Pension Pillars and Immigration

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Abstract
A strand of literature is devoted to the discussion of possible usage of immigration as mitigation for public pension problem in ageing economies. It is widely believed that immigration has positive effect on the public finances, in particular, the pay-as-you-go (PAYG) defined benefit social security schemes. This paper, however, shows that the total welfare of the population, both native and immigrant, is increased further if the migrants are kept out of the state managed first pillar PAYG scheme: PAYG system is ineffective and generates some welfare loss for participating population. Not introducing the immigrants to PAYG brings welfare increase. However the increase is not symmetric and a redistributive mechanism needs to be implemented. Computational experiments conducted on German data confirm the theoretical prediction.

1 Introduction
Many developed countries experience population ageing, a joint decrease in birth and mortality rates. This movement in population structure heavily burthens public finances through the pay-as-you-go (PAYG) defined benefit social security schemes, as the tax base is increasingly shortened while claims for benefits are increased. Over the years many possible mechanisms have been suggested to palliate the sitation. One of those suggested mechanism was ‘to import’ the necessary young population, i.e. to increase immigration in countries hit by population ageing.

In two sister papers Razin and Sadka (1999, 2000) presented the fact that introduction of even only low skilled immigrants to the economy with a PAYG

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system installed brings welfare gains to overall population. Lee and Miller (2000) in a similar fashion presented some computational experiments to show the potential benefits from the immigration. Storesletten (2000) in a detailed computational general equilibrium model once again established the fact that immigration can be used as a mitigation mechanism for ageing-troubled PAYG. Volumes of literature have followed (e.g. Uebelmesser, 2004; Krieger, 2005; Kemnitz, 2008; Lacomba & Lagos, 2010) that tackled the problem from different perspectives.

Yet another strand of literature is busy with social security reforms (e.g. Boersch-Supan, 2003; Imrohoroglu & Kitao, 2009), most working around the idea that PAYG-type social security is costly for the ageing population (Aaron, 1966) and that without the hidden efficiency gains from the reforms (e.g. removal of the distortive labour tax) the intergenerational welfare of the population will be invariant (Breyer, 1989). Based on this idea a study on increased immigration and pension reform is redundant: As the initial implicit debt should be paid either postponing to for future via continuing PAYG or turning into explicit debt by a radical reform. In either case, the immigrants as the natives pay exact same tax.

However, the immigration has one important difference, i.e. the immigrants are new to the economy and do not necessarily need to be introduced to the PAYG system, given the availability of pension pillars currently in use. Given the Aaron (1966) idea that the PAYG is costly for an ageing population, it is reasonable to assume that introduction of those immigrants to PAYG, though positive itself, is not optimal, as it is possible to avoid the costs by keeping newly arrived immigrants out of the system.

While welfare generating itself, the current paper shows, the policy of migrant exclusion is welfare diminishing for the native population, vis-à-vis the case of immigrants participating in the system: The immigrants, while participating in the system, are generating welfare losses for themselves, however are contributing to pension taxes and thus partially bare the burden of existing PAYG. Though the generated debt of PAYG is transferred to the general government budget which is financed again by taxes, the natives now have to pay more if the immigrants do not participate in the system.

The case is more evident in the very first period when the immigrants arrive. There is a group of native old people that need to be paid their social security benefits. The current young, both native and migrant, have to pay for the pensions via PAYG contribution and other taxes (to cover the generated deficit). When the
immigrants participate in PAYG they pay exactly the same taxes as the natives and share all the costs proportionally. While if the immigrants are out of the system, the natives only pay the PAYG tax, which generated much larger deficit, that has to be paid by both groups. Thus the natives are worse off, as they cannot shift the burden fully. It is noteworthy that the natives are still benefiting from the arrival of the immigrants as now they at least share the costs of deficit.

The current paper, constructing a simple two period overlapping generations model, shows that a simple tax-deduction that social security contribution brings to the personal budget, can play a redistributive mechanism, and generate effectively a Pareto improvement: The immigrants never enter to the costly PAYG and the natives get back the contribution that immigrants would make in each period. So, according to the paper, seemingly discriminatory policy towards immigrants actually generates welfare gains not as much to the natives as to the immigrants themselves, and immigrants are to share their welfare with the natives in order to generate a Pareto improvement.

The paper also has a numerical exercise conducted. The results are in line with the theoretical model. However, the computational exercise is more rich, and brings in new results, most importantly that some immigrants may actually be worse off, if directly placed in the third pillar pension, i.e. without any tax-deductable pension contribution given the tax brackets in use.

The rest of the paper is constructed as follows: The second part introduces some assumptions and theoretical framework for the study. In the third part the theoretical results are shown. The fourth part shows the computational experiment, and the final, fifth part, concludes.

2 The Model

2.1 The Economic Environment

The economy is characterized with open capital market and closed labor market. Hence the price of capital good is being taken from the world market as given while the price of the labor is being determined in the economy. However a usual CRS production function will be assumed which is identical to having open labor market and importing the wages from the world labor market.

The economy starts with installed pay-as-you-go (PAYG) unfunded social security scheme. The population grows with a non-sufficient rate for the PAYG to
be optimal. In the first period immigration rate will be increased and the policy will continue indefinitely. Two alternative scenarios will be studied - immigrants joining the PAYG, and immigrants are allowed not to join it.

2.1.1 Demographics

The economy is populated with agents who differ in age \(i\), generation in the economy \(g\), and level of education \(e\). There is a measure \(\mu_{i,g,e}^t\) defined on the age \(i\) population of generation \(g\) with level of education \(e\) at time \(t\).

Individuals start their life at age \(i = 0\) and live at most \(I\) years. The probability of surviving to age \(i\), given that the agent of generation \(g\) is alive at age \(i - 1\), is denoted by \(\pi_{i,g}\). The probability of surviving depends on the generation in the economy, as the immigrants grew in a different environment and more often had worse medical treatment and nutrition before migration which negatively influences their survival probability. However second and subsequent generations of immigrants already are disposed to better treatment and in this are identical to natives: \(\pi_{i,1} \leq \pi_{i,g \neq 1}\).

For the theoretical model in the section three, a two period life will be considered, first period agents work, consume and save; while in the second period they are old and consume only their savings and pensions. While in the computational part the model is richer, with agents living maximum of 5 periods during their lifetime. In the first period after they are born they basically do nothing but getting education and consuming transfers from the government. During the period \(i = 2\), Age 2, agents start working and make decisions on the levels of savings and consumption of goods and leisure. At the same period the immigrants are being introduced to the economy.\(^1\) Also the agents are fertile only during the period \(i = 2\). At the period \(i = 3\) the agents conduct the same economic activities as in the previous period. At periods \(i = 4\) and \(i = 5\), if survived, the agents are retired, they get pension benefits and make decisions on consumption and savings (given that there cannot be negative asset possession when retired). All the agents should have left the economy by the end of period \(i = 5\): \(\pi_{6,g} = 0\).

The immigrants are allowed to enter the country when they are at the beginning of the age \(i = 2\). They represent the generation \(g = 1\). The future generations of

\(^1\)This is a technical assumption to avoid (a) the issue of 1.5 generation in case the immigrants are possibly introduced during Age 1; (b) childlessness of the immigrants in case introduced in Age 3; (c) total inactivity of the immigrants in case introduced in Age 4 or 5.
immigrants, \( g = 1, 2, \ldots \), are born in the economy; however the generations slowly undergo some assimilation: The more generations of the ancestors have been in the economy, the more similar to natives the agent is. The natives themselves represent generation \( g = 0 \). The theoretical model, for the sake of analytical convenience, assumes that from the second generation the immigrant descendents are natives.

When born the level of education, \( e \), that agents ‘attain’ during the period \( i = 1 \) of their life is revealed. Each generation \( g \) has its own distribution for the level of education \( H_{g,t} = (\eta_{g,e,t}|e = 1, 2, \ldots) \) where \( \eta_{g,e,t} \) is the probability of having education level \( e \) for an agent from generation \( g \) at time \( t \). It is most often assumed that the immigrants on average have lower education, or in other words, their distribution is skewed towards lower educational levels. However, if the immigration policy is designed to select the desired type of immigrants, as the ‘point’ system of immigration in some countries, it is possible to have the immigrants’ distribution of education level skewed towards higher educational level.\(^2\) Again, the theoretical model will assume no differentiation for skills.

Following Card (2005), rate of assimilation is defined as ‘1 minus the intergenerational correlation (p.320),’ where the intergenerational correlation shows the effect of the parent’s education on the child’s education for generation \( g > 0 \):

\[
H_{g+1,t} = \rho H_{g,t} + (1 - \rho) \bar{H}_t
\]

where \( (1 - \rho) \) is the rate of assimilation, and \( \bar{H}_t \) is the mean of the distribution. Thus it is assumed that the immigrant generations necessarily assimilate and that all the generations have the same rate of assimilation.

The education level is one of the determinants of the efficiency level \( \varepsilon_{i,g,e} \) of the type \( (i, g, e) \) agent in the labor market. The efficiency also depend on the age (usual Mincerian model), and generation: Generation \( g = 1 \) will have different productivity as the education in the home country of the immigrants is supposedly worse than in the host economy. Low efficiency level of immigrants compared to the locals with the same education can be explained also with existing bureaucracy and discrimination against immigrants in the host economy (Krieger, 2005, p.91).

The immigrants, natives and immigrant generations also differ in their fertility rates (e.g. Lee & Miller, 2000). As in the case of labor efficiency, fertility \( \varphi_{g,e} \) is also affected by the other individual characteristics: As it was already mentioned

\(^{2}\text{This model does not include the possibility of educating the young immigrants, which may be studied in somewhat extended model.}\)
above it is assumed that the agents are fertile only during the period \( i = 2 \) of their lifespan (that is why the subscript for the age of the agent is omitted). It is also well established fact in the demographic (and economic) literature that the fertility rate is decreasing with the education: \( \frac{\partial}{\partial e} \varphi_{g,e} < 0 \).

However, the demographic literature still has not reach to a conclusion on the fertility rates of the immigrant generations. Though it is largely accepted that the immigrants have higher fertility rates than natives, the most recent studies (e.g. Milewski, 2007) claim that the second and subsequent generation of the immigrants have the same fertility rate as the natives if it is controlled for the individual characteristics, including level of education and marriage. Nevertheless, they also claim that the immigrant generations still have higher levels of nuptiality compared to the natives, viz. higher fertility rates per immigrant. Basing on the idea of Hill and Johnson (2002) that the generation ‘serves as a proxy for changes in other personal characteristics (p.59)’ (1) type of assimilation rule will be used in this model for the fertility levels of the immigrant generations:

\[
\varphi_{g+1,e} = \hat{\rho} \varphi_{g,e} + (1 - \hat{\rho}) \varphi_{0,e}
\]

where \( (1 - \hat{\rho}) \) is the rate of assimilation in fertility rates.

Depending on the generation agents are differently introduced to the economy. While the government chooses the type and age of immigrants, the others are being born at the beginning of Age 1 and draw their level of education from the distribution \( H_g \):

\[
\begin{align*}
\mu_{1,0,e}^t &= \sum_e \mu_{2,0,e}^t \cdot \varphi_{g,e} \cdot \eta_{0,e} & \text{for } g = 0 \\
\mu_{1,g+1,e}^t &= \sum_e \mu_{2,g,e}^t \cdot \varphi_{g,e} \cdot \eta_{g+1,e} & \text{for } g > 0
\end{align*}
\]

and each period immigrants are allowed with a size of a fixed percentage, \( \psi \), of the age \( i = 2 \) agents in the economy:

\[
\mu_{2,1,e}^t = \psi \cdot \sum_{g \neq 1} \sum_e \mu_{2,g,e}^t \cdot \eta_{1,e} & \text{for } g = 1
\]

On the other hand some agents of different ages will leave the economy based on the survival probability:

\[
\mu_{i+1,g,e}^t = \mu_{i,g,e}^t \cdot \pi_{i,g}
\]

For the sake of convenience again a simplified version of the model will be used for the theoretical results. The native population (including the generations of immigrants) will grow at a rate of \( (1 + \zeta) \), and the immigrants will be allowed to the country as a \( \psi \) share of the current young population.
2.1.2 The Preferences and Household’s Problem

Each agent in this economy comprises a household whose preferences are represented by a time-separable, nested CES utility function. Thus a type \((g, e)\) agent born at time \(t - 1\) has the following utility function:

\[
\mathcal{U}_t = \max \left\{ \frac{1}{1-\gamma} \sum_{i=2}^{I} \beta^{i-1} \left[ \alpha_1 \frac{1}{1-\nu} \left[ c_{t,i}^{1-\nu} + \alpha (1 - n_{t,i})^{1-\nu} \right] \right] \prod_{j=0}^{i-1} \pi(j, g) \right\}
\]

(6)

where \(c_{t,i}\) is the consumption of the agent at age \(i\) at time \(t\), while \(n_{t,i}\) is the time spent in the labor market. Here, the parameters \(\beta, \nu, \gamma\) and \(\alpha\) represent rate of time preference, the intra- and intertemporal elasticity of substitution, and the leisure preference, respectively.

The utility function does not include any activity done at the age \(i = 1\). There are two main reasons for it: First, as the first generation immigrants are absent from the economy during period \(i = 1\) of their lifespan then by default they would have lower level of utility compared to the local born agents, and, second, the agents do not optimize at the age \(i = 1\) but rather they consume the government transfers.

As it is mentioned before the agents do consume in the first period: Their consumption is, however, mere the government transfers, \(c_1 = \chi_{1,g,e}\), which are age, generation and type specific. Thus the higher the education, the more spent on the agent in the period \(i = 1\) of the lifespan. Government gives transfers also in other periods of the lifetime which, together with the labor income, \(w \in n_i\), in each period, public pension benefits when retired, and interest on savings if made, are the only income sources for the agents. On the other hand the income is spent on consumption of goods, savings, taxes and contributions to public pension:

\[
c_i (1 + \tau^n_i) + a_{i+1} \leq w \in n_i (1 - \tau^n) + (1 + r) a_i - T_i (h_i) + P_i (h^i) + \chi_i
\]

(7)

where \(a_i\) is the savings (debts) made in period \(i - 1\), \(\tau^n_i\) and \(\tau^n\) are the taxes payable to government for consumption and income; \(w\) and \(r\) are the prices from the world markets of labor and capital, respectively.

Interaction with the social security system shows up in the household budget constraint (7) with two terms - the contribution \(T_i (h_i)\) which is a function of the agent’s labor market participation at period \(i\), \(h_i = w \in n_i\), and the pension benefit \(P_i (h^i)\) which is a function of the history of the agent’s social security contributions up to period \(i\), \(h^i = \{T(h_j)\}_{j<i}\). The pension benefit is nonzero if the agent is
retired and had ever contributed to the social security system: \( P_t (h^t) = 0 \) if \( i < 4 \), or \( T (h_j) = 0 \) for all \( j \). In case of social security reform \( T_t (h_k (i, m, g, e)) = 0 \), \textit{viz.} all who already contributed to the system will get benefits, others do not contribute and thus do not get any benefit in future.

2.1.3 The Production and Firm

As it was mentioned above there is a representative firm in the economy, which produces one final good using the production function

\[
Y_t = F (K_t, N_t)
\]

where \( N_t \) is the total effective labor offered in the economy for the price \( w_t \)

\[
N_t = \sum_{i=2}^{3} \sum_{g,e} n_{i,g,e} e_i(g,e) + h_{i,g,e}
\]

and \( K_t \) is the capital borrowed from the world capital market with the offered price.

The firm maximizes its profit subject to paying for the capital and labor resources as well as for the depreciation of the capital. Initially Cobb-Douglas production function will be assumed with capital share \( \theta \).

2.1.4 The Fiscal Policy and Government

The role of the government is to maintain fiscal and immigration policies. For the fiscal policy the government is using two separate units - the social security and general government budget. Social security budget is being balanced each period by collecting public pension contribution to distribute pension benefits. Possible deficit or the proficit of the social security budget is being controlled by the interacting term with the general government budget. Thus the social security budget is

\[
\sum_{i=4}^{5} \sum_{g,e} P_t (h^t (i, g, e)) \mu_{i,g,e}^t = \sum_{i=2}^{3} \sum_{g,e} T_t (h_i (i, g, e)) \mu_{i,g,e}^t + B_t
\]

and the general budget of the government is

\[
\sum_{i=1}^{5} \sum_{g,e} \chi_{i,g,e} \mu_{i,g,e}^t + B_t + D_t = \sum_{i=1}^{5} \sum_{g,e} (\tau_i^c c + \tau^n w e n) \mu_{i,g,e}^t + D_{t-1} (1 + r)
\]
Here $D_t$ represents government savings or assets. The possibility of having assets (or debt) gives dynamic (unbalanced) nature to the general budget of the government. The definition of unfunded public security system requires the social security budget be balanced each period (Uebelmesser, 2004), conversely $D_t$ and $B_t$ in the general budget of the government virtually make the social security budget unbalanced via ‘consolidated’ budget. However, as Linbeck & Persson (2003) claim all the governments do violate the balancing condition to some extent. In order to keep the consolidated government, and thus social security, budget balanced it will be assumed that ratio of the government assets to total production is constant over time:

$$D_t = \delta Y_t$$  \hspace{1cm} (12)

and will allow the consumption tax to balance the budget.

For the analitical simplicity the theoretical part of the paper will assume zero initial government assets and zero present value of the time-infinity assets, while granting a possibility of freely borrowing or lending in the international asset market. This assumption gives a balanced long term budget, and is welfare neutral (Breyer, 1989).

2.2

3 Results: The algebraic analysis

3.1 Notation and assumptions

The population starts with $N_0$ population and grows at the rate of $(1 + \varsigma) (1 + \psi)$ for natives as well as in total. However, at time $t$ the total population will be equal to $T_t = N_0 (1 + \varsigma)^t (1 + \psi)^t$ while for the natives it is $N_t = N_0 (1 + \varsigma)^t (1 + \psi)^{t-1}$ (except for $i = 0$ case). The PAYG tax rate will be denoted with $\theta$, the benefit rate with $\phi$, the tax (again on labor income) will be $\tau$ with a subscript and superscript where appropriate to denote the cases of immigrant non-participation or participation in PAYG (viz. discrimination and non-discrimination).

As a measure for welfare only the government policy relevant parameters will be used. Based on the principle of consumption smoothing and Imrohoroglu & Kitao (2009) idea of invariant labour elasticity, the use of the following expression as a direct welfare measure can be proved:

$$W = 1 - \tau (1 - \alpha) - \theta + \frac{\phi}{1 + r}$$  \hspace{1cm} (13)
where \( \alpha \) is the tax-deduction that comes with participation in PAYG. Thus the immigrants, when decided not to participate in PAYG will have much simpler welfare function, just \( W = 1 - \tau \).

### 3.2 PAYG inclusion of immigrants

In case of full participation in PAYG the government has to decide on the tax level \( \tau \) to balance the intertemporal budget:

\[
\sum_{i=1}^{\infty} \frac{T_i \tau_{nd} (1 - \alpha)}{(1 + r)^{i-1}} + \sum_{i=1}^{\infty} \frac{T_i \theta}{(1 + r)^{i-1}} = N_0 \phi + \sum_{i=2}^{\infty} \frac{T_{i-1} \phi}{(1 + r)^{i-1}} + \sum_{i=1}^{\infty} \frac{D_i - D_i (1 + r)}{(1 + r)^{i-1}}
\]

(14)

that easily solves for the tax rate:

\[
\tau_{nd} = \left[ \frac{\phi}{1 + r} \left( 2 - \frac{(1 + \xi) (1 + \psi)}{1 + r} \right) - \theta \right] (1 - \alpha)^{-1}
\]

(15)

For the natives the welfare comparison can just be comparison of this tax rate to the one that would balance the government budget in case of migrants’ non-participation (or discrimination against migrants).

### 3.3 PAYG exclusion of immigrants

In this case the government solves for \( \tau^d \):

\[
\sum_{i=1}^{\infty} \frac{N_i \tau^d (1 - \alpha)}{(1 + r)^{i-1}} + \sum_{i=1}^{\infty} \frac{\psi N_i \tau^d}{(1 + r)^{i-1}} + \sum_{i=1}^{\infty} \frac{N_i \theta}{(1 + r)^{i}} = N_0 \phi + \sum_{i=2}^{\infty} \frac{N_{i-1} \phi}{(1 + r)^{i-1}} + \sum_{i=1}^{\infty} \frac{D_i - D_i (1 + r)}{(1 + r)^{i-1}}
\]

(16)

with the answer:

\[
\tau^d = \left[ \frac{\phi}{1 + r} \left( 1 + (1 + \psi) \left( 1 - \frac{(1 + \xi) (1 + \psi)}{1 + r} \right) \right) - \theta \right] (1 - \alpha + \psi)^{-1}
\]

(17)

which is similar to the tax rate \( \tau_{nd} \) from (15) however has more emphasis on the size of the immigration.
3.4 Welfare comparison

As it was mentioned before it is enough to directly compare the tax rates and in case $\tau^{nd} - \tau^d > 0$ the natives will prefer the immigrants to be out of PAYG. The condition is satisfied when:

$$\theta < \frac{\phi}{1 + r} \left( 1 + \alpha \left( 1 - \frac{(1 + \psi)(1 + \zeta)}{1 + r} \right) \right)$$

(18)

For the migrants, the welfare functions from (13) should be compared, having in mind that in case of PAYG exclusion, immigrants do not get tax-deduction. The welfare comparison for the migrants will give exactly the same condition as in (18).

Note that when the tax-deduction rate $\alpha = 0$, the condition (18) collapses to $\theta < \phi (1 + r)^{-1}$, which is the condition that PAYG itself is welfare improving. Basically the exact opposite of it is the lower bound for the PAYG contribution rate $\theta$, as a welfare generating PAYG makes the analysis redundant. So the values that $\theta$ can take are:

$$\frac{\phi}{1 + r} < \theta < \frac{\phi}{1 + r} \left( 1 + \alpha \left( 1 - \frac{(1 + \psi)(1 + \zeta)}{1 + r} \right) \right)$$

(19)

Thus the larger tax-deduction rate the many values of PAYG contribution rate satisfy the Pareto enhancing policy. Further, using the limit values of the contribution rate it is possible to calculate the limit values of the tax rates from (15) and (17). In case of the maximum value, that is, the RHS value from (18), the tax rates (both in case of inclusion and exclusion) is

$$\tau = \frac{\phi}{1 + r} \left( 1 - \frac{(1 + \psi)(1 + \zeta)}{1 + r} \right)$$

(20)

And in case of the minimum value, the tax rates are:

$$\tau^{nd} = \frac{\phi}{1 + r \ 1 - \alpha} \left( 1 - \frac{(1 + \psi)(1 + \zeta)}{1 + r} \right)$$

$$\tau^d = \frac{\phi}{1 + r \ 1 + \psi - \alpha} \left( 1 - \frac{(1 + \psi)(1 + \zeta)}{1 + r} \right)$$

(21)

(22)

In case of the highest possible value of PAYG contribution, the balancing tax rate is independent of the tax-deduction rate (i.e. the rate by which the natives transfer the burden on the immigrants) as the exclusion of the immigrants with that rate is too costly for the natives. On the other extreme, when the PAYG rate is so low that it almost allows the PAYG system to run without deficit, the
natives and imigrants can agree on sharing the burden of already existing PAYG, and excluding the migrants with a less tax rate: \( \tau^d < \tau^{nd} \). Moreover, the higher the \( \alpha \), the higher is the pay for non-discriminatory PAYG-inclusion tax, as in this case the immigrants get the tax-deduction, and thus, nominally, the tax rate should be higher.

4 Results: The numerical analysis

4.1 Parameterization (or the German Economy)

In this section the model will be parametrised to fit the German economy. The German economy have couple of important advantages: First, while being an immigration country for a while, the German government only recently started to address the issue and shape a policy (Schily, 2007), thus the experiment with the German data has a large practical value. Further, the German economy is open and much more fits the model. And finally, the German pension system is highly actuarial, so no significant labour-effect from the policy changes could be expected (as modelled initially). As the German data is already calibrated for a similar type of model (Akin, 2007), most of the parameters will be transferred from there directly.

4.1.1 Individuals: Demographics and Preferences

The population distribution, fertility rates and other demographic indicators are present in Akin (2007). The main difference from Akin model is the existence of the future generations of immigrants. In order to approximate the data for those generations, the results of Card (2005) is used. Thus Card claims that both macro and micro level data shows that on average the next generation’s characteristics can be explained by thirty percent of the parents characteristics, i.e. the rate of assimilation is \( (1 - \rho) = .7 \).

With this specification agents of generations \( g \geq 5 \) have their characteristics more than 99 percent converged to the natives. Thus, for the rest of the paper, only natives, immigrants and up to the fifth generation will be discussed; the sixth generation already will be native. Effectively this means that the generation can take values \( g = \{1, 2, ... 5\} \). Hence, (3) holds for generations \( g \in [1, 4] \) and for \( g = 5 \) the following is true:

\[
\mu_{1,0,\bar{e}} = \sum_{e} \left( \mu_{2,0,e} \cdot \varphi_{0,e} + \mu_{2,5,e} \cdot \varphi_{5,e} \right) \cdot \eta_{0,\bar{e}}
\]

(23)
As there is no definite information on the rate of assimilation for the fertility rates, and \emph{ad hoc} assumption of $\hat{\rho} = \rho$ will be used.

To keep in line with Akin (2007), and many other contributions to the field, two levels of education will be considered: $e = \{h, l\}$, where $h$ stands for high education and $l$ for low. Further, in order to match Akin model the survival probability is actually removed from the model, making the lifespan of the agents deterministic. At the same time in order to eliminate misbalance of working and retired life all the agents are required to leave the economy after being retired for one period. Thus only four periods remaining in the lifespan, $i = \{1, 2, 3, 4\}$ and the survival probability is $\pi_{i,g} = 1$ for $i < 5$ and $\pi_{6,g} = 0$.

For the sake of simplicity $\alpha$ in the utility function is taken to be zero. As a result the agents will not derive utility from leisure and will devote all the available time to working. Though this is divergence from Akin (2007) it is not a particularly strong assumption, as in case of open economy there is neither general equilibrium effect of increased immigration or increased savings on the wages, nor the labor leisure decision will be distorted in case of removing the almost actuarial social security system of Germany. The preference parameters $\beta$ and $\gamma$ take the value of 0.98 and 2, respectively, as in Akin (2007). The value of intratemporal elasticity of substitution $\delta$ is irrelevant now as the assumption $\alpha = 0$ cancels $\nu$ out of the utility function.

The wages are used to find the efficiency level $\varepsilon_{i,g,e}$: the wage of high skilled age $i = 2$ natives is taken as numéraire and $\varepsilon$ calculated accordingly. The data provided by Akin once again has been used in this case. A working assumption that the migrant descendants have the same efficiency level as the natives is used.

4.1.2 Government: Taxes and Social Security System

The government interaction is designed so to match the existing German system: The social security contribution is 9.75 percent of gross wage earnings with special ceilings for the highest earnings group. There is a progressive formula based income tax, payable after the social security contributions, culminating at around 42 percent for the highest earnings group (overall there are 4 bracket groups with the breaks being 7,665, 12,740 and 52,152 euros of after-social-security-contribution wages). German workers pay sickness, unemployment and long term care insurances as well.
The pension benefits of the agents are ‘point’-based which reflects their relative earnings position in the economy. During each life-period agent’s earning is being compared to the nationwide average earning: The national average is considered one point and the agent is getting points according to:

$$\vartheta = \sum \frac{h_i}{h_i}$$

(24a)

where $h_i = w_i n_i$ as before, and

$$\bar{h}_i = \frac{\sum h_i \mu}{\sum \mu}$$

(24b)

Next each year pension point $\vartheta$ is assigned a value $\Pi_t$, the ‘pension point value’ which is calculated (as simplified in Akin, 2007):

$$\Pi_t = \Pi_{t-1} \cdot \frac{\bar{h}_{t-1}}{\bar{h}_{t-2}} \cdot \left( 1 + \kappa \left( 1 - \frac{ISR_{t-1}}{ISR_{t-2}} \right) \right)$$

(24c)

where $\kappa = .25$ is the allocation factor, and the inverse support ratio is

$$ISR_t = \frac{\sum_{g,e} \mu_{1,g,e}^t}{\sum_{g,e} \left( \mu_{2,g,e}^t + \mu_{3,g,e}^t \right)}$$

(24d)

And finally the pension benefit for an agent is calculated as

$$P_t \left( h^i (i, g, e) \right) = \vartheta \cdot \Pi_t$$

(24e)

For the reformed economy, the agents will stop contributing to the social security system and thus stop generating pension points. In some of the experiments some agent’s would be opt out from the Social Security system. In this case they will neither contribute nor collect any benefits through the system.

4.2 Experiments and results

The paper intends to report on several experiments conducted. First, basic increased immigration will be studied. Second experiment will consider introduction of skill control for immigrants similar to the practice in Canada, New Zealand and some other countries. The other experiments will be following Sainsbury (2006) and considering different levels of immigrants’ participation in the public finances. And finally another group of experiments studies the economic and welfare effects
of privatising the Social Security (i.e. terminating existing unfunded system) in combination with changes in immigration policy. All the results of the experiments are compared to the Status quo (SQ) economy (no changes in either Social Security system or the Immigration policy) as well as to each other.

In the first experiment the immigration policy is increased and the economy now accommodates immigrants with a size equivalent to 0.5 percent of the current population in each period, while the Social Security is intact.

The fertility and the education level of the new immigrants are assumed to be identical to the current immigrants: they are on average less educated compared to the natives and have higher fertility. However, only the low skilled immigrants have fertility rate high enough to reproduce equal (and more) to their number children, and as a result the overall population in the economy is still decreasing. As the model does not have any other source of growth but labor, the decreasing population results in a decreasing production though the extra migration guarantees higher production compared to the SQ economy.

On the other hand, due to qualitative changes in the labor the per capita production has different path when SQ is compared to the reformed economy. While the SQ economy is in a steady state, the introduction of new immigrants brings a leap up which is followed by constant decrease to a below SQ level: In the reformed economy the average agent has lower productivity, and thus the lower level of per capita production, while the leap is generated by the change in the proportion of working age population. Once the initial immigrants get older, retire and produce generations, the source of high per capita production disappears.

As for the welfare: All types of agents of all the generations are better off under reformed economy. There are two channels for the increased welfare, the social security system and the decreased consumption tax. The increased immigration of (on average) low-skilled immigrants decreases the ‘pension-point-value’ in the economy, however on the other hand it decreases the average wage and thus increases ‘individual points’ collected by the agents. Overall the social security benefits grow higher for each and all agents.

The consumption tax also goes down with the extra immigrants introduced to the economy. This decrease also result from two almost orthogonal sources: first though there are more people to share the burden of the public budget debt, the immigrants increase the burden itself with ageing and producing (costly) children. On the other hand, the immigrants make the social security system less costly for the government. In result, the consumption tax decreases from 17.5 per cent to 16.9.
Figure 4 illustrates an experiment where ‘point-system’ is used to choose the immigrants’ skill level. In this case the number of immigrants are allowed in, however, half of them are skilled as opposed to the previous 10 per cent only. The ‘points system’ is beneficial to all the groups. However the mechanism of generating the welfare gains is slightly different, though the sources are still the same social security benefits—vs.—contribution and the consumption tax. The effect on the social security is the opposite of the previous case. Here agents collect less points however the value of each point is higher, resulting again in high benefits. The high skilled immigration contribution to the social security is comparable to the previous case of with the lower skilled immigrants (as the social security system mostly generates losses due to population misbalance). The ‘point system’ also mitigates the burden on the public budget as the skilled migrants pay more taxes and collect less benefits.

The last experiment conducted (illustrated in Fig. 5) compares the economy described in the first experiment, i.e. increased (mostly low-skill) immigration, to an economy where those immigrants are not allowed to participate in the Social Security system. Note that the natives and future generations of the immigrants do participate in the system, as well as the immigrants still use other public funds. This experiment corresponds to the analytical analysis in the previous section.

This case also guarantees higher welfare to all the agents if compared to SQ. The new policy is also beneficial to almost all the agents with the exception of high skilled immigrants. The low-skilled immigrants are benefiting as they do not participate in the costly Social Security scheme. The natives’ welfare increase is mostly coming from eliminating costly participation of the immigrants from the Social Security system, while they still pay for it via consumption tax (sharing with the natives).

The high-skilled immigrants almost are unaffected by this change. However they are suffering a very small welfare losses: Before the tax on labor income and the like were calculated based on the after-social-security-contribution income, while in this case the high-skilled is paying the highest taxes. Thus the high tax brings the most of the burden to the high-skill immigrant who not having a tax-deduction appears in a higher bracket of taxation, and thus suffers the most.

5 Conclusion

The literature prior to this have been discussing the social security reformes, pension pillars and immigration separately. However, in this paper the issues are
combined. The paper showed that actually given the circumstances all the population, both native and immigrant, will be better off (Pareto improvement) if natives are not participating in the PAYG system.

Though initially the reform sounds discriminative, the immigrants themselves will choose not to participate in the PAYG system as it is costly in an ageing economy. However, the natives will prefer the immigrants to participate in the system, as they will take away some of the initial burden from the natives. The solution that this paper suggests is the tax-deducting strategy for the PAYG participation, that exists virtually in all countries. Though it may sound double negative, that the immigrants are not participating in the social security system and that they have to pay higher taxes than the natives, still once the immigrants have the chance they will choose the third pillar, where they would save their own retirement funds and still enjoy the extra welfare of not participating in the costly PAYG.

The paper consisted of two parts: In the first part a simple analytical model was presented with the result, while in the second part a richer model was numerically analysed (based on German data). Results on several experiments are reported: economy with larger number of immigrants, with chosen immigrants, immigrants who are not participating in PAYG and all of those are compared to the case of no changes at all. The results suggest that it is all beneficial to allow more immigrants in, and it is better for all to control the immigrants and to choose the higher qualified immigrants. The effects are due to internal arrangements of the Social Security system, and of immigrants being able to contribute to the public finances more than what they get. In the experiment where the immigrants do not take part in PAYG all the agents but high-skilled immigrants are being better off. Given that the immigrants are not directly participating in an arrangement which is inefficient by construction, and the fact that the other immigrants did not make the losses of the system larger, it might be expected that the high skilled immigrants would also be better off. However in this case the fact that the social security contribution is deductible for the other taxes plays crucial role.
Appendix A.

Figure 1: The welfare of (g,s) agents in SQ economy (in black) and reformed economy (in blue, cyan is the unadjusted version)
Figure 2: Figure 3. The welfare of (g,s) agents in SQ economy (in blue) and reformed economy with low-skilled migrants (in black) and more high skill migrants (dotted)
Figure 3: Figure 4. The welfare of (g,s) agents in reformed economy where migrants are out of SS (blue with diamonds) vis-a-vis reformed economy with SS including migrants.
Figure 4: Figure 5. The welfare of (g,s) agents in reformed economy where migrants are out of any insurance scheme (blue with diamonds) vis-a-vis reformed economy with immigrants excluded only from SS
References


