the welfare gains from the immigration may go to the future cohorts. Thus, this work shows that lacking other intergenerational welfare smoothing mechanism, the current population may prefer to abandon the PAYG system in favour of fully funded pension systems.<sup>8</sup> And as opposed to the previous literature (*i.e.* Cohen, Razin & Sadka, 2009; Razin, Sadka & Suwakiri, 2011) the need to reform the social security comes not because the immigrants are a drain on the system but rather from the fact that the immigration delays the benefits into future and social security reform is a mechanism for redistribution of those gains. The paper starts with a small open economy environment where Bismarckian (actuarially fair, intra-generationally non-redistributive) PAYG system already exists and a structural reform is studied. The paper leads to the conclusion that the immigration reforms may lead to social security reforms once the initial population generates welfare gains from the reform. Further, this work also allows to study joint determination of social security and immigration policies, *i.e.* simultaneous reforms in both policies if immigration control is possible.

## The Economic Environment

A small-open-economy environment is used. The model implicitly assumes the existence of a firm that locally hires all the available labour. No financial institution is modelled: Savings and borrowings are made based on constant (world) prices. Explicitly the model is populated by heterogeneous agents and a government that manages the social security budget.

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<sup>6</sup>Aslanyan (2014) showed that even in the case of closed economy, while the redistributive pensions decrease with an increase in unskilled immigration, the non-redistributive part of the pensions, or the Bismarckian pensions, increase.

<sup>7</sup>For instance, Vikhrov (2013) discusses the EU free labour movement policy; Belot & Hatton (2012) emphasise the often-ineffectiveness of selective policies because of the refugees and tied family migration, Razin & Wahba (2011) highlight the inability of the selective policies to control for the quality of education.

<sup>8</sup>An alternative to abandoning the balanced PAYG system in favour of funded system can be retaining PAYG in combination with borrowing, *i.e.* moving to the debt-financed PAYG economy. While these alternatives yield identical welfare (according to the Breyer equivalence result), still the new system cannot be considered an unfunded any more.

## Population

The population differ in age (young,  $i = 0$ , and retired,  $i = 1$ ), in skill level (skilled,  $s = 1$ , and unskilled,  $s = 0$ ) and number of previous generations in the economy. Immigrants,  $m$ , are introduced to the economy while young and are considered the first generation in the economy,  $g = 1$ . Descendants of immigrant dynasties can be of any generation,  $g = 2, 3, \dots$ , and the natives,  $n$ , belong to dynasties that are present in the economy at time  $t = 0$ .

While the share of skilled among immigrants,  $\lambda \in (0, 1)$ , is a 'choice variable' for the government, the locals ( $l$ , descendants of native and immigrant dynasties) are born skilled with some probability: The share of skilled among descendants of native dynasties is  $\theta \in (0, 1)$ , and the share of skilled among immigrant dynasties is either  $\lambda$  or  $\theta$  depending on the assimilation process.<sup>9</sup> (For notation also  $\gamma(s, g)$  will be used as the probability for generation  $g$  agent having skill level  $s$ .)

In the labour market the skill level directly translates into efficiency level,  $\varepsilon(s)$ . The skill level also co-determines, together with generational background, the fertility rate  $\varphi(s, g)$  of the agents. Natives, for the sake of simplicity, are assumed to reproduce with unit fertility.<sup>10</sup> Thus, if  $\mu_t(i, s, g)$  is defined as a measure on type  $(i, s, g)$  agents, the introduction of the new generation of (type  $s$ ) natives can be presented as:

$$\mu_{t+1}(0, s, 0) = \gamma(s, 0) \sum_{s'} \mu_t(0, s', 0) \quad (1)$$

and the introduction of the immigrant dynasty descendants as:

$$\mu_{t+1}(0, s, g + 1) = \gamma(s, g + 1) \sum_{s'} \mu_t(0, s', g) \cdot \varphi(s', g) \quad (2)$$

The size and quality of the first generation immigrants is a government policy:

$$\mu_t(0, s, 1) = \gamma(s, 1) \cdot \psi \sum_{g \neq 1} \sum_{s'} \mu_t(0, s', g) \quad (3)$$

<sup>9</sup>The immigrant dynasties will either assimilate fully and have the skill distribution of the natives,  $\theta$ , or fully inherit the ancestral skill distribution (with  $\lambda$  as the share of skilled and  $(1 - \lambda)$  as the share of unskilled). The skill distribution is commonly believed (e.g. Card & Rothstein, 2007; Heath, ROTHON & Kilpi, 2008) to be between the two, but for the sake of analytic simplicity only the extreme cases are considered.

<sup>10</sup>The constant population, though very optimistic for ageing societies, already makes unfunded pension systems costly for the participating population. Principally, welfare losses arise once the sum of the growth rates for the real wage and population is less than the real interest rate (Aaron, 1966).

where  $\psi$  is the size of the immigrant population compared to the local-born population. The government chooses not only the size of the group but also the share of skilled among the immigrants:  $\lambda = \gamma(1, 1)$ .

Each agent stays in the model for two periods (except the initial retired population,  $\mu_0(1, s, n)$ , that are present only for the second period of their lives), *viz.* individual ageing is deterministic:

$$\mu_{t+1}(1, s, g) = \mu_t(0, s, g) \quad (4)$$

*i.e.*, everybody ages, stays retired for one period and leaves the model afterwards (also no return migration is allowed).

The size of the effective labour force depends on the absolute size of the population and their average efficiency:

$$N_t = \sum_{s,g} \varepsilon(s) \mu_t(0, s, g) \quad (5)$$

At the start of the economy half of the native population is young (age  $i = 0$ ), and the other half is retired (age  $i = 1$ ). Thus the effective labour force at time  $t = 0$  is

$$N_o = \mathcal{L}(\theta \varepsilon_s + (1 - \theta) \varepsilon_u) = \mathcal{L}E_\theta \quad (6)$$

where  $\mathcal{L}$  is exactly half of the total population in the country, and  $E_\theta$  is the average efficiency of a native worker.

## Government

The government regulates immigration (as presented in eq. (3)) and implements the fiscal constitution: The fiscal constitution includes taxation, pension benefits and sustainability of public debt (if it exists), and can be presented as:

$$\sum_s \rho w \varepsilon(s) \mu_1(1, s, n) + B_0(1+r) = \sum_{s,g} \bar{\tau} w \varepsilon(s) \mu_1(0, s, g) + B_1 \quad (7)$$

$$\text{(for all the other cohorts)} \quad B_{t-1}(1+r) = \sum_{s,g} \bar{\tau} w \varepsilon(s) \mu_t(0, s, g) + B_t \quad (8)$$

and

$$\lim_{t \rightarrow \infty} B_t \cdot (1+r)^{-t} = 0 \quad (9)$$

where  $\rho_t$  is the pension replacement rate<sup>11</sup>,  $\tau_t$  is the tax rate, and  $B_t$  is the debt at year  $t$  (a borrowing from time  $t-1$  due at time  $t$ ). For the sake of comparability there will be no initial debt under any type of the government budget discussed below, i.e.  $B_0 = 0$ .

In the case of standard unfunded PAYG system the government debt will stay 0 for all times  $B_t = 0$  as the budget will balance in each period. The debt-financed government budget will be used while analysing social security reforms *i.e.* the case when the government terminates the PAYG system, converts the implicit debt of the unfunded PAYG pension (the pension claims of the initial retired population) into explicit debt, and uses taxes to service the debt. In this case the initial pension replacement rate will remain on the usual level, *i.e.*  $\rho_1 = \rho$ , as the claims of the initial retired is being honoured, while for all the other periods the population will not receive any pensions, *i.e.*  $\rho_t = 0$  for any  $t > 1$ . These reforms are usually politically non-feasible (Yamada, 2011) as the policy reforming generation would have to finance the reform, *i.e.* contribute for the PAYG while saving for own retirement, and thus create a double-paying generation. In order to avoid these double-paying generation problem a single tax will be considered that will evenly distribute the debt over the generations and the sustainability condition (9) will be utilised to avoid perpetual debt-generating situations.

<sup>11</sup>This specification follows Bismarckian (earnings-related) social security system that allows intergenerational redistribution and excludes intragenerational redistribution.

## Households and Welfare

Each household is represented an individual agent that maximises lifetime utility, which is derived from consumption in both periods:

$$U(c_t^t, c_{t+1}^t) = v(c_t^t) + \beta v(c_{t+1}^t) \quad (10)$$

where  $c_j^i$  is the consumption of an agent born at time  $i$  during time  $j$ ;  $U(\cdot, \cdot)$  is a time-separable utility function with  $\beta \in (0, 1)$  being the time-discount coefficient and  $v(\cdot)$  being a continuous, twice continuously differentiable, strictly increasing, strictly concave function that satisfies the Inada conditions.<sup>12</sup> To finance consumption an agent uses labour income net of taxes and savings in the first period, and in the second period savings and social security benefits are used. Thus, at time  $t$  a  $(0, s, g)$  type agent faces the following budget constraints:

$$c_t^t + a_t \leq w\varepsilon(s)(1 - \tau_t) \quad (11)$$

$$c_{t+1}^t \leq \rho_t w\varepsilon(s) + a_t(1 + r) \quad (12)$$

where  $a_t$  is the savings. The agent's efficiency is  $\varepsilon_s = \varepsilon(1)$  and  $\varepsilon_u = \varepsilon(0)$ .

Further, as there are no borrowing constraints, (11) and (12) can be combined into one intertemporal budget constraint:

$$c_t^t + \frac{c_{t+1}^t}{1 + r} \leq w\varepsilon(s) \left( 1 - \tau_t + \frac{\rho_t}{1 + r} \right) \quad (13)$$

Essentially the intertemporal budget constraint (13) shows that only the present discounted value of the lifetime after-tax income, *i.e.* the expression on the right-hand side of (13), matters for the consumption choice of an agent type  $(s)$ . This observation is the base of the following lemma:

<sup>12</sup>*Leisure* is not considered in the utility function for notational simplicity: All of the following results hold with the conventional time-separable, CRRA utility function:

$$U(c_t^t, c_{t+1}^t, n_t, n_{t+1}) = \sum \beta^i \frac{\left[ (c_{t+i}^t)^\alpha (n_{t+i})^\gamma \right]^{1-\delta}}{1 - \delta}$$

and the budget constraint (11)-(12): The agent's decision on leisure depends not on other (own or government) policy variables, only on parameters and interest rate.



**Lemma 1** Let  $\mathcal{U}_s(\tau_t)$  be the lifetime utility as a function of the size of the social security system:

$$\mathcal{U}_s(\tau_t) = \max U(c_t^t(s), c_{t+1}^t(s)) \quad (14)$$

and also denote

$$\mathcal{W}_t \equiv 1 - \tau_t + \frac{\rho_t}{1+r} \quad (15)$$

then  $\mathcal{U}_s(\tau_t)$  is strictly increasing in  $\mathcal{W}_t$ .

**Proof.** The first order conditions of the optimisation problem (10) subject to (13) give implicit functions of consumptions in the both periods depending only on  $W_t$  defined in (15):  $c_t^t = c_t^t(W_t)$  and  $c_{t+1}^t = c_{t+1}^t(W_t)$ . On the other hand  $U(c_t^t(s), c_{t+1}^t(s))$  is strictly increasing in both arguments and thus is strictly increasing in  $\mathcal{W}_t$ . Then from the definition,  $\mathcal{U}_s(\tau_t)$  is also strictly increasing in  $\mathcal{W}_t$ . ■

Further, all type  $s$  agents of the same generation face identical optimisation problem (10)-(13), viz.  $\mathcal{U}_s(\tau_t)$  is independent of the agents' generation in the country  $g$  and represents the utility of all the type  $s$  agents of generation  $t$ . Moreover, according to the lemma 1  $\mathcal{U}_s(\tau_t)$  for  $s = 0, 1$  are both strictly growing in  $\mathcal{W}_t$  at the same time. However  $\mathcal{W}_t$  is independent of agents' skill type  $s$  and thus is a valid measure of welfare for the entire generation under different government policies. Thence, based on Lemma 1, the effect of the public policies on the lifelong income of the population,  $\mathcal{W}_t$ , shall be considered the **measure of welfare** and will be used to compare the welfare of the agents under different policies. Still, the welfare of the initial retired population is invariable: Independent on the policy changes the retired population consumes own savings and due pension benefits.

## The Status Quo Economy

The *Status Quo* economy starts with an established social security system and no immigration and those policies are maintained without changes. Thus the population dynamics (1)-(6), in

combination with zero-immigration policy

$$\psi = 0, \quad (16)$$

takes the following form:

$$N_{t+1} = N_t = N_0. \quad (17)$$

The government also maintains balanced PAYG system at any time, i.e. runs (7)-(9) with no borrowings:

$$B_t = B_0 = 0 \quad (18)$$

The initial retired population, or the generation  $t = 0$  at period  $t = 1$ , did contribute to the PAYG system in the previous period and thus anticipate social security benefits with replacement rate  $\rho$  during the period  $t = 1$ . The subsequent generations will contribute with a tax rate  $\tau_t$  and claim social security benefits with replacement rate  $\rho$  during the next period.

**Definition 1** *Given the (world) prices for labour and capital,  $w$  and  $r$ , the replacement rate for public pension  $\rho$ , the rate of time preference  $\beta$ , skill and fertility rates  $\varepsilon_s$  and  $\varphi_{g,s}$ , skill distribution  $\gamma_{s,g}$ , the initial value of public debt,  $B_0$ , and the size of the immigrant population compared to the native population  $\psi$ , the Status Quo equilibrium in the economy is an allocation*

$$\left\{ c_0^1; \left\{ c_t^t, c_{t+1}^t, a_t, \mu_t(i, s, g), \tau_t, B_t \right\}_{s=0,1;g=1,2\dots}^{i=0,1;t=1,2\dots} \right\}$$

*such that the initial (time  $t = 0$ ) retired agents consume their savings and pension benefits, households optimise (10)-(12), the government budget (7)-(9) is balanced according to (18), and the population evolves according to (17).*

**Lemma 2** *In the Status Quo economy the tax rate is constant*

$$\tau_t = \rho \quad (19)$$

*and the Welfare is unity*

$$\mathcal{W}^{SQ} = 1 \quad (20)$$

for all generations.

**Proof.** Using (18) the government budget (7)-(9) can be re-written as

$$\rho N_t = \tau_t N_{t+1} \quad (21)$$

that in combination with (17) results in (19). Combination of (19) and (15) results in (20). ■

The equal welfare for all the agents is a result of balanced budget and constant population growth rate. Moreover, the unity welfare for the Status Quo economy makes it an ideal *numéraire* — a benchmark to be compared with.

## The *Laissez Faire* Economy

In the *Laissez Faire* economy the government allows in immigrants with exogenously given skill level, cancels the PAYG social security system, finances the initial retired population's pension claims through borrowing, and sets a tax rate so that the government budget is balanced over infinite-time horizon.

**Definition 2** Given the (world) prices for labour and capital,  $w$  and  $r$ , the replacement rate for public pension  $\rho$ , the rate of time preference  $\beta$ , skill and fertility rates  $\varepsilon_s$  and  $\varphi_{g,s}$ , skill distribution  $\gamma_{s,g}$ , the initial value of public debt,  $B_0$ , and the relative size of the immigrant population  $\psi$ , the equilibrium in the *Laissez Faire* economy is an allocation

$$\left\{ c_0^1; \left\{ c_t^t, c_{t+1}^t, a_t, \mu_t(i, s, g), B_t \right\}_{s=0,1;g=1,2\dots}^{i=0,1;t=1,2\dots} \right\}$$

such that the initial (time  $t = 0$ ) retired agents consume their savings and pension benefits, households optimise (10)-(12), the government budget sets a tax rate,  $\bar{\tau}$  to balance the budget (7)-(9), and the population evolves according to (1)-(5).

The equilibrium defined above starts from a point with an existing restricted (defined benefit) PAYG social security system and no-migration policy, *i.e.* the start in the *Status Quo* economy.

The definition suggests that there is one level of tax for all times, thus making the reform a mechanism for intergenerational welfare redistribution (smoothing). Also, one tax rate makes convenient comparison between *Laissez Faire*, *Status Quo* and Debt-financed PAYG economies: While the comparison between the *Status Quo* and Debt-financed PAYG economies shows the effect of immigration, the comparison of *Laissez Faire* and Debt-financed PAYG economies shows the effect of the social security reforms. Similarly, comparison of the *Laissez Faire* with zero-migration, i.e.  $\psi = 0$ , and *Status Quo* also describes the welfare effects of social security reforms:

**Proposition 1** *In equilibrium Laissez Faire and Debt-financed PAYG economies yeild equal welfare:*

$$\bar{\mathcal{W}} = \widetilde{\mathcal{W}} \quad (22)$$

where  $\bar{\mathcal{W}} = \mathcal{W}_t$  in the *Laissez Faire* economy, and  $\widetilde{\mathcal{W}} = \mathcal{W}_t$  in the economy with debt-financed PAYG for all  $t$ .

**Proof.** The government budget constraint in the debt-financed PAYG economy (7), divided on both sides by  $(1+r)^t$  and summed over all periods, can be rewritten as

$$\tilde{\tau}w \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} = \rho w \sum_{t=1}^{\infty} \frac{N_{t-1}}{(1+r)^t} + \sum_{t=1}^{\infty} \frac{B_t - B_{t-1}(1+r)}{(1+r)^t} \quad (23)$$

that, using (9) and  $B_0 = 0$ , solves for

$$\tilde{\tau} = \frac{\rho N_0}{1+r} \left( \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} \right)^{-1} + \frac{\rho}{1+r} \quad (24)$$

Plugging  $\tilde{\tau}$  into the definition of the welfare measure

$$\widetilde{\mathcal{W}} \equiv 1 - \frac{\rho N_0}{1+r} \left( \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} \right)^{-1}. \quad (25)$$

The government budget constraint in *Laissez Faire* economy (7)-(8), divided on both sides by  $(1+r)^t$  for each period and summed over all periods, can be rewritten as

$$\bar{\tau}w \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} = \rho w \frac{N_0}{1+r} + \sum_{t=1}^{\infty} \frac{B_t - B_{t-1}(1+r)}{(1+r)^t} \quad (26)$$

that, using (9) and  $B_0 = 0$ , solves for

$$\bar{\tau} = \frac{\rho N_0}{1+r} \left( \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} \right)^{-1} \quad (27)$$

Plugging  $\bar{\tau}$  into the definition of the welfare measure and using the fact that  $\rho_t = 0$

$$\bar{W} \equiv 1 - \frac{\rho N_0}{1+r} \left( \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} \right)^{-1} \quad (28)$$

Direct comparison of (25) and (28) yields the results. ■

The proposition is based on the fact that once the social security has been introduced one generation received benefits without contributing to the system. This implicit debt stays within the PAYG system and needs financing through contributions. This part of the contribution is what Nishiyama & Smetters (2007) refer to as the *effective tax* of the system - that is the difference between the tax contribution and pension benefit under the (debt-financed) PAYG system where all the generations contribute to cover the internal debt of the system. In the case of *Laissez Faire* economy the internal debt is turned into an external debt and is financed through the taxation and thus is equal to the *effective tax* of the system. Further, as the effective population growth rate is assumed to be less than the interest rate throughout this work, no welfare gains are possible under either system: should there be extra borrowing in the *Laissez Faire* economy the future cohorts will face higher taxes for financing it (as the interest on the debt will be more than the population growth rate). Similarly, the redistribution through the PAYG system generates losses.

In essence, *Proposition 1* is an extended version of the Breyer (1989) equivalence result, which states that PAYG is (Pareto-)efficient or that social security reforms do not generate welfare gains themselves. Similar equivalence results in the framework of social security reforms are presented in Fenge (1995), Lindbeck & Persson (2003) and Conesa & Garriga (2008). They claim that the government can conduct a Pareto-neutral reforms using appropriate debt financing. Accordingly, *Proposition 1* extends the Breyer equivalence result to show that Pareto-neutral reforms are possible while there are demographic changes in the number and skill level of the population, *i.e.* where the heterogeneity and migration are incorporated to the model.

**Proposition 2** *Increased immigration policy is welfare enhancing independent of social security policy:*

$$\bar{\mathcal{W}} = \widetilde{\mathcal{W}} > \mathcal{W}^{SQ} \quad (29)$$

**Proof.** With positive immigration  $\psi > 0$ , according to (1)-(5),  $N_t$  grows compared to  $N_0$ , that results in  $\widetilde{\mathcal{W}} = \bar{\mathcal{W}} > \mathcal{W}^{SQ}$ .

■

As the *proposition 2* shows the increased immigration policy is Pareto-superior independent of social security policy. Again, the result is majorly based on the assumption of an open economy with a fully actuarial social security system (as the sole channel for immigrants to affect the welfare of the locals).

## Welfare Analysis

As the population dynamics is different under each version of the dynastic assimilation, each case needs to be studied separately. Thus,

**Lemma 3** *In Laissez Faire economy the welfare is*

- *in the case of full assimilation*

$$\bar{\mathcal{W}}^{fa} = 1 - \frac{\rho}{1+r} \frac{E_\theta (r - \psi \Phi_\lambda)}{(E_\theta + \psi E_\lambda)} \quad (30)$$

- *in the case of partial assimilation*

$$\bar{\mathcal{W}}^{pa} = 1 - \frac{\rho}{1+r} \frac{E_\theta r (1+r - \Phi_\theta - \psi \Phi_\lambda)}{(E_\theta + \psi E_\lambda) (1+r - \Phi_\theta)} \quad (31)$$

- *in the case of no assimilation*

$$\bar{\mathcal{W}}^{na} = 1 - \frac{\rho}{1+r} \frac{E_\theta r (1+r - \Phi_\lambda (1+\psi))}{(E_\theta + \psi E_\lambda) (1+r) - \Phi_\lambda E_\theta (1+\psi)} \quad (32)$$

**Proof.** Follows from (28) and (5). ■

### The Case of Full Assimilation

In the case of full assimilation of the immigrant dynasties, unfunded PAYG economy reaches new steady state after the first post-migration period. Once the first cohort immigrants father children (that are identical to the natives) and a new cohort of immigrants are allowed to enter, the per-period change in the effective labour stabilises and yeilds constant welfare for everyone. Thus, the initial  $t = 1$  young population has a welfare level different from all the other cohorts:

**Proposition 3** *In the case of full assimilation of the immigrant descendants, the welfare*

$$\widehat{\mathcal{W}}_1^{fa} < \bar{\mathcal{W}}^{fa} < \widehat{\mathcal{W}}_{t>1}^{fa} \quad (33)$$

if, and only if,

$$\lambda < \frac{E_\theta \varphi_u - \varepsilon_u}{E_\theta (\varphi_u - \varphi_s) + \varepsilon_s - \varepsilon_u} \quad (34)$$

and  $\widehat{\mathcal{W}}_1^{fa} > \bar{\mathcal{W}}^{fa} > \widehat{\mathcal{W}}_{t>1}^{fa}$  if the inequality (34) holds with the opposite sign. If the inequaltiy (34) holds as equality, the welfare for all cohorts in the unfunded PAYG economy and Laisser Faire economy is identical.

**Proof.** Direct comparison of the welfare yeilds:

$$\bar{\mathcal{W}}^{fa} - \widehat{\mathcal{W}}_1^{fa} = -\frac{\rho}{1+r} \frac{\psi}{\psi E_\lambda + E_\theta} (E_\lambda - E_\theta \Phi_\lambda) \quad (35)$$

which is positive if, and only if,

$$E_\theta \Phi_\lambda > E_\lambda, \quad (36)$$

negative if  $E_\lambda > E_\theta \Phi_\lambda$ , and zero if  $E_\lambda = E_\theta \Phi_\lambda$ . Similarly,

$$\mathcal{W}_{LF}^{fa} - \mathcal{W}_{t>1}^{fa} = \frac{\rho}{1+r} \frac{\psi}{(\psi E_\lambda + E_\theta)(1 + \psi \Phi_\lambda)} (E_\lambda - E_\theta \Phi_\lambda) \quad (37)$$

the sign of which also depends on the sign of  $(E_\lambda - E_\theta \Phi_\lambda)$ . Further, subsituting for  $E_\lambda$  and  $\Phi_\lambda$  in (36) yeilds (34). ■

The proposition can be understood in the terms of Breyer equivalence result, *i.e.* the welfare in *Laissez Faire* economy is the ‘weighted average’ of the welfare under unfunded PAYG across cohorts. The proof of the proposition also suggests that the difference between the welfare levels of the initial young and the *Laissez Faire* is larger than difference between the welfare levels of farther cohorts and the *Laissez Faire*, *i.e.* the gains or losses from terminating the PAYG is larger for the initial young population than any other agent in later periods.

Moreover, the proposition claims that if the skill distribution of the immigrants is skewed towards the lower end, the initial young population will have higher welfare under the *Laissez Faire* economy, *i.e.* the initial young will initiate social security reforms (in order to redistribute the welfare gains from immigration in their own benefit).

Using the data from the previous chapter (based on Akin, 2012) the inequity (34) can be quantified. Thus,  $E_\theta = 1 + 0.31 \cdot 0.46 = 1.1426$ , and the nominator is  $1.14 \cdot 1.1426 - 1 = 0.3025$ , while the denominator is  $0.46 + 1.1426 \cdot 0.3 = 0.8028$ . Hence, when  $\lambda < 0.3025/0.8028 = 0.377$ , the initial young, *i.e.* the first (policy-setting) cohort, has lower welfare than all the others. Furthermore, the calculated value for share of skilled among immigrants that makes the first cohort better off,  $\lambda > 0.377$ , is well above the calculated share of skilled among natives from the data  $\theta = 0.31$  and the share of the skilled among immigrants  $\hat{\lambda} = 0.06$  (thus suggesting that according to the model the current German population is in favour of full termination of the PAYG system).

## The Case of Partial Assimilation

In the case when the immigrant dynasties partially assimilate, *i.e.* the local born population adopt the skill distribution of the native population while inheriting the fertility levels of the ancestors, the effective labour force changes the composition each period, wherefore, causing intergenerational welfare disbalance in unfunded PAYG economy. This welfare disbalance generates a possibility for welfare redistribution if the PAYG is terminated, *i.e.* the *Laissez Faire* economy is installed.

**Proposition 4** *In the case of partial assimilation of the immigrant descendants, the welfare*

$$\widehat{\mathcal{W}}_1^{pa} < \bar{\mathcal{W}}^{pa} \quad (38)$$



if, and only if,

$$\lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{\varepsilon_u(\Phi_\theta - 1)}{r}}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - \frac{(\varepsilon_s - \varepsilon_u)(\Phi_\theta - 1)}{r}} \quad (39)$$

and  $\widehat{\mathcal{W}}_1^{pa} > \bar{\mathcal{W}}^{pa}$  if the inequality (39) holds with the opposite sign. If the inequality (39) holds as equality, the welfare for the initial young in the unfunded PAYG economy and Laissez Faire economy is identical.

**Proof.** Direct comparison of the welfare yeilds:

$$\widehat{\mathcal{W}}^{pa} - \widehat{\mathcal{W}}_1^{pa} = \frac{\rho}{1+r} \frac{\psi}{\psi E_\lambda + E_\theta} \frac{E_\theta \Phi_\lambda r - E_\lambda (1+r - \Phi_\theta)}{1+r - \Phi_\theta} \quad (40)$$

the sign of which equals to the sign of the nominator of the third ratio on the right-hand side of the eq (40). ■

Thus, similar to the case with full assimilation, there is a possibility that the welfare of the initial young population is higher in the *Laissez Faire* economy *vis-à-vis* unfunded PAYG economy, *viz.* the initial young cohort will prefer to terminate the PAYG system and shift some of the burden of the initial retired population's pension benefit claims to be 'financed' with the immigration-generated gains for the future cohorts.

Further, in the case of partial assimilation the share of skilled among the immigrants needs to be higher than in the case of full assimilation in order for the unfunded PAYG (*vis-à-vis* the *Laissez Faire* economy) to bring higher welfare to the initial young population. As can be seen in the previous chapter the level of welfare in the case of partial assimilation is identical to the welfare in the case of full assimilation for the first two periods. However, according to proposition (6) the welfare under partial assimilation grows after the second period towards a new steady state level. Thus, the welfare in the *Laissez Faire* economy, as the 'weighted average' of the unfunded PAYG levels, guarantees higher welfare. The same relationship can be seen by comparing (34) and (39): The ratio on the right-hand side of the inequality (39) has an additional positive term added at the nominator and a negative term at the denominator, thus yeilding a larger number. Again, the data from the previous chapter can be used to quantify the inequality (39):  $\lambda < 0.63$ . As with the case of full assimilation, the share of skilled among migrants  $\lambda$  – that makes unfunded PAYG preferable for the initial young – is much higher than the current level  $\hat{\lambda} = 0.06$ . Hence, according to the model and the German data any new immigration will make the initial young

population to prefer a social security reform in the case of partial assimilation as well.

**Proposition 5** *In the case of partial assimilation of the immigrant descendants, the welfare*

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} > \bar{\mathcal{W}}^{pa} \quad (41)$$

if, and only if,

$$\lambda < \frac{E_{\theta} \varphi_u - \varepsilon_u + \frac{\varepsilon_u (\Phi_{\theta} - 1)}{r} + \frac{E_{\theta} (1+r) (\Phi_{\theta} - 1)}{\psi r}}{E_{\theta} (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - \frac{(\varepsilon_s - \varepsilon_u) (\Phi_{\theta} - 1)}{r}} \quad (42)$$

and  $\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} < \bar{\mathcal{W}}^{pa}$  if the inequality (42) holds with the opposite sign. If the inequality (42) holds as equality, the welfare for the initial young in the unfunded PAYG economy and Laissez Faire economy is identical.

**Proof.** Direct comparison of the welfare yields:

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} - \bar{\mathcal{W}}^{pa} = \eta \cdot [(E_{\theta} \Phi_{\lambda} r - E_{\lambda} (1 + r - \Phi_{\theta})) \psi + E_{\theta} (1 + r) (\Phi_{\theta} - 1)] \quad (43)$$

where  $\eta = \frac{\rho(1+r-\Phi_{\theta}-\Phi_{\lambda}\psi)}{(1+r)(E_{\theta}-\psi E_{\lambda})(\psi E_{\lambda}+E_{\theta})(1+r-\Phi_{\theta})} > 0$ . Thus, the sign of (43) is determined by the sign of the second term, which is positive if the inequality (42) holds, negative if the inequality has the opposite sign and equal when both sides of the inequality are equal. ■

In effect, the proposition claims that most of the cohorts receive higher welfare in the unfunded PAYG economy as opposed to the Laissez Faire economy if the share of the skilled among migrant population is not very large. However, comparison of the inequalities (39) and (42) shows the threshold value of the share is higher than that in the Proposition 4, *viz.* there is a possibility that the share of skilled is between the two values so that the initial young and the distant future cohorts are better off in the unfunded PAYG economy at the ‘expence’ of the intermediate cohorts. Correspondingly, there are values of  $\lambda$  that make initial young and the immediate future cohorts prefer the Laissez Faire economy, *i.e.* prefer to terminate existing PAYG system.

**Proposition 6** *In the case of partial assimilation*

- (a) *the initial young have the highest welfare in the unfunded PAYG economy compared to all the other cohorts and to the Laissez Faire economy, while all the other cohorts have lower welfare compared to*

the Laissez Faire economy:

$$\widehat{\mathcal{W}}_1^{pa} > \bar{\mathcal{W}}^{pa} > \widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} \quad (44)$$

if, and only if,

$$\lambda > \frac{E_\theta \varphi_u - \varepsilon_u + \frac{\varepsilon_u(\Phi_\theta - 1)}{r} + \frac{E_\theta(1+r)(\Phi_\theta - 1)}{\psi r}}{E_\theta(\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - \frac{(\varepsilon_s - \varepsilon_u)(\Phi_\theta - 1)}{r}} \quad (45)$$

- (b) the initial young have the highest welfare in the unfunded PAYG economy compared to all the other cohorts and to the Laissez Faire economy, while some immediately following cohorts have lower welfare compared to the Laissez Faire economy

$$\widehat{\mathcal{W}}_1^{pa} > \widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} > \bar{\mathcal{W}}^{pa} \quad (46)$$

if, and only if,

$$\frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta(\Phi_\theta - 1)}{\psi}}{E_\theta(\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)} < \lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{\varepsilon_u(\Phi_\theta - 1)}{r} + \frac{E_\theta(1+r)(\Phi_\theta - 1)}{\psi r}}{E_\theta(\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - \frac{(\varepsilon_s - \varepsilon_u)(\Phi_\theta - 1)}{r}} \quad (47)$$

- (c) the initial young have the higher welfare in the unfunded PAYG economy compared to the Laissez Faire economy, while some future cohorts have higher welfare compared to the initial young and to the Laissez Faire economy

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} > \widehat{\mathcal{W}}_1^{pa} > \bar{\mathcal{W}}^{pa} \quad (48)$$

if, and only if,

$$\frac{E_\theta \varphi_u - \varepsilon_u + \frac{\varepsilon_u(\Phi_\theta - 1)}{r}}{E_\theta(\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - \frac{(\varepsilon_s - \varepsilon_u)(\Phi_\theta - 1)}{r}} < \lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta(\Phi_\theta - 1)}{\psi}}{E_\theta(\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)} \quad (49)$$

- (d) the initial young have the lower welfare in the unfunded PAYG economy compared to the Laissez Faire economy, while some future cohorts have higher welfare compared to the initial young and to the Laissez Faire economy

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} > \bar{\mathcal{W}}^{pa} > \widehat{\mathcal{W}}_1^{pa}$$

if, and only if,

$$\lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{\varepsilon_u(\Phi_\theta - 1)}{r}}{E_\theta(\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - \frac{(\varepsilon_s - \varepsilon_u)(\Phi_\theta - 1)}{r}}$$

**Proof.** Follows directly from the *Propositions 4 and 5*. ■

Proposition 6 cases (a) - (c) jointly characterise those cases when the initial young have higher welfare under unfunded PAYG compared to *Laissez Faire* equilibrium, when the share of immigrants is higher than the threshold level given by inequality (39). This case is divided into three subcases. In the case of (a), when the share of skilled in the migrant population is large and the inequality (45) holds, the initial young population prefers the unfunded PAYG economy while all the other cohorts have lower level of welfare compared to *Laissez Faire* economy. In cases of (b) and (c) there are some future cohorts who also benefit from being in the Balanced PAYG economy.

Figures 1 - 7 illustrate different cases that may arise in the case of partial assimilation. The vertical axis shows the level of welfare that the measure takes in each case. The horizontal axes show the time (or the post-migration period, *i.e.* the cohort starting from the initial young) and the share of skilled among migrant population,  $\lambda$ . That is, in each time period the welfare of agents is shown in the unfunded PAYG economy and *Laissez Faire* economy. In the case of *Laissez Faire* economy the welfare does not change over time and thus is represented with the white checkered flat surface: Each cross-point on the surface shows the level of welfare for generation  $t$  (invariable) when the share of skilled among migrant population is  $\lambda$ . The coloured surface shows the time-variant (cohort-dependent) welfare in the *Laissez Faire* economy. The darker shades represent lower levels of welfare

Figure 1 depicts the case of the data that has been used in this and previous chapter (*i.e.* based on Akin, 2012, an assumption that the immigrant population is equal to 5 per cent in each period, and pension replacement rate of 22 per cent calculated based on Eichhorst *et al.*, 2011). The results that the welfare with *Laissez Faire* decreases with the increase in the share of skilled among immigrant population, can be seen in the angle of the white surface: The higher the share of skilled the lower the welfare is. Similar observation for the time 1 (the first post-migration period) in the unfunded PAYG economy illustrates the results of the proposition that the welfare of the initial young increases with the share of skilled among immigrant population. For all the other periods on the unfunded PAYG surface selective immigration corresponds to lower welfare. Further, the results of Proposition 2 (any immigration is Pareto-improving in the given environment) can be seen comparing the welfare levels to the welfare levels to 0.9496 of the *Status Quo* economy.

In figure 1, however, only the cases (c) and (d) of the proposition 6 can be observed, i.e. there are always some later cohorts that have higher welfare than the initial young. Furthermore, for some low values of  $\lambda$  the welfare of the initial young is lower than that of any other cohort in the unfunded PAYG economy and of the own welfare in the *Laissez Faire* economy. (The value of the skilled in the immigrant population from the German Socio-Economic Panel data, as calculated in Akin (2012), is 0.06 or 6 per cent of the entire immigrant population in Germany also belongs to the case when the initial young have the lowest welfare.) Again, it is expected that when the welfare of the initial young is higher in the *Laissez Faire* economy compared to the unfunded PAYG, that is in any case when the share of skilled among migrant population is less than 63 per cent of the total, the initial young will initiate a termination of the unfunded PAYG system.

Figure 2 and 3 illustrate the same results with higher or lower size of immigration. In figure 2 the share of migrants in the total population is assumed one per cent in each period, while figure 3 illustrates the case of 15 per cent. Thus, when the size of immigration is low the welfare gains are also small (in line with Proposition 2). Moreover the magnitude of the effect of the increase in the share of the skilled among immigrant population is smaller with smaller share of the immigrant population in the total. Further, as can be seen from figure 3 even with the large (15 per cent) migration the PAYG system generates welfare losses for the population and only for the very late cohorts the welfare measure reaches 0.99, i.e. the system still generates welfare loss about 1 per cent. Also, with larger share of immigrant population and larger share of skilled among the immigrant population the welfare of the initial young is larger than that of any other cohort in the unfunded PAYG or *Laissez Faire* economy (in line with case (a) in proposition 6), however, the initial young still prefers *Laissez Faire* economy over unfunded PAYG once the share of skilled among the migrant population is lower than 63 per cent (as predicted by Proposition 4).

Figure 4 and 6 illustrate the effects of the skill and fertility premiums. Thus, figure 4 corresponds to the case when the skill level does not translate into an efficiency level, but just define the share of population with high and low fertility. In this case the welfare of the initial young gets immune to the share of skilled among immigrants (as they do not affect the tax base of the first post-migration period, and in the second period when the initial young get retired they receive the promised pensions regardless) and thus makes the *Laissez Faire* economy more attractive (in this case the share of skilled among migrants should be higher than the previously calculated 63

per cent, and actually with the current data the *Laissez Faire* economy guarantees higher level of welfare to the initial young with any level of  $\lambda$ ).

Figure 5 is based on an assumption that there is no difference in fertility levels between immigrants, *i.e.* skilled and unskilled immigrants have equally high fertility rates (replacement rate 1.4 per person based on Akin, 2012). In this case the initial young benefit from higher share of immigrants in the unfunded PAYG economy, but the welfare for all the future cohorts does not change with the share of skilled among migrant population. As opposed to the other cases discussed above the welfare in the *Laissez Faire* economy now increases with the share of skilled among migrant population as the inequality does not hold. By the Breyer-equivalence-result logic if the welfare of the initial young population increases while the welfare of all the other cohorts does not change with the increase in the share of skilled among migrant population then the average welfare (the welfare in the *Laissez Faire* economy) will increase. Alternatively, from the definition of the *Laissez Faire* economy, the new debt created from termination of PAYG is financed equally by all the agents, *viz.* the larger the tax base the smaller the tax rate and the larger the welfare is. Thus more skilled immigrant population increases the tax base and thus the welfare. Furthermore, welfare in the *Laissez Faire* economy is higher than in the unfunded PAYG economy for the initial young cohort, again implying that a social security reform will be initiated.

Figure 5 illustrates a similar case of no fertility premium. However, in this case all the agents in the economy are assumed to have the same unit fertility rate, *i.e.* each period the population is fathering a young-age population of the same size and new migrants enter to form a constant share of total population. In this case, similar to the previous equal fertility scenario of Figure 6, the first generation benefits from the higher share of skilled among migrants while all the other cohorts are indifferent. Furthermore, the equality of fertility rates of migrant and native population makes the tax rate stabilised already from the second period on. Again, the inequality does not hold and the welfare in the *Laissez Faire* economy grows with the increase in the share of skilled among migrant population. However, in this case the welfare in the *Laissez Faire* economy is lower (because of the smaller tax base) than the welfare under unfunded PAYG for the initial young once the share of the skilled among migrants is more than 30 per cent.

Finally figure 7 illustrates a case where migrant fertility is lower than initially assumed. As opposed to previously used (based on Akin, 2012) fertility rate of 1.14 for the unskilled immigrants

a unit rate is assumed, and for the skilled immigrants previously used (data implied) rate of 0.84 is still used. Though this is against the commonly used assumption of immigrants having higher fertility than the natives, the logic of the results still apply. The welfare of the initial young is increasing with the increase in the share of the skilled among migrant population, and the welfare for other cohorts in the Balanced PAYG economy decreases. Further the welfare in the *Laissez Faire* economy decreases with the increase in the share of skilled among migrant population. As opposed to the result of the proposition 6, here the welfare in the unfunded PAYG economy is decreasing over the time to its new steady state, though (in accordance with proposition 2) the welfare is still above the *Status Quo* level of 0.9496.

### The Case of No Assimilation

In the case of no assimilation of the migrant descendants, *i.e.* when the immigrant generations inherit the skill and fertility levels of their migrant ancestor, the welfare in the unfunded PAYG economy starts at the same level as in the cases of full or partial assimilation and either increases or decreases over the cohorts to reach a new level. Meanwhile the welfare in the *Laissez Faire* economy is constant over time: Thus

**Proposition 7** *In the case of no assimilation of the immigrant descendants, the welfare*

$$\widehat{\mathcal{W}}_1^{na} < \bar{\mathcal{W}}^{na} < \widehat{\mathcal{W}}_{t \rightarrow \infty}^{na} \quad (50)$$

*if, and only if,*

$$\lambda < \frac{E_\theta \varphi_u (1 + \psi) - \psi \varepsilon_u - E_\theta}{E_\theta (\varphi_u - \varphi_s) (1 + \psi) + \psi (\varepsilon_s - \varepsilon_u)} \quad (51)$$

and  $\widehat{\mathcal{W}}_1^{na} > \bar{\mathcal{W}}^{na} > \widehat{\mathcal{W}}_{t \rightarrow \infty}^{na}$  if the inequality (51) holds with the opposite sign. If the inequality (51) holds as equality, the welfare for all cohorts in the unfunded PAYG economy and *Laissez Faire* economy is identical.

**Proof.** Direct comparison of the welfare yields:

$$\bar{\mathcal{W}}^{na} - \widehat{\mathcal{W}}_1^{na} = \frac{\rho}{1 + r} \frac{\psi E_\lambda (1 + r) [\Phi_\lambda (1 + \psi) E_\theta - E_\theta - \psi E_\lambda]}{(\psi E_\lambda + E_\theta) ((\psi E_\lambda + E_\theta) (1 + r) - \Phi_\lambda (1 + \psi))} \quad (52)$$

which is positive if, and only if,

$$E_{\theta} (\Phi_{\lambda} (1 + \psi) - 1) > \psi E_{\lambda}, \quad (53)$$

negative if  $\Phi_{\lambda} (1 + \psi) E_{\theta} - E_{\theta} - \psi E_{\lambda} < 0$ , and zero if  $\Phi_{\lambda} (1 + \psi) E_{\theta} - E_{\theta} = \psi E_{\lambda}$ . Similarly,

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{na} - \bar{\mathcal{W}}^{na} = \frac{\rho}{1+r} \frac{\Phi_{\lambda} (1 + \psi) E_{\theta} - E_{\theta} - \psi E_{\lambda}}{(\psi E_{\lambda} + E_{\theta}) (1 + \psi) \Phi_{\lambda}} \quad (54)$$

the sign of which also depends on the sign of  $(\Phi_{\lambda} (1 + \psi) E_{\theta} - E_{\theta} - \psi E_{\lambda})$ . Rearranging the terms will result in the inequality (51). ■

As in the unfunded PAYG economy the welfare of the initial young does not depend on the assimilation that the immigrant descendants undergo, the welfare of the initial young increases with an increase in the share of skilled among migrant population in the case of no assimilation as with the cases of full and partial assimilation. Similarly the welfare in the *Laissez Faire* economy, as in the cases of full and partial assimilation, decreases with an increase in the share of skilled among migrant population, however, it decreases faster than in the other two cases (as can be seen from the shape of the white surface in the figure 8). This also decreases the threshold level of the share of skilled among migrant population that makes the welfare in the unfunded PAYG economy above the welfare in the *Laissez Faire* economy for the initial young. Using the data from Akin (2012), as in the previous chapters, the quantified version of the inequality (51) is  $\lambda < 0.46$ . Furthermore, as the distribution of the welfare over the cohorts is smooth in the unfunded PAYG economy, the threshold value of  $\lambda$  also corresponds to the saddle point of the welfare surface, *i.e.* to the line where either welfare starts on higher levels for the initial cohorts and decreases to the lower levels for the future cohorts or starts with a lower levels for the initial cohorts and increases to higher welfare levels for the future cohorts figure 8. From the smoothness of the surface also follows that once the welfare in the *Laissez Faire* economy is higher compared to the unfunded PAYG and the initial young initiate social security reform, the second and some subsequent cohorts will also be better off.



## Joint Analysis: Selective Migration and Social Security Reform

Political economy of reforms assumes that population, at least the median voter, in an economy has higher welfare after the reform. Current environment assumes that the initial old (or any old) will get their pension benefits once they have contributed to the PAYG system<sup>13</sup>. Hence the initial young is the decisive voter. Further, it is assumed that the initial young makes the voting based on welfare obtained.

Figure 10 shows the welfare of initial young as a function of the share of skilled among migrant population under different conditions: As it was established previously, and can be observed in figure 9, in the unfunded PAYG economy the welfare of the initial young does not depend on the characteristics of migrant descendants (and thus on the dynastic assimilation) as they already receive fixed pension benefits in the second post-migration period when the first local-born immigrants enter the economy. In the *Laissez Faire* economy, however, the entire population of the economy matters and thus the welfare of the initial young also depends on the migrant dynastic assimilation. The results of the previous chapter suggested that the initial young will prefer larger share of immigrants (as can be seen from upward-sloping starred line in figure 10) while the entire economy, as summarised by debt-financed balanced PAYG system, is worse off. Contrariwise, if the debt-financed system is understood as a *Laissez Faire* economy, the initial young will prefer to reform the social security (terminate the unfunded PAYG system) if the share of skilled among migrants is relatively small.

However, in some cases the initial young may reform both policies - the social security and the immigration - at the same time to achieve the highest possible welfare. Thus, if the 'melting pot' system prevails in the economy and the immigrant descendants fully assimilate into the native population, the initial young will strongly prefer selective immigration policy in combination with sustaining the PAYG system. If the 'cultural mosaic' system prevails and the immigrant dynasties do not fully assimilate into the native population the initial young may prefer non-selective immigration policy combined with social security reform (*Laissez Faire* economy). In the case of partial assimilation, as also can be observed from the line with down-pointing triangle

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<sup>13</sup>Perfect commitment (Persson and Tabellini, 2000) to the PAYG system is assumed and not analysed in the current text: The *Laissez Faire* economy suggests that the initial old receive their pension benefits though the PAYG system is terminated, and the Debt-financed PAYG assumed that each cohort will receive pension benefits once they turn old. In no case the old population may face a risk of financing consumption only by own savings.

in Figure 10, in the *Laissez Faire* economy with small share of skilled among migrant population the welfare is comparable to the one in the unfunded PAYG economy with large share of skilled. Thus the result will mostly depend on the comparison of the skill and fertility premiums (and the possibility of perfect migrant selection, which is usually constrained by joint family migration, asylum seekers or the quality of education), and with the benchmark data, where the skill and fertility premiums are rather comparable, the *Laissez Faire* economy with smaller share of skilled among migrants will possibly prevail. In the extreme case when the descendants of migrants do not assimilate into the native population the *Laissez Faire* economy with smaller share of skilled among migrant population will generate higher welfare for the initial young compared to the unfunded PAYG economy with the higher share of skilled among migrant population. Still as the results in the first chapter suggest the preferred policy for the initial young (policy setting cohort) will be welfare depriving for (at least distantly) future cohorts. As can be observed from the figures 1, and 8 the future cohorts and the initial young always prefer the exact opposites.

Further, comparing the welfare under full assimilation to partial assimilation, also seen in figure 9 and figure 10, partial assimilation is preferred in the *Laissez Faire* and in the unfunded PAYG economy, as in the case of partial assimilation the descendants of the migrants retain the fertility rates of their ancestors (assumed to be higher) and thus improve the PAYG system if in the unfunded PAYG economy or increase the tax base if in the *Laissez Faire* economy. In the limit, when the economies get into a steady state, in the unfunded PAYG economy the welfare in the case of full assimilation is

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{fa} = 1 + \frac{\rho}{1+r} - \frac{\rho}{1+\psi\Phi_\lambda} \quad (55)$$

and this level is constant from the second post-migration cohort on, while in the case of partial assimilation the welfare grows further to reach the value

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} = 1 + \frac{\rho}{1+r} - \frac{\rho}{\Phi_\theta + \psi\Phi_\lambda} \quad (56)$$

which is larger than

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{fa} < \widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} \quad (57)$$

as  $\Phi_\theta > 1$ , or the fertility rates of the migrant is larger than the native fertility, by assumption.

## CONCLUSION

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In 9 the welfare in the cases of full and partial assimilation is depicted by the two white planes (that continue from the kink for the welfare of the second post-migration cohort), and as can be observed, the partial assimilation is above the full assimilation for each cohort and  $\lambda$ . Meanwhile, in the case of no assimilation the welfare, starting from the same level as in the other cases (from the level in 10), smoothly converges to the limit value

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{na} = 1 + \frac{\rho}{1+r} - \frac{\rho}{(1+\psi)\Phi_\lambda} \quad (58)$$

which is larger than

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{fa} < \widehat{\mathcal{W}}_{t \rightarrow \infty}^{na} \quad (59)$$

as  $\Phi_\theta > 1$ , or the fertility rates of the migrant is larger than the native fertility, by assumption, and is larger than

$$\widehat{\mathcal{W}}_{t \rightarrow \infty}^{pa} < \widehat{\mathcal{W}}_{t \rightarrow \infty}^{na} \quad (60)$$

if  $\lambda > \theta$ , and reverse if otherwise. Thus meaning that the ‘melting pot’ system is Pareto-inferior to the ‘cultural mosaic’ system (independent of the extent of the assimilation - partial or no assimilation) and full assimilation does cause welfare losses *vis-à-vis* partial or no assimilation. However, in figure 9 the welfare in the case of full assimilation (the coloured surface) is above the one for the case of no assimilation once the share of skilled among migrant population is large. This is due to the data used with small fertility rate for skilled migrant and the assumption of unit fertility in the model (while in the data the native skilled are less fertile compared to the migrants).

## Conclusion

Given the popularity of social security reforms and the general inefficiency of selective migration policies (EU free labour movement, tied family migration, quality of foreign education and the like), this work claims that (unskilled) migration may cause or facilitate a social security reform. As opposed to the usual claim that the unskilled migrants are drain on the social security system and thus threaten the existence of the system, the present work claims that the unskilled migrants

threaten the unfunded social security system even when it is purely earnings-related with no intra-generational redistribution (*i.e.* Bismarkian pension system).

The results are based on the public choice logic, *i.e.* the policy that guarantees higher welfare for the majority of the current population prevails. However, the welfare of the initial retired population is assumed to be constant (as they receive the promised pensions and the interest on the savings is unchanged in the open economy environment). Hence, the results are based on the comparison of the welfare of the initial young. Two stylised environments are considered - an environment with unfunded pay-as-you-go (PAYG) social security and an environment with no pension system (while the claims of the initial old are satisfied and the generated debt is evenly distributed over all the generations). In essence, the idea of unskilled-migration-induced intergenerational-conflict is used to claim that the policy-setting cohort may prefer to abandon the social security in order to redistribute the gains from unskilled migration towards themselves.

The work further highlights the importance of the assimilation of the migrant dynasties into the native population. For instance, in case of complete no assimilation the initial cohort will prefer to have only low skilled migrants in combination with a reform in the social policy, *i.e.* moving towards a fully funded system. Further, the work claims that the fully assimilative 'melting pot' policy of some countries is actually inferior to the 'cultural mosaic' policy, where the descendants of migrant families can inherit, at least some, characteristics of their ancestors.

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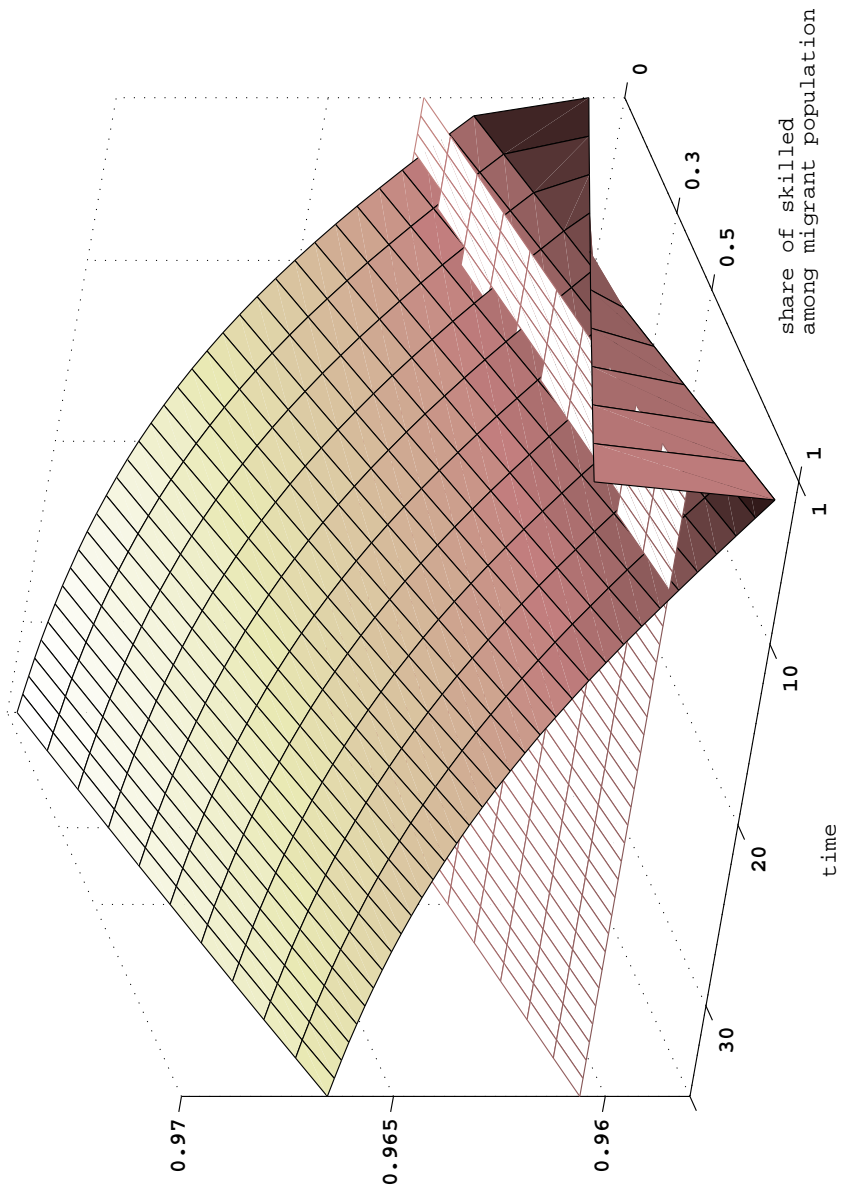
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## Appendix: Figures

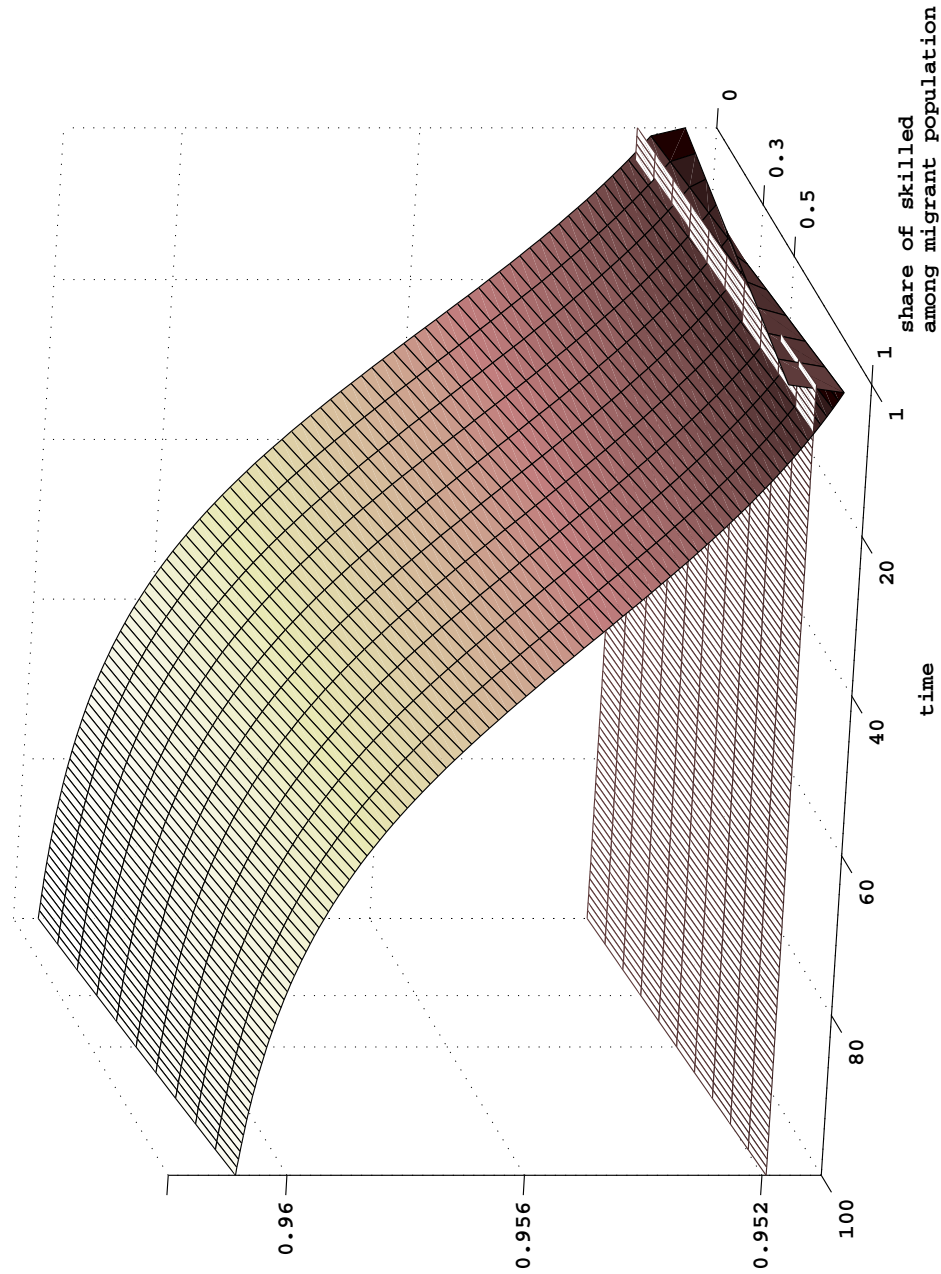
Figure 1: Partial Assimilation with Benchmark Parametres and 5 Per Cent Migration



Partial assimilation of migrant dynasties is parametrised here based on Akin (2012) data. The share of migrant population in the total is assumed constant 5 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

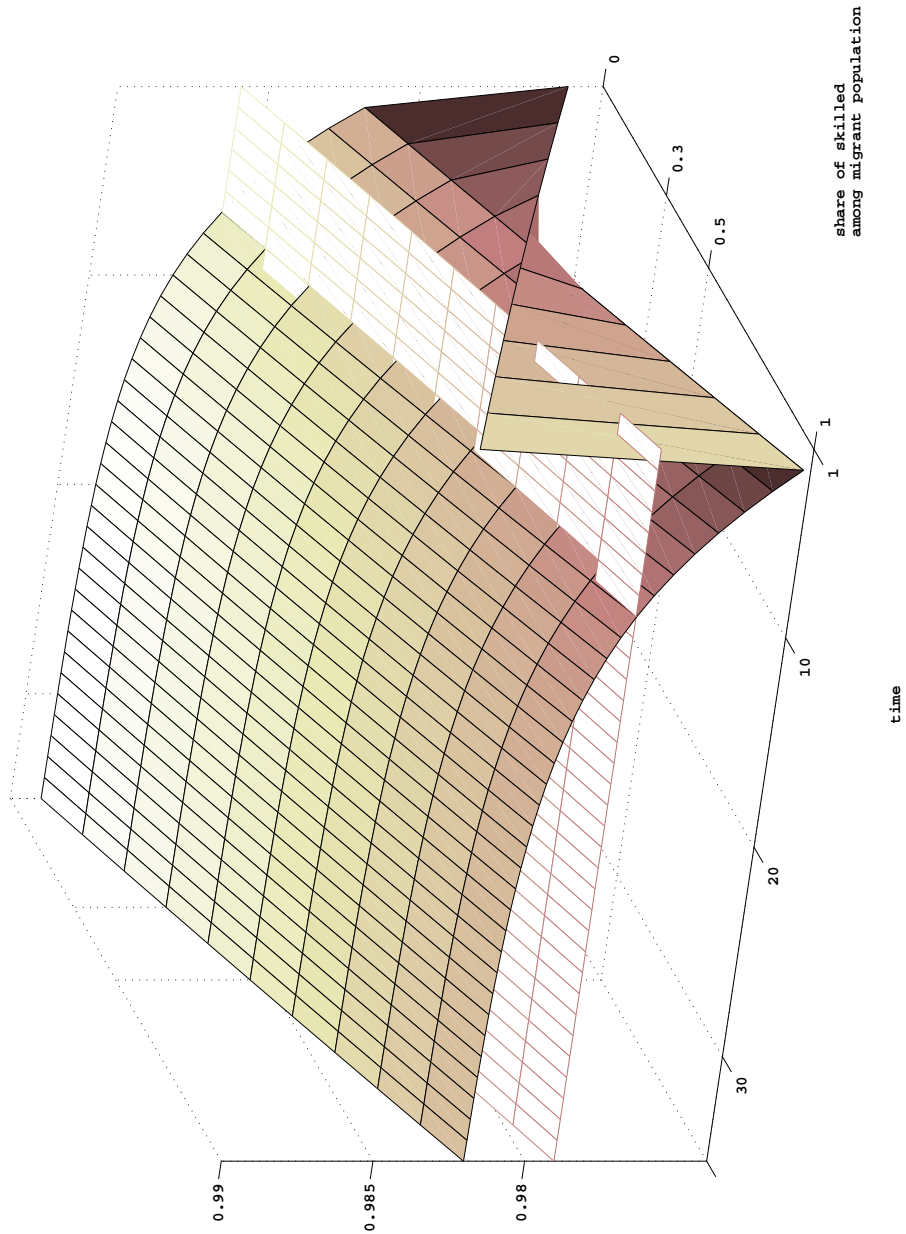


Figure 2: Partial Assimilation: Lower Migration



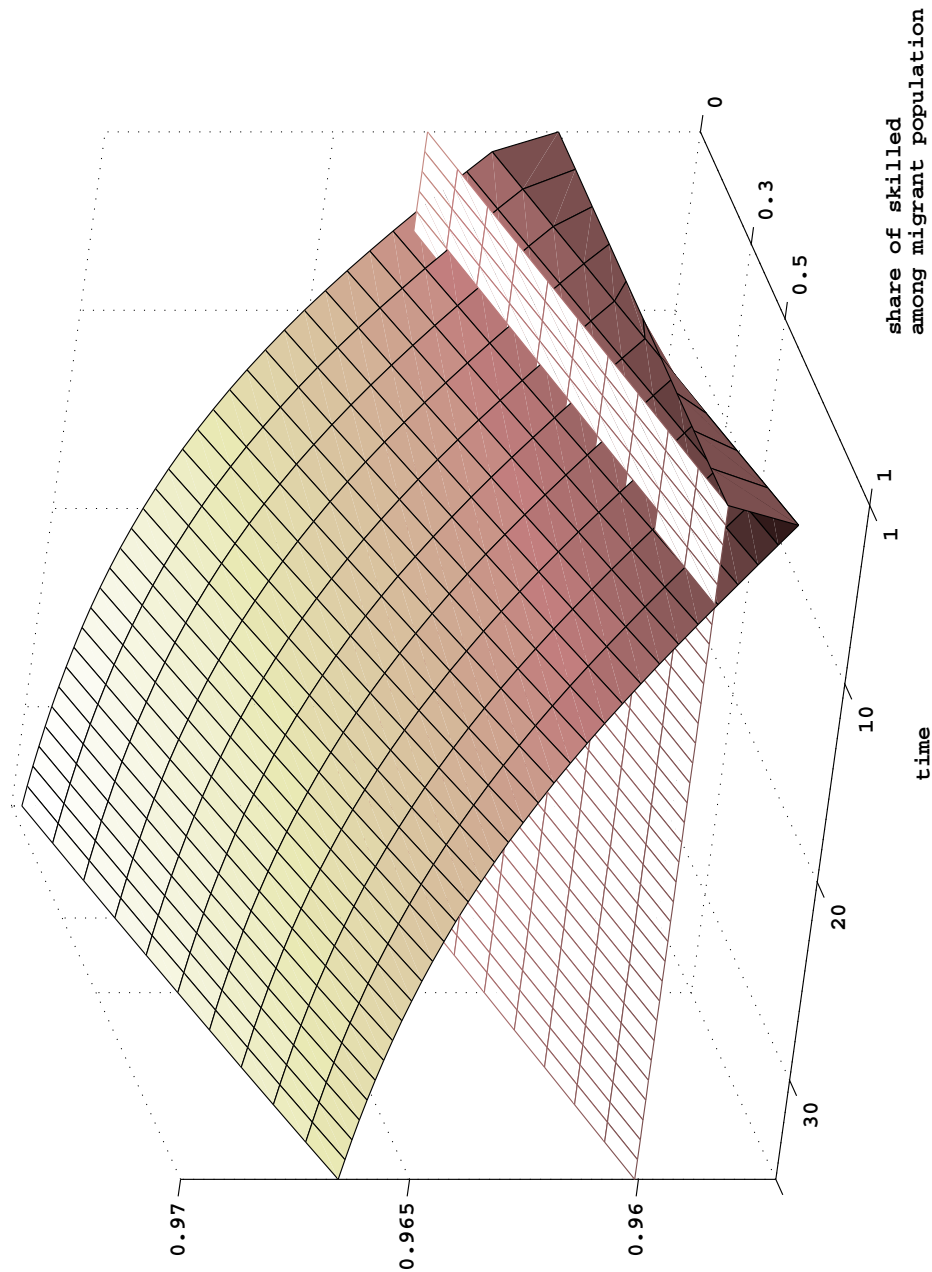
Partial assimilation of migrant dynasties is parametrised here based on Akin (2012) data. The share of migrant population in the total is assumed constant 1 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

Figure 3: Partial Assimilation: Higher Migration



Partial assimilation of migrant dynasties is parametrised here based on Akin (2012) data. The share of migrant population in the total is assumed constant 15 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

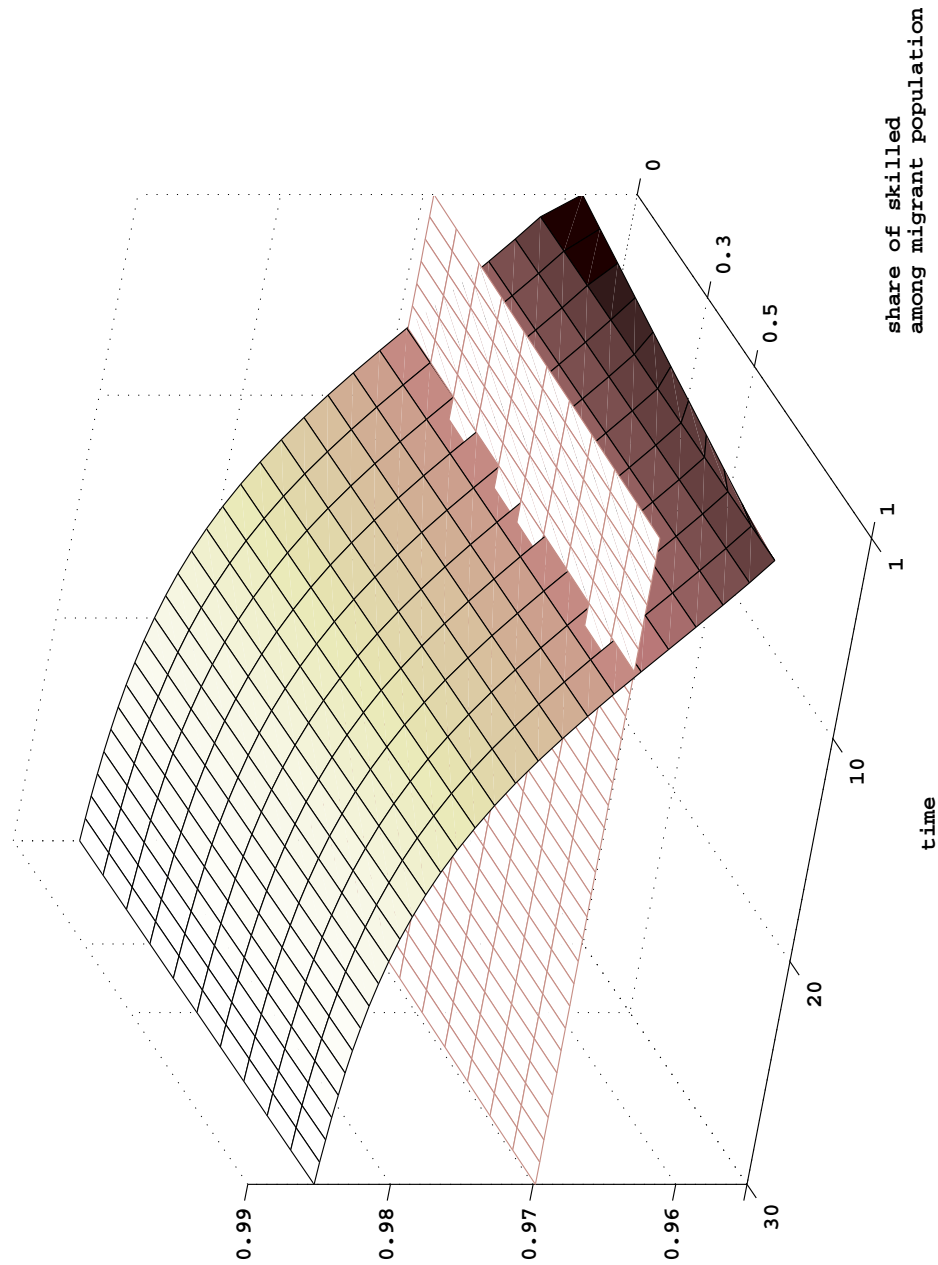
Figure 4: Partial Assimilation: Zero Skill Premium



Partial assimilation of migrant dynasties is parametrised here with 5 per cent migration rate and Akin (2012) data with the exception of the efficiency level (the skill levels are assumed to translate into equal efficiencies, but different fertility rates).

The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

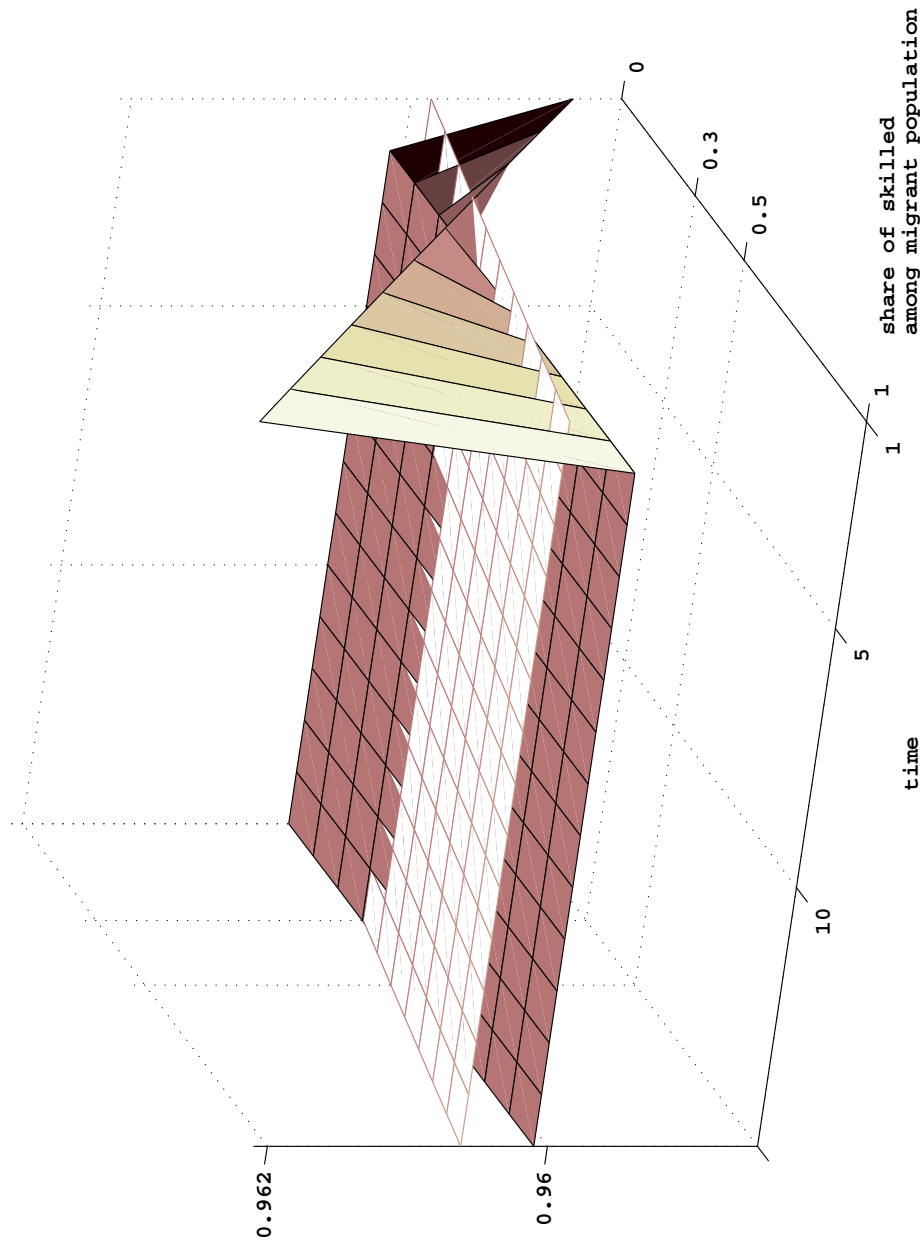
Figure 5: Partial Assimilation: High Fertility



Partial assimilation of migrant dynasties is parametrised here with 5 per cent migration rate and Akin (2012) data with the exception of the fertility level: Immigrants have equally high fertility rates.

The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

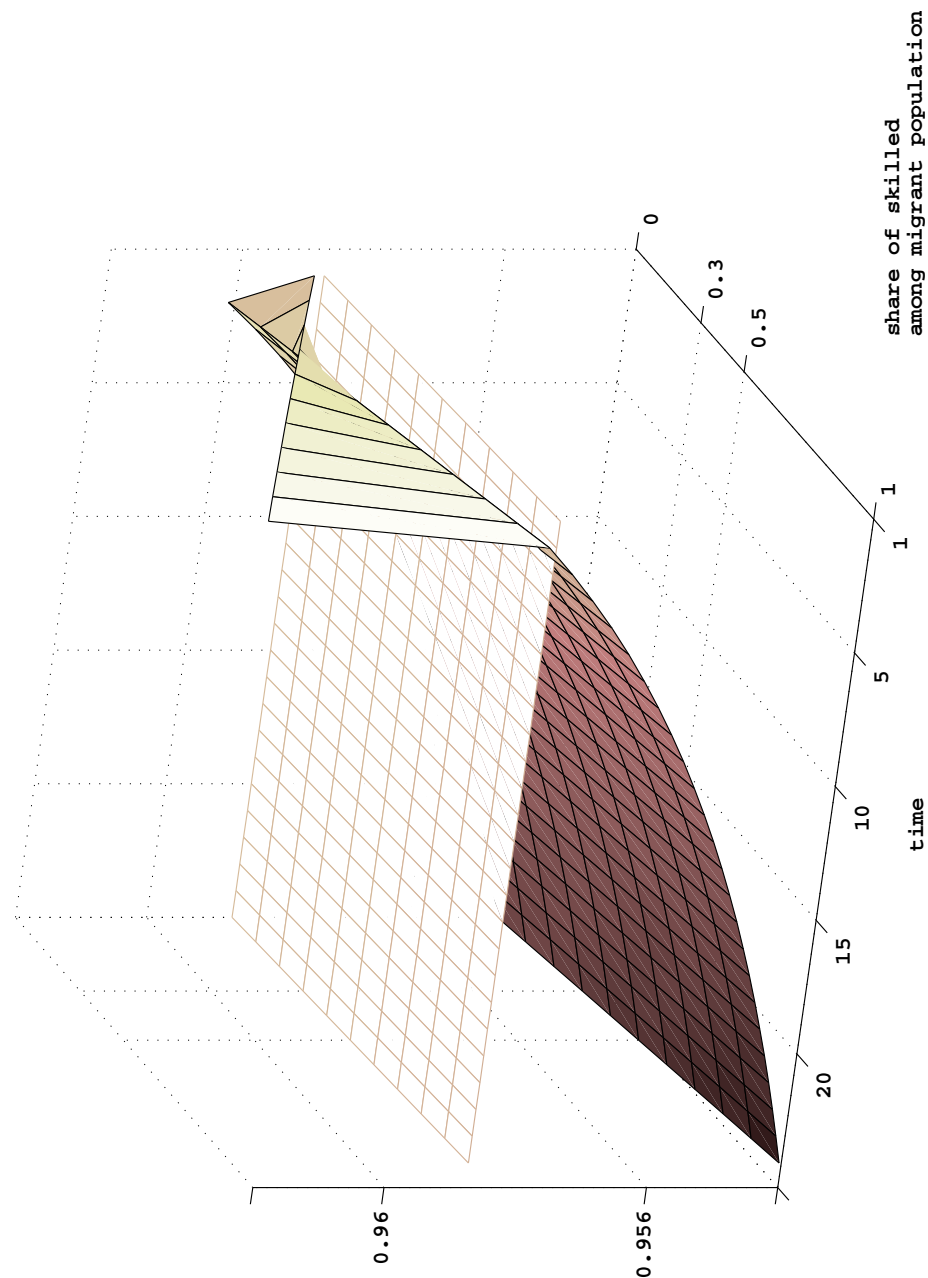
Figure 6: Partial Assimilation: Zero Fertility Premium



Partial assimilation of migrant dynasties is parametrised here with 5 per cent migration rate and Akin (2012) data with the exception of the fertility level: All the population, native and migrant alike, have the same unit fertility.

The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

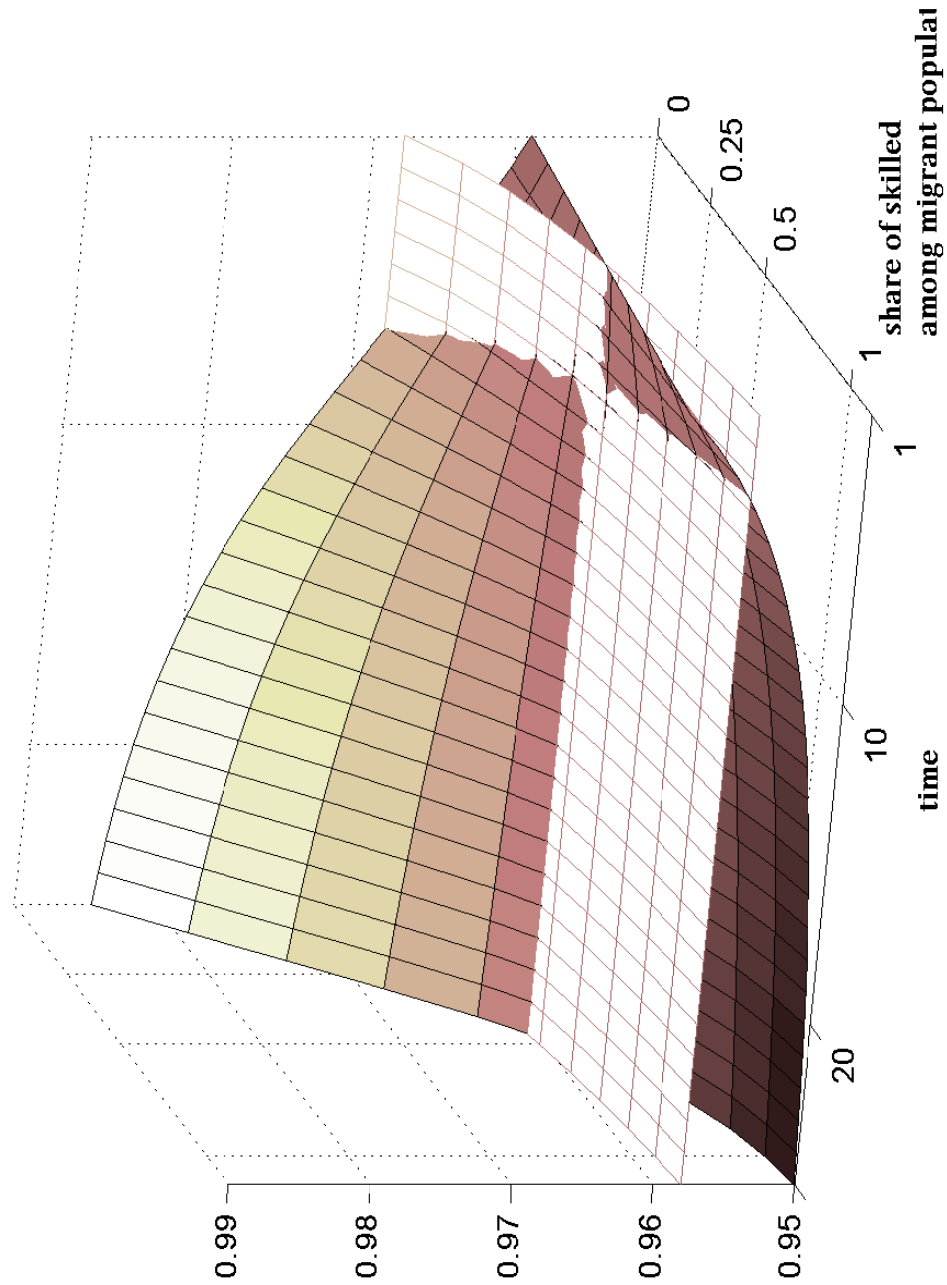
Figure 7: Partial Assimilation: Low Fertility



Partial assimilation of migrant dynasties is parametrised here with 5 per cent migration rate and Akin (2012) data with the exception of the fertility level: While the unskilled immigrants are assumed to have equal to natives fertility rates, the skill migrants are assumed to have smaller rate.

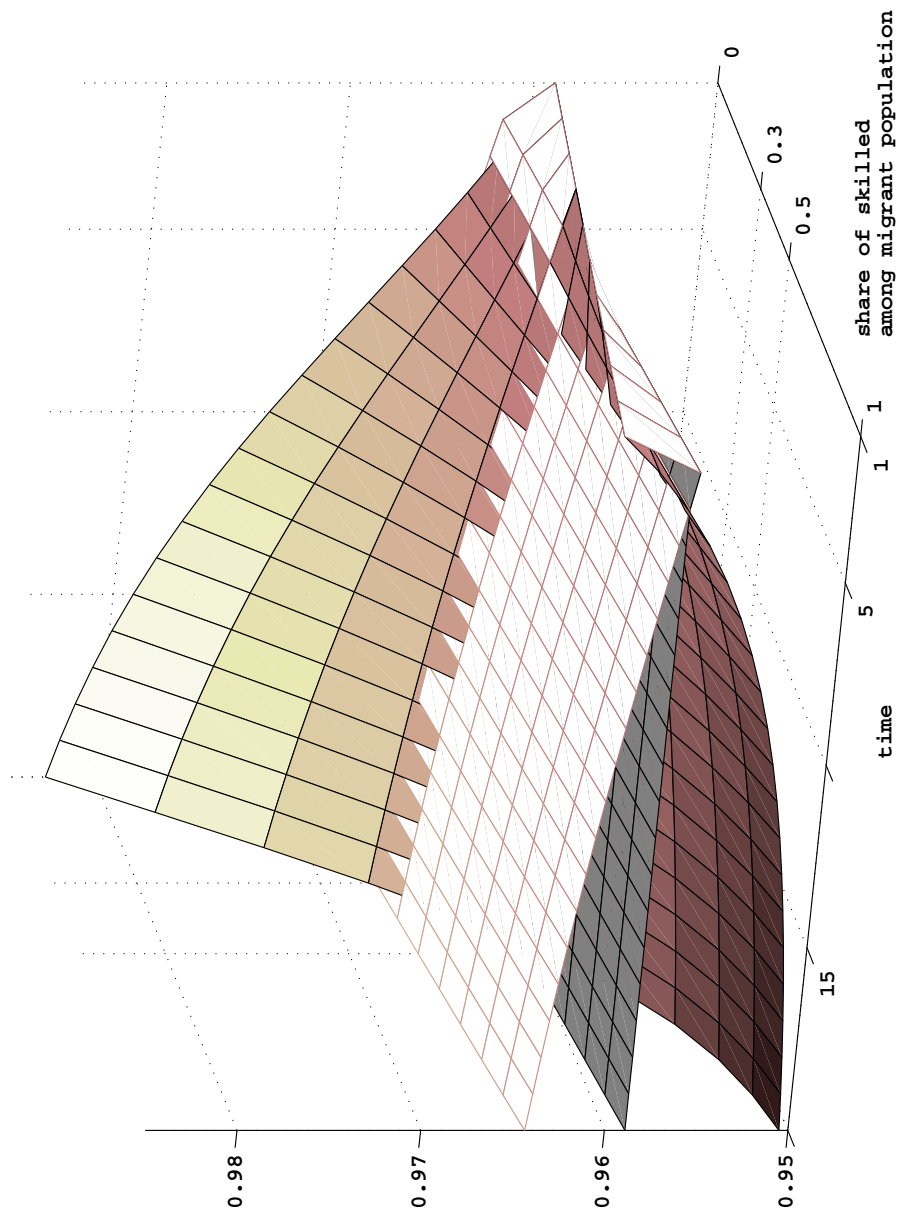
The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

Figure 8: No Assimilation: Benchmark Data



Here no assimilation of migrant dynasties is parametrised based on Akin (2012) data. The share of migrant population in the total is assumed constant 5 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.

Figure 9: PAYG and Dynastic Assimilation

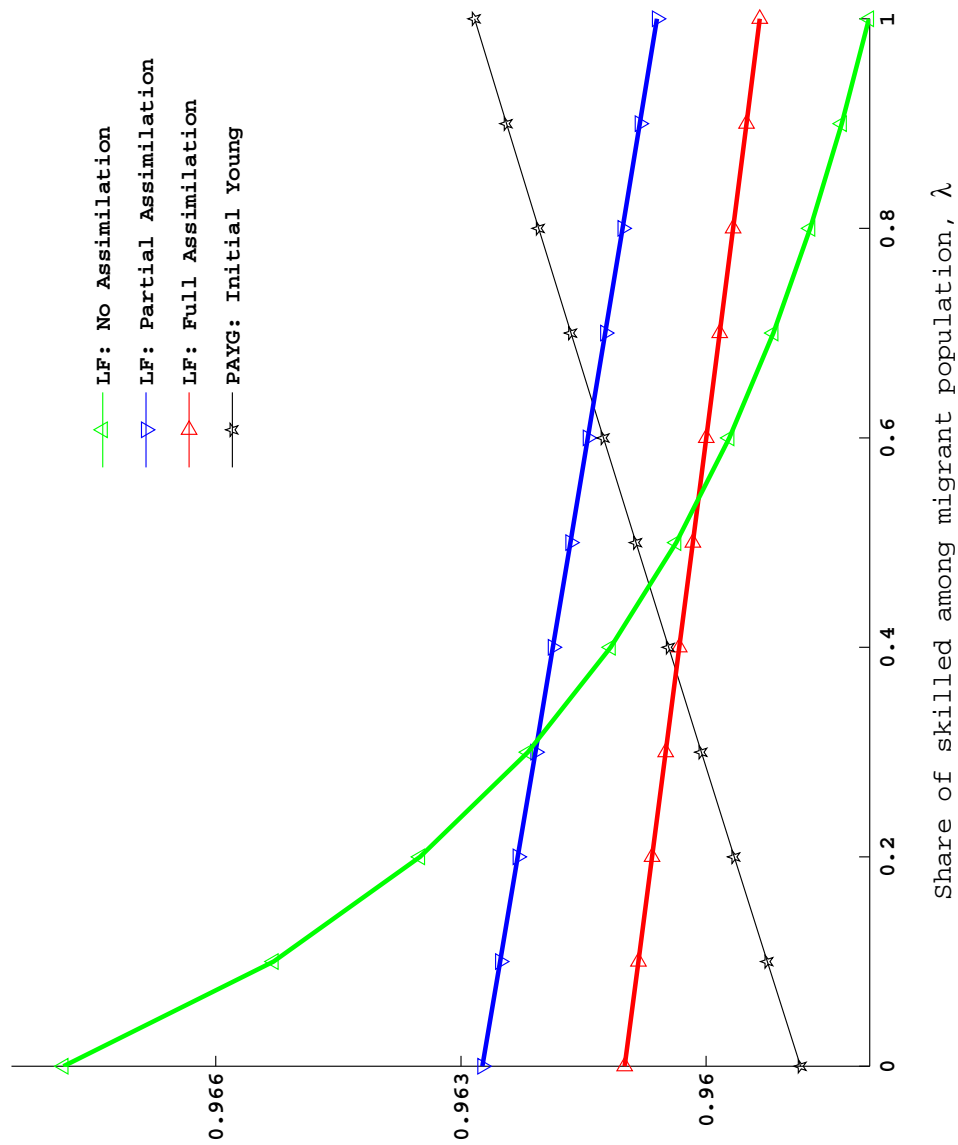


Here the welfare in PAYG economy is parametrised for different assimilation cases based on Akin (2012) data. The share of migrant population in the total is assumed constant 5 per cent at any period.

The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents in case of no assimilation. The white plain surfaces represents the cases of full and partial assimilation (full assimilation is always below the partial assimilation).



Figure 10: Initial Young



Here the welfare for the initial young is presented in various cases: While in the unfunded PAYG the welfare is growing with the share of skilled among migrants, the welfare decreases in case of the *Laissez Faire* economy and is different for each assimilation case.