Melting Pot *vs.* Cultural Mosaic
Dynamic Public Finance Perspective

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Abstract

The traditional immigrant countries can be characterised as either supporting a *melting-pot* or *cultural-mosaic* system: While melting-pot system idealises full assimilation of immigrants, the cultural-mosaic system favours multiculturalism. The current study shows that full-assimilation, *i.e.* the melting-pot system, is inferior to cultural-mosaic system if looked through the lenses of public finance (chiefly, unfunded social security) and hence the population welfare. At the same time, the economic comparison of no-assimilation and partial-assimilation systems depends on the existence of immigrant skill control in the economy.

1 Introduction

Immigration debate always includes some discussion on the 'quality' of the immigrants allowed to enter the country. Very often, if not always, it is assumed that the high quality immigrants are more beneficial for the economy (e.g. Storesletten, 2000). However, economic literature has works (e.g. Razin & Sadka, 1999) that claim low skill immigrants are already beneficial to the economy. Another strand of literature, also, highlights the fact that in case low-skilled immigration is compliment to high skill native work, the low skill immigration is beneficial to the economy (e.g. Cortes, 2008).

The current work discusses the regulation of immigration through the public finances and the prespective of assimilation of the immigrant dynasties: The immigrants are different in their characteristics, and thus when introduced they bring

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distortions to the social security budget. While in the funded system the government is able to transfer the debt over the generations, the pay-as-you-go system has to absorb each generation separately. Moreover, if the future generations of the immigrants do not assimilate they bring further distortions.

The demographic literature (*e.g.* Milewski, 2007) does not have a unique answer to how exactly the immigrant generations assimilate. Thus for the countries with high immigration (*e.g.* US, Germany, Spain) it is useful to know the possible scenarios that the immigration can cause to the social security. In what follows it is shown that depending on the nature of the assimilation the immigrant generations undergo the welfare of the present and cohorts is different. Depending on the nature of the assimilation different cohorts are being gainers or losers of the possible reform.

The demographic side of the paper is mostly standard in the literature. However, two main differences can be highlighted: first, the immigrants do not necessarily have lower skills compared to the natives, and second the immigrant generations do not necessarily assimilate. Three alternative assimilation scenarios are studied: the immigrant generations fully assimilate (a usual assumption in the literature), partially assimilate (keeping the fertility rates of the ancestors but converging to the educational level of the natives), and non-assimilation (neighborhood effect and the like). The theoretical possibility of partial assimilation with keeping the educational level of the ancestor while adapting the fertility rates of the natives is not discussed as it is not observable in real life.

The rest of the paper is organized as follows: Section 2.1 describes the economic environment and gives basic definitions of the terms. Sections 2.2 to 2.6 define status quo, unrestricted pay-as-you-go and fully funded economies, and give the basic results of Pareto-improving property of increased immigration policy, as well as the equivalence result discussed above. Section 2.7 describes 'classic' unfunded social security system and conducts welfare comparison with other economies under diferent assimilation scenarios. Finally, Section 3 concludes the paper.

2 The Model

2.1 The Economic Environment

The economy is presented by a small-open-economy model where different types of agents and the government interact. The model is abstracted from the existence of a firm, though it is not an exchange-economy model. The model assumes that the government and the agents take the world prices of the consumption good and resources - labor and capital. The price of the consumption good is taken as the unity, and the price of using the capital resource and the labor resource for one period is denoted by r and w respectively.

However this approach is equivalent of assuming the existence of a firm which takes the world prices, utilizes all the labor supply in the economy to produce good using the world technology. At the same time it is assumed that the supply of the capital good is infinitely large at the world price. If either of the resources has constant price over time, the other resource's price is also constant given CRS world production function.

2.1.1 Demographics

The economy is populated with agents who differ in their age (i), skills (s) and the generation in the country (g). For simplicity, there are only two groups for age and skills: the agents are either *young* (0) *age* or *old* (1) *age*, and are either *unskilled* (0) or *skilled* (1). Generation can take any value, $g = 0, 1, 2...$, where $g = 0$ for the locals and their generations, $g = 1$ for the newly immigrated agents, and $g = 2, 3...$ for the subsequent generations of the immigrants. A measure of population, denoted by $\mu_t(i, s, g)$, is defined over the agents of type (i, s, g) at time t.

The agents live two periods. During the first period of their lives the agents are young: They supply labor to the market, get wages, make savings and contributions to the government budget. In the next stage of their lives, the agents are non-productive, thus their work is not paid, and however, they get back their savings with the interest and some government benefits. All the agents live both periods, *i.e.* there is no life-uncertainty. For the notation $i = 0$ will be used for the young age, and $i = 1$ for the old age agents (Table 1).

Table 1. Individual Characteristics	
Age $i = \begin{cases} 0 & \text{for } \textit{young} \text{ age} \\ 1 & \text{for } \textit{old} \text{ age} \end{cases}$	*fertility* $\zeta(i,g) = \begin{cases} 0 & \text{for } i = 0 \\ 1 + \zeta(g) & \text{for } i = 1 \end{cases}$
Skills $s = \begin{cases} 0 & \text{for } \textit{unskilled} \\ 1 & \text{for } \textit{skilled} \end{cases}$	*productivity* $\varepsilon(i,s) = \begin{cases} \varepsilon(s) & \text{for } i = 0 \\ 0 & \text{for } i = 1 \end{cases}$
Generation $g = \begin{cases} 0 & \text{for } \textit{locals} \\ i & \text{for } \textit{generation } i \end{cases}$	*skill distribution* $\gamma(s,g) = \begin{cases} \gamma(0,g) & \text{for } s = 0 \\ \gamma(1,g) & \text{for } s = 1 \end{cases}$

The immigrants are allowed to the country when they are young. The further generations are born in the country. Their behavior during the two periods of the lifetime is similar to the natives, *viz.* they work, save and pay taxes while they are young, and get back the savings and some government benefits while old. The immigrants and their generations may be different in their reproductive behavior from the natives (*e.g.* Alders, 2000).

When the locals turn to the second period of their lives they produce young age local agents equal to them in the number, $\zeta(0) = 0$.[1] The fertility rate of the immigrants and their generations are larger or equal to that of the locals: $\zeta(g) \geq 0$.

When born the agents observe their skill level, s. Only two levels will be considered - the unskilled workers denoted by $s = 0$, and skilled workers with $s = 1$.The locals draw their skill level from a distribution which is constant on the aggregate level: The θ part of the local young age agents are skilled and the remaining $(1-\theta)$ part are unskilled. The skill distribution of migrants is assumed to be part of the government policy. Thus, the government chooses $\lambda \in [0,1]$ part of the migrants to be skilled and $(1-\lambda)$ unskilled. The migrant generation may inherit their parents' distribution of skills or the local distribution (Table 1). Both versions will be considered later. For the young age agents the skill level is translated into efficiency level, $\varepsilon(s)$, in the labor market.

2.1.2 Population Dynamics

Agents are differently introduced to the economy depending on their generation. While the government chooses the type and age of immigrants, the others are being

[1] It is worth noting that, though ageing supposes decreasing population rate, unit fertility is still a valid assumption for studying problems connected to unfunded social security system: Aaron (1966) claims that the unfunded social security generates welfare losses for the agents if the natural rate of return is larger than the social security system's rate of return (which is equal to the population growth rate in this model). For that reason only those levels of migration will be studied with which the natural rate of return will still exceed the system's rate of return.

born at the age zero and draw their type from the distribution. The θ part of the newborn locals is skilled, while the rest of the newborns are unskilled. Children of the migrants and their descendants draw their productivity from their distribution, $\gamma(s,g)$, thus the evolution of population can be presented as follows:

$$\mu_{t+1}(0,1,0) = \sum_s \mu_t(0,s,0) \cdot \theta \tag{1}$$

$$\mu_{t+1}(0,0,0) = \sum_s \mu_t(0,s,0) \cdot (1-\theta) \tag{2}$$

$$\mu_{t+1}(0,1,g) = \sum_s \mu_t(0,s,g-1) \cdot (1+\zeta(g-1)) \cdot \gamma(1,g) \tag{3}$$

$$\mu_{t+1}(0,0,g) = \sum_s \mu_t(0,s,g-1) \cdot (1+\zeta(g-1)) \cdot \gamma(0,g) \tag{4}$$

$$\mu_{t+1}(1,s,g) = \mu_t(0,s,g) \tag{5}$$

On the other hand, each period the migrants are being introduced into the economy when they are young. It is assumed that the government policy for the migrants is such that the young age migrants will be equal to a constant fraction of the young age locals:

$$\mu_t(0,1,1) = \psi \sum_{g\neq 1} \sum_s \mu_t(0,s,g) \cdot \lambda \tag{6}$$

$$\mu_t(0,0,1) = \psi \sum_{g\neq 1} \sum_s \mu_t(0,s,g) \cdot (1-\lambda) \tag{7}$$

The effective labor at period t, N_t, is the sum of the young age agents according to their efficiency level:

$$N_t = \sum_s \sum_g \mu_t(0,s,g) \cdot \varepsilon(s) \tag{8}$$

thus the effective labor per average local young age agent can be represented as

$$N_0 = \theta \cdot \varepsilon(1) + (1-\theta) \cdot \varepsilon(0) \tag{9}$$

and the effective labor per migrant:

$$\Lambda = \lambda \cdot \varepsilon(1) + (1-\lambda) \cdot \varepsilon(0) \tag{10}$$

Hence, at the first period the effective labor is the weighted average of the locals at the period zero and the migrants:

$$N_1 = \theta \cdot \varepsilon(1) + (1-\theta) \cdot \varepsilon(0) + \Lambda \cdot \psi \tag{11}$$

where the local young age population at time zero is assumed to be unity.

2.1.3 The Government

The government in this economy conducts fiscal, social and immigration policies. The fiscal constitution includes governing the public debt, B_t, and collecting labor income tax, τ_t. Government also distributes social security benefits to the old age agents, $\varphi_t w\varepsilon(s)$ for the type s agents, where φ_t is the replacement rate[2]. Immigration is under the government control. The government decides on the skill distribution of the migrants, Λ. However, the government policy does not allow for the old age immigrants, as that policy is the same as to increase the population at no cost. In the equilibrium the migration will be set at a constant fraction level, ψ.

It is assumed that the government can also borrow and lend any amount of money any time it needs:

$$\sum_{s,m} \varphi_t w\varepsilon(s)\mu_t(1,s,g) + B_t = \sum_{s,m} \tau_t w\varepsilon(s)\mu_t(0,s,g) + B_{t-1}(1+r) \tag{12}$$

However the government policies should be feasible, thus the following sustainability, no-perpetual-debt-financing condition must be satisfied:

$$\lim_{t \longrightarrow \infty} B_t \cdot (1+r)^{-t} = 0 \tag{13}$$

Further two type of government policies will be discussed and compared - a PAYG system and a *Laissez Faire* system - where the PAYG system is characterized with active government intervention and active social security policy as opposed to the *Laissez Faire* system. In both cases it will be assumed that the economy starts with a zero migration, active social security policies, as well as some initial government assets (debts).

[2]The replacement rate is the ratio to the effective wage that the agents get as social security benefit.

2.1.4 The Households and Welfare

The household in this economy is presented by an individual agent who maximizes lifetime utility, which is derived from consumption in the both periods:

$$U\left(c_t^t, c_{t+1}^t\right) = v(c_t^t) + \beta v(c_{t+1}^t) \tag{14}$$

where c_j^i is the consumption of an agent born at time i during time j; $U(\cdot,\cdot)$ is a time separable utility function with $\beta \in (0,1)$ being the time discount coefficient and $v(\cdot)$ being a continuous, twice continuously differentiable, strictly increasing, strictly concave function that satisfies the Inada conditions[3]. To finance consumption agent uses labor income net of taxes and savings in the first period, and for the second period the agent uses the capital income (from savings) and government benefits. Thus, at time t a $(0, s, g)$ type agent faces the following budget constraints:

$$c_t^t + a_t \leq w\varepsilon(s)\left(1 - \tau_t\right) \tag{15}$$

$$c_{t+1}^t \leq \varphi_t w\varepsilon(s) + a_t(1+r) \tag{16}$$

where a_t is the savings, τ_t is the tax rate, and $\varphi_t w\varepsilon(s)$ is the benefit.

Further, as there are no borrowing constraints, (15) and (16) can be combined into one intertemporal budget constraint:

$$c_t^t + \frac{c_{t+1}^t}{1+r} \leq w\varepsilon(s)\left(1 - \tau_t + \frac{\varphi_t}{1+r}\right) \tag{17}$$

Essentially the intertemporal budget constraint (17) shows that only the present discounted value of the lifetime after-tax income, *i.e.* the expression on the right-hand side of (17), matters for the consumption choice of an agent type (s, g). This observation is the base of the following lemma:

[3] *Leisure* is not considered for a notational simplicity: All of the following results would hold if the utility was also a function of leisure: $U(c_t^t, c_{t+1}^t, n_t, n_{t+1})$. Further, if a conventional time-separable, CRRA utility function is assumed:

$$U(c_t^t, c_{t+1}^t, n_t, n_{t+1}) = \sum \beta^i \frac{\left[\left(c_{t+i}^t\right)^{\alpha}(n_{t+i})^{\eta}\right]^{1-\theta}}{1-\theta}$$

and given the budget constraint (15)-(16), the agent's decision on leisure does not depend on other (own and government) policy variables.

Lemma 1 *Let $\mathcal{U}_s(\tau_t, \varphi_t)$ be the lifetime utility as a function of the size of the social security system:*

$$\mathcal{U}_s(\tau_t, \varphi_t) = \max U\left(c_t^t(s), c_{t+1}^t(s)\right) \tag{18}$$

and also denote

$$W_t \equiv 1 - \tau_t + \frac{\varphi_t}{1+r} \tag{19}$$

then $\mathcal{U}_s(\tau_t, \varphi_t)$ is strictly increasing in W_t.

Proof. The first order conditions of the optimization problem (14) subject to (17) give implicit functions of consumptions in the both periods depending only on W_t defined in (19): $c_t^t = c_t^t(W_t)$ and $c_{t+1}^t = c_{t+1}^t(W_t)$ .On the other hand $U\left(c_t^t(s), c_{t+1}^t(s)\right)$ is strictly increasing in both arguments and thus is strictly increasing in W_t. Then from the definition, $\mathcal{U}_s(\tau_t, \varphi_t)$ is also strictly increasing in W_t. ■

As it was mentioned above, *Lemma 1* firstly states the obvious fact that the lifetime utility of an agent depends on the values of the government policies, τ_t and φ_t, maintained under given equilibrium. In addition the lemma shows that the utility is proportional to the specific form of the effect of the government policies on the lifetime income of the agent, W_t. This result is due to the assumption of small open economy where the prices of capital and labor are given, and provides a possibility to study the first order effects of the government policy on the welfare.

Further, all type s agents of the same generation face identical optimization problem (14)-(17), *viz.* $\mathcal{U}_s(\tau_t, \varphi_t)$ is independent of the agents' generation in the country g and represents the utility of all the type s agents of generation t. Moreover, according to the lemma 1 both $\mathcal{U}_s(\tau_t, \varphi_t)$ for $s = 0, 1$ are strictly growing in W_t at the same time. However W_t is independent of agents' skill type s and thus is a valid measure of welfare for the entire generation under different government policies. Therefore, in what follows W_t will be referred to as the welfare of generation t and will be used to compare the welfare of the agents under different policies.

2.2 The *Status Quo* Economy

The economy starts with an installed PAYG system and no immigration policy. The initial old, or the generation $t = 0$ at period $t = 1$, did contribute to the PAYG system in the previous period and thus anticipate social security benefits with replacement rate:

$$\varphi_t = \phi \tag{20}$$

during the period $t = 1$. Under status quo economy no social security reform is undertaken and thus the government continue maintaining PAYG system with the same benefit level given at (20) and levies a tax, constant over time, to balance government budget constraint (12)-(13):

$$\tau_t = \tau^{SQ} \tag{21}$$

The *Status quo* economy assumes that the government does not alter the immigration policy as well, *i.e.* the zero immigration policy is continued:

$$\psi = 0 \tag{22}$$

Thus the population dynamics from the Section 2.1.2, from the assumption of one-to-one reproduction rate of the natives in combination with (22), takes the following form:

$$N_t = N_{t-1} = N_0 \tag{23}$$

The equilibrium in the *Status quo* economy is an allocation $\{c_1^0(s), c_t^t(s); c_{t+1}^t(s); a_t; \mu_t(i,s,g); \tau^{SQ}; B_t\}_{s=0,1;g=0,1\ldots}^{t=1,2,\ldots}$, such that the old age agents consume their savings and social security benefits, households optimize (14)-(17), the government budget (12)-(13) is balanced, and the population dynamics follows (23), given the world prices of labor and capital resources, w and r, and the values of B_0, $\{a_0\}_{s=0}^{1}$, N_0, constant rates of replacement (the benefit ratio), ϕ, and migration, ψ.

In the *Status quo* economy the agents of all the generations get equal welfare:

Lemma 2 *In the Status quo economy the welfare $W_t = W^{SQ}$ for all t where:*

$$W^{SQ} = 1 - \phi \frac{r}{1+r} \tag{24}$$

Proof. Dividing (12) by $(1+r)^t$ and taking sum over all periods results in

$$\tau^{SQ} w \sum_{t=1}^{\infty} \frac{N_0}{(1+r)^t} = \phi w \sum_{t=1}^{\infty} \frac{N_0}{(1+r)^t} + \sum_{t=1}^{\infty} \frac{B_t - B_{t-1}(1+r)}{(1+r)^t} \tag{25}$$

which with conditions (13) and $B_0 = 0$ solves for

$$\tau^{SQ} = \phi \tag{26}$$

Plugging (26) into (19) will give (24). ■

The equal welfare for all the agents is a result of the constant tax rate assumption as the benefits are equal by definition. However, the same result would be obtained if the social security budget was being balanced each period for the population evolves according to (23).

In the following sections the welfare under different policies will be considered. All the policies (equilibria) discussed below consider immigration policy reform, *i.e.* positive number of immigrants is allowed into the economy. Thus the comparison of the welfare under *Status quo* with the other equilibria will indicate the effect of the immigration policy on the natives.

2.3 PAYG equilibrium

The economy starts with existing PAYG system with replacement rate ϕ, initial government assets B_0, and zero migration, *i.e.* in the *Status quo* economy. Under PAYG equilibrium the government continues with active social security policy, where the replacement rate is on a constant level:

$$\varphi_t = \phi \tag{27}$$

Besides continuing on the social policy, constant flow of immigrants (with average skill level Λ) is introduced to the economy following (6) from the first period onward. At the same time the government sets a constant tax level:

$$\tau_t = \tau^P \tag{28}$$

so that its budget constraint (12) is satisfied.[4]

Given the world prices of labor and capital resources, w and r, and the values of B_0, $\{a_0\}_{s=0}^{1}$, $\{\mu_0(i,s,g)\}_{i,s,g}$, constant rates of replacement (the benefit ratio), ϕ, and migration, ψ, and the parameters of population skill distribution θ and λ, an equilibrium is $\{c_1^0(s), c_t^t(s); c_{t+1}^t(s); a_t; \mu_t(i,s,g); \tau^P; B_t\}_{s=0,1;g=0,1...}^{t=1,2,...}$, such that

1. The old age agents at period 1 consume:

$$c_1^0(s) = \phi w \varepsilon(s) + a_0(s) \cdot (1+r) \tag{29}$$

[4]The constant tax rate is one of the many solutions that would balance the government budget constraint (12) given constant migration and replacement rates, (6) and (27). However, there is no Pareto-superior alternative to constant tax policy: Constant tax charges each generation equally and distributes all the (current and future) liabilities over all the generations equally. Thus if any one generation pays less tax (and yields higher welfare according to (19)) then some other generation should pay higher tax to compensate for that (and so educing lower welfare according to (19)).

2. Households maximize (14) subject to (15)-(16):

$$\max U\left(c_t^t, c_{t+1}^t\right), \qquad (30)$$
$$s.t. \quad \begin{array}{l} c_t^t + a_t \leq w\varepsilon(s)\left(1-\tau^P\right) \\ c_{t+1}^t \leq \phi w\varepsilon(s) + a_t(1+r) \end{array}$$

3. The population sequence $\{\mu_t(i,s,g)\}_{t=0}^{\infty}$ is generated by (1)-(5)

4. The government policy satisfies (12), (13), (27) and (28).

Hence in this economy the initial old agents, *i.e.* generation born at time 0, are unaffected by the policy change and get the welfare they have been promised in the *Status quo* economy. On the other hand, all the generations born after time 0 get equal to each other welfare because the tax and benefit rates are set to be constant. Thus the PAYG equilibrium corresponds to the case when government undertakes immigration policy reforms however the social security policy is unaltered.

2.4 Welfare comparison: *Status quo vs.* PAYG

The definition of PAYG equilibrium in case of nil immigration is identical to the definition of the equilibrium in the *Status quo* economy. Thus the comparison of the PAYG and *Status quo* shows the effect of the immigration policy reform on the welfare of the agents:

Proposition 1 $W^P > W^{SQ}$ *for any* $\psi > 0$.

Proof. From the sum of (12) divided by $(1+r)^t$ over all the periods, and conditions (13), $B_0 = 0$ it follows that under PAYG equilibrium:

$$\tau^P - \frac{\phi}{1+r} = \frac{\phi N_0}{1+r}\left(\sum_{t=1}^{\infty}\frac{N_t}{(1+r)^t}\right)^{-1} \qquad (31)$$

and in the *Status quo*

$$\tau^{SQ} - \frac{\phi}{1+r} = \frac{\phi N_0}{1+r}\left(\sum_{t=1}^{\infty}\frac{N_0}{(1+r)^t}\right)^{-1} \qquad (32)$$

As according to (1)-(11) $N_t > N_0$ for any $\psi > 0$ and $t \geq 1$, it is true that

$$\frac{\phi N_0}{1+r}\left(\sum_{t=1}^{\infty}\frac{N_t}{(1+r)^t}\right)^{-1} < \frac{\phi N_0}{1+r}\left(\sum_{t=1}^{\infty}\frac{N_0}{(1+r)^t}\right)^{-1} \tag{33}$$

Hence from (19) and (31)-(33) follows the claim. ■

Virtually, *Proposition 1* shows that increased immigration is a Pareto-improving policy independent of the size and skill distribution of the migrants and their descendants. In other words, increased immigration does not have any channel to decrease the welfare of the locals. On the contrary, immigrants and their descendants increase the rate of return of the PAYG system and thus the welfare of the agents under the PAYG equilibrium.

2.5 *Laissez faire* equilibrium

In the first period the government terminates existing PAYG system in a *Status quo* economy setting the replacement rate on zero level for all the upcoming generations:

$$\varphi_t = 0 \tag{34}$$

for $t \geq 1$. However the government satisfies the benefit claims of the generation born at $t = 0$. Thus the government turns the implicit debt of the terminated PAYG system into an explicit debt . At the same time the government introduces constant (fraction ψ of natives) flow of migrants with average skill level Λ. In order to finance the debt the government sets a tax on constant level τ^L so that the following constraints are satisfied:

$$\begin{aligned}
\sum_s \phi w\varepsilon(s)\mu_1(1,s,g) + B_1 &= \sum_s \tau^L w\varepsilon(s)\mu_1(0,s,g) + B_0(1+r) && (35)\\
\text{(for all the other periods)} \qquad B_t &= \sum_s \tau^L w\varepsilon(s)\mu_t(0,s,g) + B_{t-1}(1+r) && (36)\\
\lim_{T\to\infty} B_T\,(1+r)^{-T} &= 0 && (37)
\end{aligned}$$

Again the constant tax rate is one of the many solutions that would balance (35) - (37). However, there is no other tax policy which is Pareto-superior to this policy: By terminating the existing PAYG system the government turns the implicit debt of the system into an explicit debt which is added to the initial debt, $B_0 = 0$, to form total government liabilities. After the government levies taxes to service the liabilities, and the fixed tax rate distributes those liabilities equally over the generations. Thus if the tax rate is lowered for any one generation then the other generations have to pay higher taxes in order to finance the government debt.

Given the world prices of labor and capital resources, w and r, and the values of $B_0, \{a_0\}_{s=0}^{1}$, $\{\mu_0(i,s,0)\}_{i,s}$, ϕ, the constant rate of migration ψ, and the parameters of population skill distribution θ and λ, an *equilibrium* is a sequence $\{c_1^0(s), c_t^t(s); c_{t+1}^t(s); a_t(s); \mu_t(i,s,g); \tau^L; B_t\}_{t=1,2...}^{i,s,g}$, such that

1. The old age agents at period 1 consume:

$$c_1^0(s) = \phi w \varepsilon(s) + a_0(s) \cdot (1+r) \tag{38}$$

2. Households maximize (14) subject to (15):

$$\begin{aligned} &\max U_t\left(c_t^t, c_{t+1}^t\right), \\ s.t. \quad & c_t^t + a_t \leq w\varepsilon(s)\left(1-\tau^L\right) \\ & c_{t+1}^t \leq a_t(1+r) \end{aligned} \tag{39}$$

3. The population sequence $\{\mu_t(i,s,g)\}_{t=0}^{\infty}$ is generated by (1)-(5)

4. The government policy satisfies (35)-(37).

The equilibria described above start from a point with existing (defined benefit) PAYG social security system and no migration policy, *i.e.* they start in the *Status quo* type economy. In the above described PAYG equilibrium the existing (unfunded) social security system is preserved and positive immigration flow is introduced to the economy. This case essentially corresponds to a government immigration policy reform. Correspondingly, the *Laissez Faire* equilibrium terminates the existing social security system and introduces a positive immigration flow. Thus *Laissez Faire* equilibrium presents a case of reform in both of the government policies.

2.6 Welfare comparison: *Laissez Faire vs.* PAYG (and *Status quo)*

In essence the comparison of *Laissez Faire* and PAYG equilibria is a study of social security reform: From the definitions of the equilibria under each level of immigration the PAYG and *Laissez Faire* equilibria correspond to a reform with specific population dynamics. Agents have welfare losses in participating in the

PAYG system where the rate of return is lower than the real rate of return. On the other hand, under the *Laissez Faire* equilibrium they have to pay extra taxes for financing debt generated by termination of existing PAYG system. Thus the agents face higher tax than benefits in both of the equilibria. Meantime the next claim shows that welfare under these equilibria are identical:

Proposition 2 *Laissez Faire and PAYG equilibria yield equal welfare:*

$$W^L = W^P \tag{40}$$

where $W^L = W_t$ *under Laissez Faire equilibrium, and* $W^P = W_t$ *under PAYG equilibrium.*

Proof. From (34) and (35)-(37) can be obtained that

$$\tau^L = \frac{\phi N_0}{1+r} \left(\sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} \right)^{-1} \tag{41}$$

Plugging τ^P from (31) and τ^L from (41) into (19) and using (27) and (34) will prove the claim. ■

Corollary 1 *Increased immigration policy is welfare enhancing independent of social security policy:*

$$W^L = W^P > W^{SQ} \tag{42}$$

Proof. Follows directly from the *Proposition* 1 and *2*. ■

In essence, *Proposition 2* shows that in an open economy where the benefits are proportional to the income level, *i.e.* the taxation is non-distortionary, Pareto-improving transition to a funded system is not possible by itself. Similar equivalence results in the framework of social security reforms are presented in Fenge (1995), Lindbeck & Persson (2003) and Conesa & Garriga (2007). They claim that the government can conduct a Pareto-neutral reforms using appropriate debt financing. Accordingly, *Proposition 2* extends their result to show that Pareto-neutral reforms are possible while there are demographic changes in the number and skill level of the population, *i.e.* where the heterogeneity and migration are incorporated to the model.

While it is possible to compare PAYG and *Laissez Faire* equilibria with different levels of immigration rate, most of those cases are of no economic meaning. However, it is worth comparing *Laissez Faire* with positive immigration to PAYG with nil immigration, *i.e.* to *Status quo* economy. This comparison shows the welfare effect of the reforms in both policies, immigration and social security. As the

Corollary 1 shows the increased immigration policy is Pareto-superior independent of social security policy.

It should be noted as well that the claim in *Proposition 2* does not depend on the size of immigration rate ψ, and most importantly includes the case of zero immigration. Hence the proposition suggests that sole social security reform, *i.e.* *Laissez Faire* with no immigration, is Pareto-neutral.

The claim in this section is thoroughly based on the idea that the government can freely manage the social security budget. However the unfunded social security systems assume some restrictions. Next section discusses the issues arising from those restrictions.

2.7 PAYG equilibrium with restrictions

While some authors (*e.g.*, Attanasio, Kitao & Violante, 2007; Nishiyama & Smetters, 2007) describe PAYG system as it is defined in the in the section 2.3, *i.e.* balancing social security budget over infinite horizon, others (*e.g.*, Sand & Razin, 2007; Conesa & Gargia, 2007; Fuster, Imrohoroglu & Imrohoroglu, 2007; Hong & Rios-Rull, 2007; Krueger & Kubler, 2006) consider 'classical' PAYG with period-per-period balanced social security budget constraint[5]:

$$\sum_{s,g} \varphi_t w \varepsilon(s) \mu_t(1,s,g) = \sum_{s,g} \tau_t^{RP} w \varepsilon(s) \mu_t(0,s,g) \tag{43}$$

Thus government sets τ_t^{RP} in each period in order to satisfy the budget constraint for a fixed replacement rate.

The definition of the *Restricted PAYG equilibrium* is same as in section 2.3 with only changes being (43) replacing (12) and (28), and τ_t^{RP} being part of the fiscal constitution instead of τ^P.[6]

Under restricted PAYG the initial old generation is getting what they were promised as it was before, however, as opposed to the equilibria defined before, each generation may face different contribution rate and thus different welfare level. The difference in the tax level can be caused by the changes in the effective labor force in the economy. Unless the change of the effective labor force from one generation to the other grows with constant rate (including zero growth) the tax level will be different for each generation.

[5] By definition unfunded social security system has period-by-period balanced budget constraint (Uebelmesser, 2004).

[6] As the government might have inherited initial debt, B_0, another tax would have been introduced for serving it. As before a constant tax rate τ^B would be considered so that (36)-(37) type dynamic budget constraint would be satisfied. However, as before for simplicity $B_0 = 0$ will be assumed.

2.8 Welfare comparison: Restricted PAYG *vs. Status quo*, PAYG and *Laissez Faire*

In this section the welfare under restricted PAYG equilibrium is compared to all the previous cases with unrestricted government budget constraint, *Status quo,* PAYG and *Laissez Faire.* First, the restricted PAYG will be compared to the *Status quo* economy. Afterwards, restricted PAYG will be compared to *Laissez Faire* and to PAYG (by transitivity from the equivalence result of the *Proposition 2*).

It is important to note that the *Status quo* economy would deliver the same welfare was it employing restricted budget constraint. Thus the comparison of restricted PAYG to *Status quo* shows the effects of immigration policy reform while the social security budget is balanced each period.

Proposition 3 *Increased immigration brings higher welfare:*

$$W_t^{RP} > W^{SQ} \tag{44}$$

for any $\psi > 0$ *and any* $t \geq 1$ *where* $W_t^{RP} = W_t$ *under restricted PAYG equilibrium.*

Proof. From (5) and (43) for PAYG, and (26) for *Status quo*, follows that

$$\tau_t^{RP} = \phi \frac{N_{t-1}}{N_t} \tag{45}$$

$$\tau^{SQ} = \phi \tag{46}$$

At the same time from (1)-(11) follows that $\frac{N_{t-1}}{N_t} < 1$ and hence $\tau^{SQ} > \tau_t^{RP}$ for any $\psi > 0$ and $t > 0$. Under both, *Status quo* and restricted PAYG equilibrium, the welfare depends only on the contribution rate (the rest is constant) and thus the smaller the contribution rate the higher is welfare. Thus $W^{SQ} < W_t^{RP}$. ■

Basically *Proposition 3* shows that if the parameters of the existing the restricted PAYG system are untouched any positive immigration is Pareto-improving policy. The intuition behind it is very simple: Higher immigration rate brings higher population growth rate which increases the rate of return of the existing social security system. Thus, *Proposition 3* combined with *Corollary 1* shows that the increased immigration policy is always beneficial for the agents independent of the social security policy (including the social security budget balancing problem).

Up to now welfare comparison is possible in general terms without specifying in details the population dynamics. However, in order to compare restricted PAYG to *Laissez Faire* or to PAYG the immigration policy space needs to be studied in quantitative and qualitative dimensions. Depending on the skill level of the immigrants the welfare path over the generations will be different.

Another important factor for the welfare is the assimilative behaviour of the future generations. Below three possible cases of assimilation is studied: full assimilation, partial assimilation and non-assimilation. In case of the full assimilation the future generations take both of the discussed parameters, fertility and skills, from the natives. In case of partial assimilation the generations inherit the fertility levels but get the same skill distribution as the natives. The last case to be discussed is the non-assimilation when the future generations are identical to their parents in skills and reproductive behavior. The other case of partial assimilation, inheriting the skill level while taking the fertility of natives, is not discussed for two reasons: it is generally not observed and it is theoretically a case of non-assimilation.

2.8.1 Full assimilation: Uninherited fertility and uninherited skills

In this section the simplest form of the population dynamics will be considered where the future generations of the immigrants are identical to natives in skills:

$$\eta(s,g) = \eta(s,0) \text{ for } g \geq 2 \tag{47}$$

and in fertility rates (while the immigrants have higher fertility rates):

$$\begin{array}{ll} \zeta(g) = 0 & \text{for } g \neq 1 \\ \zeta(g) = \zeta & \text{for } g = 1 \end{array} \tag{48}$$

With this specification the law of motion of the population is

$$N_t = N_{t-1}\left(1 + (1+\zeta)\,\psi\right) \tag{49}$$

$$= N_1\left(1 + (1+\zeta)\,\psi\right)^{t-1} \tag{50}$$

Next two lemmas will give the size of the welfares under PAYG and *Laissez Faire* equilibria and the *Proposition 4* will compare them:

Lemma 3 *In case of the full assimilation the welfare under Laissez Faire and PAYG equilibria is:*

$$W^P = W^L = 1 - \frac{\phi\left(r - \psi\left(1+\zeta\right)\right)}{1+r}\frac{N_0}{N_1} \tag{51}$$

Proof. The unique tax rate which balances the government budget under the *Laissez Faire* equilibrium can be obtained from the constraints (35) - (37):

$$\tau^L = (r - \psi(1+\zeta)) \left(\frac{\phi}{1+r} \frac{N_0}{N_1} - \frac{B_0}{wN_1} \right) \left(1 - \lim_{T\to\infty} \left(\frac{1+\psi(1+\zeta)}{1+r} \right)^T \right)^{-1} \tag{52}$$

Plugging (52) with limit values and condition $\varphi_t = 0$, for $t > 0$, into (19), and taking into consideration the fact that $B_0 = 0$, will result in (51). The welfare level under PAYG is found by transitivity. ■

Lemma 3 shows that in the *Laissez Faire* and PAYG economy with low immigration rate the welfare depends on the skill shift of the first period, *i.e.* on the quality of migrants and on their size, as well as on the fertility rates of the immigrants.

Lemma 4 *Under the restricted PAYG equilibrium with full assimilation*

$$W_1^{RP} = 1 - \phi \frac{N_0 r - \Lambda\psi}{(N_0 + \Lambda\psi)(1+r)} \tag{53}$$

$$\tilde{W}^{RP} = 1 - \phi \frac{r - \psi(1+\zeta)}{(1+\psi(1+\zeta))(1+r)} \tag{54}$$

where $W_1^{RP} = W_t$ *for* $t = 1$ *and* $\tilde{W}^{RP} = W_t$ *for* $t > 1$.

Proof. Form (43) and (9)-(11) and (50) it follows that PAYG tax level τ_t^P will be constant for $t > 1$ and is equal to:

$$\tau_t^{RP} = \begin{cases} \tau_1^{RP} = \phi \frac{N_0}{N_1} & \text{for } t = 1 \\ \tau_t^{RP} = \frac{\phi}{1+\psi(1+\zeta)} & \text{for } t > 1 \end{cases} \tag{55}$$

Plugging (27) and (55) into (19) will result in (53)-(54). ■

As it was expected *Lemma 4* claims that the welfare under restricted PAYG W_t^{RP} changes over generations. This result is due to the skill-shift in the first period (Equations (9) - (11)). However from the second generation on the welfare does not change as the effective labor in the economy gets a proportionally balanced growth according to the rule (50).

Proposition 4 .

a. *If average immigrant is relatively more skilled* $\lambda > \theta$ *then*

$$W_1^{RP} > W^L = W^P > \tilde{W}^{RP}$$

b. *if average immigrant is as skilled as local* $\lambda = \theta$ *then*

$$W_1^{RP} = W^L = W^P = \tilde{W}^{RP}$$

c. *if average immigrant is relatively less skilled* $\lambda < \theta$ *then*

$$W_1^{RP} < W^L = W^P < \tilde{W}^{RP}$$

Proof. The difference between the welfare measures from (51), (53) and (54) when $r > \psi$ is:

$$W_1^{RP} - W^L = \phi \frac{\psi (\Lambda - N_0)}{(1+r)(N_0 + \Lambda r)} \tag{56}$$

$$\tilde{W}^{RP} - W^L = \phi \frac{\psi (N_0 - \Lambda)}{(1+r)(N_0 + \Lambda \psi)} \frac{r - \psi (1+\zeta)}{1 + \psi (1+\zeta)} \tag{57}$$

It is obvious that (56) and (57) have different signs as $r > \psi$. When $\lambda > \theta$ the (56) is positive, *i.e.* unrestricted PAYG yields higher welfare, for the first period and (57) is negative, *i.e.* unrestricted PAYG yields lower welfare, for the subsequent periods. The reverse is true for $\lambda < \theta$ case. When $\lambda = \theta$ both (56) and (57) get equal to zero. The welfare under PAYG follows directly from *Proposition 2* by transitivity. ■

Effectively *Proposition 4* says the equilibria are Pareto-incomparable. The only exception is when immigration does not change the initial skill distribution in the country, *i.e.* case *b*. Evidently the quality of immigration brings distortions to the welfare of the agents. Thus if the average immigrant is more skilled compared to the local average, $\lambda > \theta$, the agents of the first generation have higher welfare under the PAYG equilibrium, while *Laissez Faire* and PAYG equilibria bring higher welfare for all the other generations (born at $t > 1$). Otherwise, when the average immigrant is less endowed, the agents of the first generation have welfare losses under the PAYG equilibrium, while it brings higher welfare for all the other generations.

The result of *Proposition 4* is due to the fact that the *Laissez Faire* and PAYG equilibrium smooth the welfare across the generations while the restricted PAYG equilibrium, balancing the budget periodically, allows inequalities across the generations. According to (9)-(11) the immigrants, when introduced, change the skill distribution in the economy and thus the agents of generation $t = 1$ have to pay tax different from those paid by the future generations where the population dynamics is controlled by (50). Thus if under the *Laissez Faire* and PAYG a constant tax is deployed which an average of the tax rates under constrained PAYG, then it is obvious that either the first generation is paying higher tax and other generation pay less compared to the constant tax rate of the unconstrained equilibrium or *vice versa.* On the other hand the benefits that the agents get are identical under both equilibria. Thus under the constrained equilibrium the welfares of first and all the subsequent generations are going to be on different directions from the welfare of unconstrained PAYG or *Laissez Faire* equilibria.

2.8.2 Partial assimilation: Inherited fertility and uninherited skills

In this case it is assumed that the generations inherit the reproduction rate of their parents

$$\zeta(g) = \zeta \tag{58}$$

while the locals still have low fertility, $\zeta(g) = 0$. At the same time it is assumed that the generations of the immigrants get similar to the locals with their skill distribution:

$$\eta(s, g) = \eta(s, 0) \text{ for } g \geq 2 \tag{59}$$

With this specification the law of motion of the population, for $t > 1$, is

$$\begin{aligned} N_t &= N_{t-1}(1+\psi)(1+\zeta) - \zeta N_1 && (60) \\ &= N_1 \left([(1+\psi)(1+\zeta)]^{t-1} - \zeta \frac{[(1+\psi)(1+\zeta)]^{t-2} - 1}{(1+\psi)(1+\zeta) - 1} \right) && (61) \end{aligned}$$

Unproportionally growing population makes the contribution rate to change in each period under the PAYG system according to (45). The unique tax rate for each generation also makes the welfare levels to be different for all these generations. At the same time contribution rate and thus the welfare under *Laissez Faire* equilibrium is constant:

Lemma 5 *In case of partial assimilation with inherited fertility and uninherited skills the welfare is:*

Under PAYG equilibrium:

$$W_1^P = 1 + \frac{\phi}{1+r} - \phi \frac{N_0}{N_1} \tag{62}$$

$$W_2^P = 1 + \frac{\phi}{1+r} - \frac{\phi}{1+\psi(1+\zeta)} \tag{63}$$

$$W_t^P = 1 + \frac{\phi}{1+r} - \phi \frac{p^{t-1} - p^{t-2} - \zeta p^{t-3} - \zeta}{p^t - p^{t-1} - \zeta p^{t-2} - \zeta} \tag{64}$$

and under Laissez Faire equilibrium:

$$W^L = W^P = 1 - \phi \frac{N_0}{N_1} \cdot \frac{(p-1)(1+r-p) + \zeta p}{(p-1)(1+r-p)} \tag{65}$$

where $p \equiv (1+\psi)(1+\zeta)$ *.*

Proof. The proof is similar to the proofs of *Lemma* 3 and *4.* ■

From (63)-(64) can be observed that under restricted PAYG equilibrium the welfare of the agents of generations $t \geq 2$ is independent of the skill difference of the native and immigrated agents. At the same time, as it is shown in the Appendix A, from the second period on under the restricted PAYG the welfare grows for the generations over time converging to some maximum limit value.

On the other hand, (62) and (65) show that the welfare under restricted PAYG of the first generation and the welfare under *Laissez Faire* of all the generation do depend on the relative skill level of the immigrants compared to the locals. For instance, high average skill level of the immigrants makes the welfare of the first generation under the PAYG and all generations under *Laissez Faire* higher while the generations $t \geq 2$ under PAYG have unaltered welfare. The following proposition discusses the possible cases in detail:

Proposition 5 *In case of partial assimilation the welfare of the generation* $t = 1$ *increases in the average skill of immigrant,* λ*, and there is a* $\bar{\lambda} > \theta$ *such that:*

$$\begin{array}{ll} W_1^{RP} > W^L = W^P & \lambda > \bar{\lambda} \\ W_1^{RP} = W^L = W^P & \lambda = \bar{\lambda} \\ W_1^{RP} < W^L = W^P & \lambda < \bar{\lambda} \end{array}$$

Proof. The proof is given in the Appendix A. ■

Proposition 5, in line with *Proposition 4*, describes the behavior of the $t = 1$ generation's welfare under restricted PAYG. As before the welfare level compares to the counterpart under *Laissez Faire* equilibrium depending on the skill level.

However, in the case of partial assimilation the average immigrant is to be higher skilled (compared to the full assimilation case) in order with social security budget balancing restriction the agents are better off (Appendix A).

As it was mentioned in the *Lemma 5*, from the second generation on the welfare of the agents under restricted PAYG does not depend on the skill level of the immigrants. At the same time *Lemma 5* claimed that the welfare of the agents under *Laissez Faire* and PAYG do depend on the skill level of the immigrants. The next proposition establishes the relationship between welfare under restricted PAYG to *Laissez Faire* and PAYG for the generations $t \geq 2$.

Proposition 6 *The welfare under restricted PAYG grows over generations for $t > 1$ and there is a unique $\bar{t}(\lambda) \in (1, \infty)$ such that*

$$\begin{array}{ll} W_t^{RP} < W^L = W^P & \text{for } t < \bar{t}(\lambda) \\ W_t^{RP} = W^L = W^P & \text{for } t = \bar{t}(\lambda) \\ W_t^{RP} > W^L = W^P & \text{for } t > \bar{t}(\lambda) \end{array}$$

Proof. The proof is given in the Appendix A. ■

As the *Proposition 6* suggests the welfare follows growing path with second generation having the lowest welfare. From (53)-(54) and (62)-(63) is obvious that the first two generations have identical welfare in case of full and partial assimilation under restricted PAYG equilibrium. (This explains the kink in the Figures 1-4.) However, if in case of full assimilation the welfare of all the generations $t > 2$ is identical (and thus is a straight line on the level of $W_2^{RP} = W_t^{RP}$), in case of partial assimilation the welfare of the generations $t > 2$ grows constantly (the derivative of W_t^{RP} with respect to time is positive) according to (64) converging to the limit value

$$\lim_{t \to \infty} W_t^{RP} = 1 + \frac{\phi}{1+r} - \frac{\phi}{(1+\psi)(1+\zeta)} \tag{66}$$

which would be in the case when the population was growing with a constant rate $p = (1+\psi)(1+\zeta)$, *i.e.* the growth rate connected to immigration.

In essence only W_1^{RP} and W^L depend on the skill level of the immigrants while W_t^{RP} being fixed for $t > 1$. Thus Figures 1-4 illustrate main possible cases where W_1^{RP} and W^L vary around W_t^{RP}, for $t > 1$, derived from the combined of results *Proposition 5* and *6*. Fig.1 presents the case when the average immigrant is extreme highly skilled so that under restricted PAYG neither of the generations $t \geq 2$ ever reach the welfare level of the *Laissez Faire* and PAYG equilibria. Meanwhile the first generation agents enjoy the highest welfare compared to the other generations

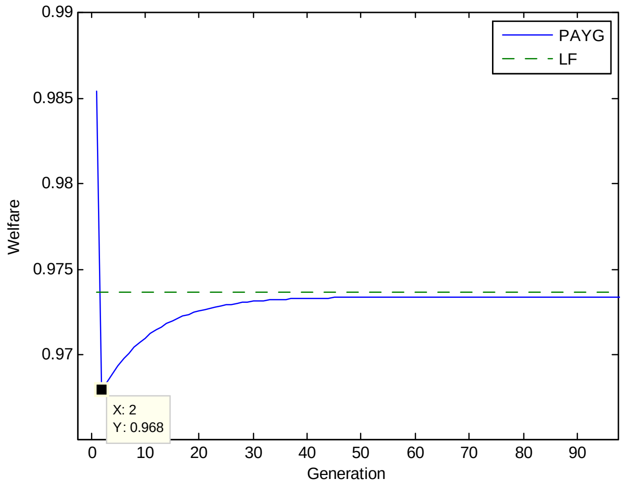

Figure 1: Inherited fertility and uninherited skills
Case: $\lambda > \bar{\lambda}$ and $\bar{t} = \infty$

as well as the *Laissez Faire* or PAYG equilibrium. Thus in this case $\bar{t}(\lambda)$ from the *Proposition 6* is infinitely large.

Fig. 2 illustrates the case where $\bar{t}(\lambda)$ from the *Proposition 6* is a finite number and thus under restricted PAYG some initial generations from the group $t \geq 2$ are worse off compared to the *Laissez Faire* and PAYG equilibria and all the subsequent generations are better off. Though the average skill level of the immigrants is low enough to guarantee finite $\bar{t}(\lambda)$, as opposed to the case illustrated in the Fig.1, however it is still high enough to bring higher welfare to the first generation under restricted PAYG compared to the unconstrained equilibria.

Fig. 3 already illustrates the case when $\lambda < \bar{\lambda}$. From *Proposition 5* it follows that in this case under restricted PAYG the first generation have welfare below the level under *Laissez Faire* and PAYG. It should be noted that in this case, as it is shown in the Appendix A, it is impossible to have infinite $\bar{t}(\lambda)$.The intuition behind it is simple: it is impossible to have all the generations being worse off under either of the equilibria. The same it true for the case illustrated in the Fig. 4 if all the generations $t \geq 2$ have higher welfare compared to the Laissez Faire, then the first generation 'ought to be' worse off. This case corresponds to average skill of immigrant being very low, at the same time the reproduction rate ζ should also be low in order the case illustrated in the Fig.4 to emerge.

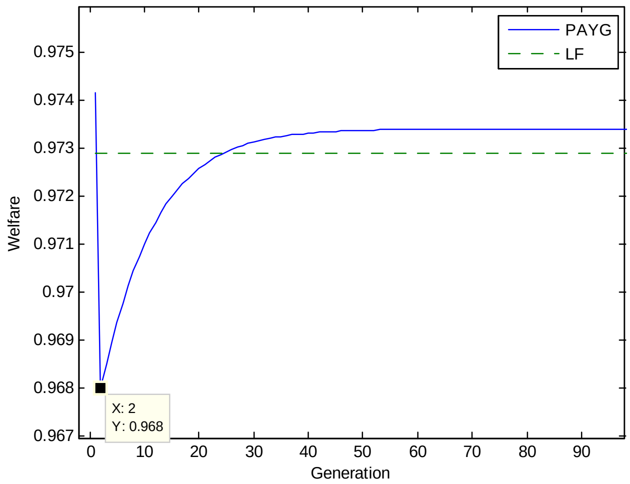

Figure 2: Inherited fertility and uninherited skill
Case: $\lambda > \bar{\lambda}$ and $2 < \bar{t} < \infty$

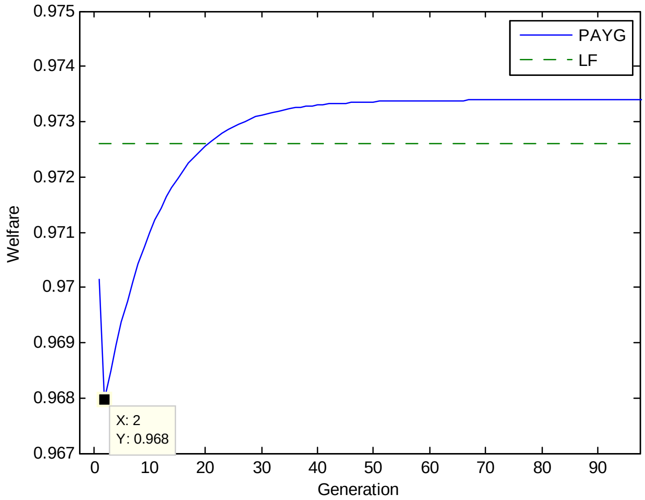

Figure 3: Inherited fertility and uninherited skill
Case: $\lambda < \bar{\lambda}$ and $2 < \bar{t} < \infty$

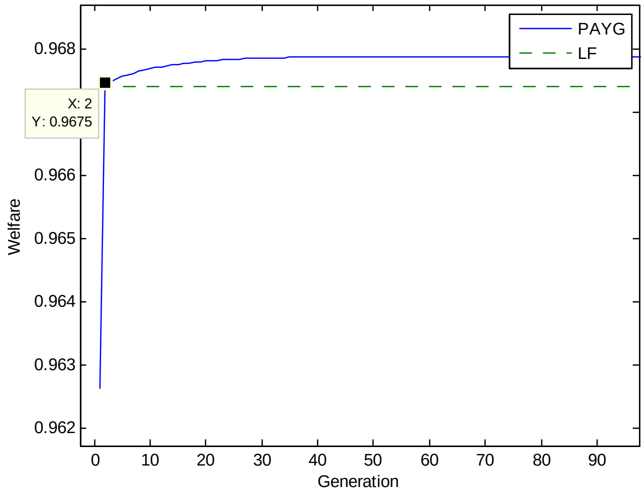

Figure 4: Inherited fertility and uninherited skill
Case: $\lambda < \bar{\lambda}$ and $\bar{t} < 2$

2.8.3 Non-assimilation: Inherited fertility and inherited skills

In this case it is assumed that the generations inherit the reproduction rate of their parents

$$\zeta(g) = \zeta \tag{67}$$

while the locals still have low fertility, $\zeta(g) = 0$. At the same time it is assumed that the generations of the immigrants inherit the skill distribution of their parents:

$$\eta(s, g) = \eta(s, 1) \text{ for } g \geq 2 \tag{68}$$

With this specification the law of motion of the population, for $t > 1$, is

$$\begin{aligned} N_t &= (N_{t-1} - N_0)(1 + \psi)(1 + \zeta) + N_1 & (69) \\ &= N_1 + \Lambda\psi \frac{(1 + \psi)(1 + \zeta)}{1 - (1 + \psi)(1 + \zeta)} - \Lambda\psi \frac{[(1 + \psi)(1 + \zeta)]^t}{1 - (1 + \psi)(1 + \zeta)} & (70) \end{aligned}$$

Unproportionally growing population makes the contribution rate to change in each period under the PAYG system according to (45). The unique tax rate for each generation also makes the welfare levels to be different for all these generations. At the same time contribution rate and thus the welfare under *Laissez Faire* equilibrium is constant:

Lemma 6 *In case of non-assimilation, i.e. inherited fertility and inherited skills,*

the welfare is:
Under PAYG equilibrium:

$$W_t^{RP} = 1 + \phi \frac{1}{1+r} - \phi \frac{N_0 (p-1) + \Lambda\psi \left(p^{t-1} - 1\right)}{N_0 (p-1) + \Lambda\psi \left(p^t - 1\right)} \tag{71}$$

and under Laissez Faire equilibrium:

$$W^L = W^P = 1 - \phi \frac{N_0}{1+r} \cdot \frac{r(1+r-p)}{N_1 (1+r) - pN_0} \tag{72}$$

where $p \equiv (1+\psi)(1+\zeta)$.

Proof. The proof is similar to the proofs of *Lemma* 3 and *4.* ■

Lemma 6, in contrast to *Lemma 5*, claims that under restricted PAYG the average skill level of the immigrants affects the welfare of all the generations. The result is due to the fact that the generations inherit the skill level and the immigrants and their generations are the only source for the population increase, i.e. the total population skill distribution is approaching to the one immigrants have. At the same time, as it could be expected, the welfare under Laissez Faire and PAYG also depend on the average skill level of the immigrants. The following proposition is the counterpart of the *Proposition 4* to *5* for the case of non-assimilation.

Proposition 7 *In case of non-assimilation the welfare under restricted PAYG is higher, equal or lower than the welfare under Laissez Faire and PAYG depending on the average skill level of the immigrants and generation of the agent. Thus there is a unique* $\bar{\lambda} > \theta$ *and a unique* $\bar{t} \in (0, \infty)$ *such that*

a. if $\lambda > \bar{\lambda}$ *then*

$$\begin{array}{ll} W_t^{RP} > W^L = W^P & t < \bar{t} \\ W_t^{RP} = W^L = W^P & t = \bar{t} \\ W_t^{RP} < W^L = W^P & t > \bar{t} \end{array}$$

b. if $\lambda = \bar{\lambda}$ *then*

$$W_t^{RP} = W^L = W^P$$

c. if $\lambda < \bar{\lambda}$ *then*

$$\begin{array}{ll} W_t^{RP} < W^L = W^P & t < \bar{t} \\ W_t^{RP} = W^L = W^P & t = \bar{t} \\ W_t^{RP} > W^L = W^P & t > \bar{t} \end{array}$$

Proof. The proof is given in the Appendix A. ■

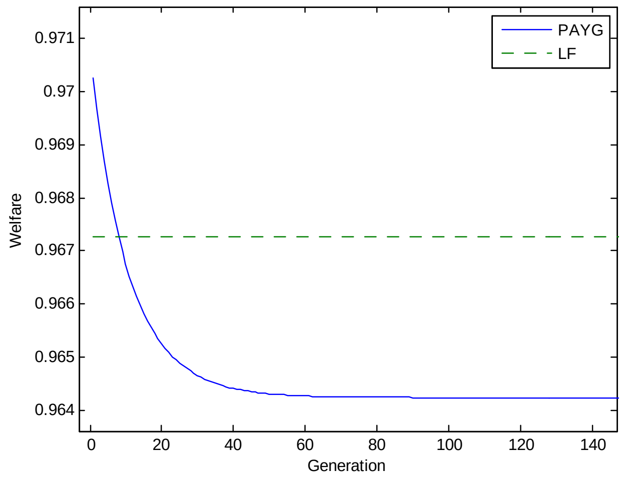

Figure 5: Non-assimilation (case $\lambda > \bar{\lambda}$)

In essence Proposition 7 is based on the idea (proved at the Appendix A) that the welfare under restricted PAYG either grows (case *c.*), stays constant (case *b.*), or decreases (case *a.*) over the generations depending on the average skill level of the immigrants. At the same time from the *Lemma 6* it is obvious that the welfare under Laissez Faire and PAYG is constant. Thus the welfare under restricted policy intersects with the welfare under unrestricted policies only once while increasing or decreasing, or brings equal welfare to all the generations.

Meanwhile *Proposition 7* is not explicit on the fact that in case of non-assimilation the locals are slowly becoming minority and the immigrants and their generations are virtually becoming the entire population. Thus after some point the effective labor starts to grow proportionally according to the growth rate of immigrants and their generations. This is revealed when the limit value of the welfare under restricted PAYG is calculated:

$$\lim_{t\to\infty} W_t^{RP} = 1 + \frac{\phi}{1+r} - \frac{\phi}{(1+\psi)(1+\zeta)} \tag{73}$$

However before that under the restricted PAYG the welfare follows uneven path over the generations. The only exception, when the welfare under the restricted PAYG is identical for all the generations, is the case when the average skill level of the immigrants already is such that the effective labor grows proportionally from the generation $t = 0$ on, *i.e.* when $\lambda = \bar{\lambda}$.

The case illustrated in the Fig. 5 is where the immigrants are on average highly

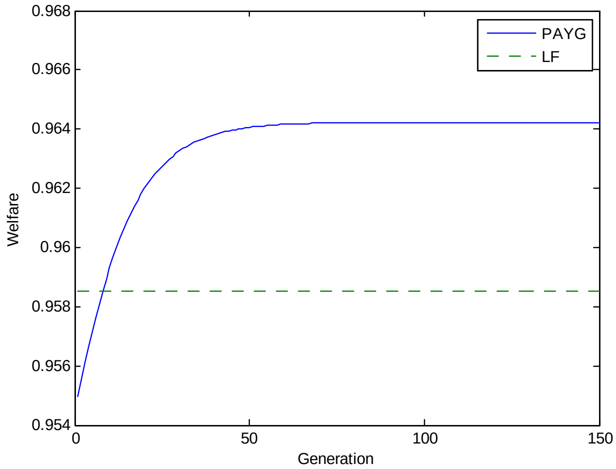

Figure 6: Non-assimilation (case $\lambda < \bar{\lambda}$)

skilled. Thus the first generation is having the highest welfare as the effective tax base is much higher then the required contribution. The second generation has less welfare compared to the first generation as the lower skilled natives still had larger portion in the population. The story for the subsequent generations is the same and their welfare slowly decreases compared to the welfare of their parents. On the other hand the welfare under Laissez Faire and PAYG equilibria is higher than the one for the future generations under restricted PAYG as now there tax base is larger compared to the case of $\lambda = \bar{\lambda}$.

The case illustrated in the Fig. 6 is similar to the case of the Fig. 5 with the opposite pattern. Here the relative tax base shrinks with each period though still tends to the same level as in the case of the Fig. 5.

2.8.4 Joint analysis

In the previous sections it was shown that under restricted PAYG the welfare of the generations follow different paths depending on the type of assimilation that the immigrant generations undergo. At the same time from (66) and (73) can be seen that the welfare under restricted PAYG in case of partial- and non-assimilation converge to the same limit value

$$\lim_{t\to\infty} W_t^{RP} = 1 + \frac{\phi}{1+r} - \frac{\phi}{(1+\psi)(1+\zeta)} \tag{74}$$

which would be the case when the effective labor was growing with a constant rate $p = (1+\psi)(1+\zeta)$ in the economy. However, part of the labor, namely the local population, evolves with different growth rate: meanwhile the share of that part is decreasing in the total population over the generations, which brings the level of the welfare closer to the one in (74). This change in the effective labor shapes the path of the welfare over the generations.

At the same time, depending on the type of the distortion that the immigration brings to the effective labor the path of the welfare is changing. Thus in case of the partial assimilation from the second generation on (according to *Proposition 6*) the welfare increases to reach the limit value given by (74), while under the non-assimilation (according to *Proposition 7*) the welfare decreases or increases to reach the same limit value depending on the average immigrant skill level.

On the other hand the welfare under restricted PAYG with full assimilation reaches its limit value $\tilde{W}^{RP}$, given by (54), already for the second generation and maintains that level for all the generations $t > 1$. Nonetheless the limit value of the welfare in case of full assimilation, $\tilde{W}^{RP}$, is lower than the limit value of the welfare in case of partial- and non-assimilation given by (74): While assimilated the immigrant generations lose the higher fertility rate which, according to (45), translates to higher tax rate which, according to (19), translates into lower welfare for the generations. In order to verify that the welfare in case of full assimilation is lower than welfare in other cases (54) can be compared to (63). Thus it shows that the limit value of the welfare in case of full assimilation is equal to the welfare of the generation $t = 2$ in case of partial assimilation which is lower than the limit value of the welfare according to *Proposition 6*.

Meanwhile in case of the restricted PAYG the welfare of the generation $t = 1$ is the same independent of the type of assimilation, given by (53), (62) and (71). According to *Propositions 4, 5* and *7* the welfare of the generation $t = 1$ can be on the either side of the limit values of the welfare under each of the types of assimilation. If the average skill level of the immigrants is high then in case of the non-assimilation the welfare of the agents decreases to the limit value, while in case of partial assimilation the welfare is increasing to reach the limit value, i.e. the generations $t \geq 2$ have higher welfare in case of non-assimilation compared to the partial assimilation. At the same time as it was mentioned above the welfare in case of the full assimilation is the lowest independent of generation (excluding $t = 1$) and skill level. This case is illustrated in Fig.7 and has a simple explanation that if the immigrants are on average higher skilled and have higher reproduction rate then the more immigrant characteristics future generations inherit the higher

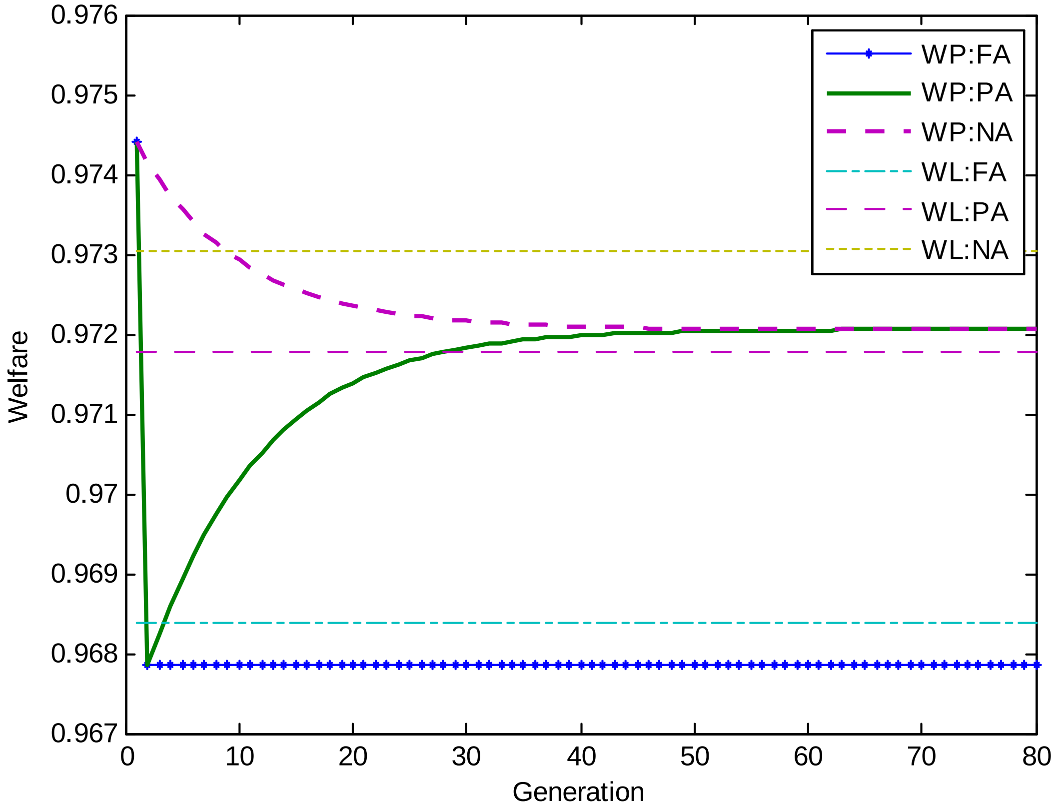

Figure 7: Full-, partial- and non-assimilation combined ($\lambda > \theta$)

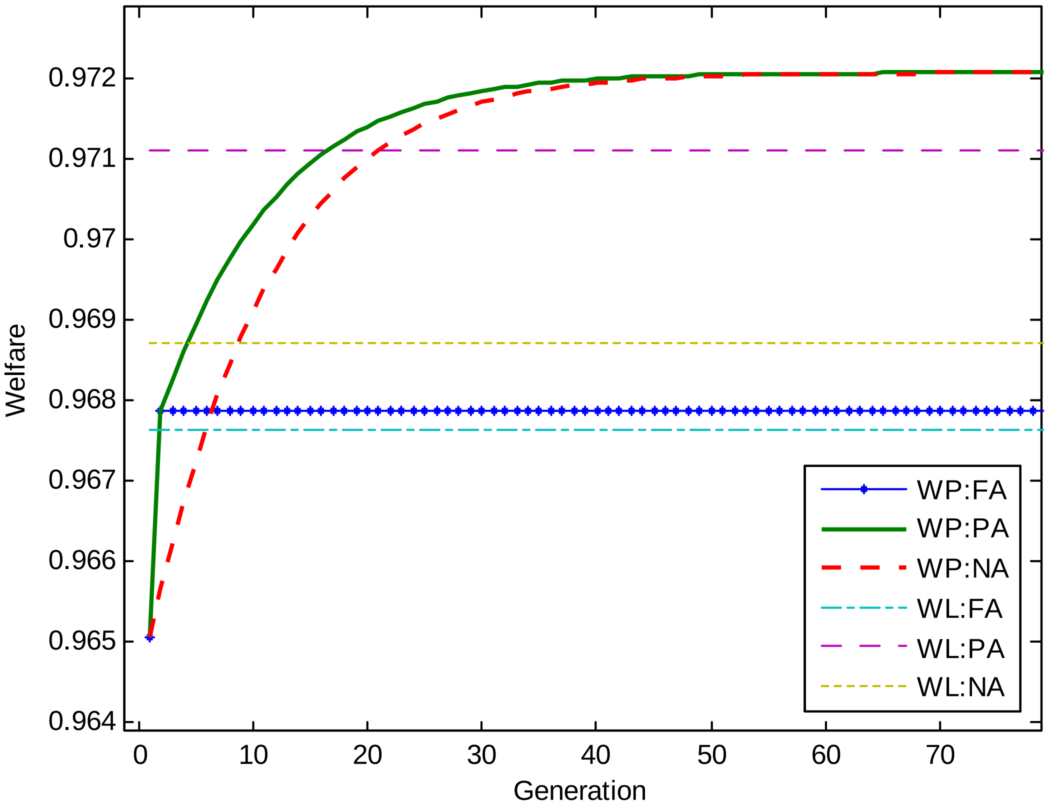

Figure 8: Full-, partial- and non-assimilation combined ($\lambda < \theta$)

the welfare.

Fig.8 illustrates the case when the average skill level of the immigrants is lower then that of locals. The according to *Propositions 4, 5* and *7* the welfare of the generation $t = 1$ is the lowest in all the cases and grows for all the other generations: In case of the full assimilation the welfare reaches the highest level for the generation $t = 2$ and stays constant on that level, while in case of partial assimilation the welfare reaches the same level for generation $t = 2$ and grows further to reach the limit value (74). Meantime in case of non-assimilation the welfare grows slower than in other cases, and reaches the welfare level of the case of full assimilation at generation $\tilde{t}$ which solves $\tilde{W}^{RPFA} = W_{\tilde{t}}^{RPNA}$, and grows after to reach the limit value (74), though being lower than the one in case of partial assimilation.

Fig.7-8 also show the welfare level in different assimilation cases under *Laissez Faire* and PAYG equilibria. Though under *Laissez Faire* equilibrium there is a fixed amount of debt (initial implicit debt of the PAYG system turned into explicit by terminating the system) that all the generations pay for, the welfare differ depending on the assimilation the immigrant generations undergo as the population in each of the assimilation cases is different quantitatively and qualitatively.

3 Conclusion

In last decades demographic developments have been challenging the national budgets in most of the developed economies: Ageing, increased longevity and decreased fertility, shrinks the tax base while increasing the number of beneficiaries of the existing social security systems. Thus the governments of those countries are increasingly facing policy sustainability problems. In recent years many solutions have been proposed to solving these problems. The most discussed sustainability-solving possibilities are parametric reforms of the existing policies (tax-increases or benefit-cuts), current deficit financing with increased immigration, and social security reforms. The current work offers an analysis of the combination of increased immigration policy with social security reforms. The work is a welfare study, where alternative policies are compared with a specially developed welfare measure.

In the current work a small open economy with overlapping generations (with demographic misbalance) of heterogeneous agents is modeled where initially a pay-as-you-go social security scheme is installed. Three possible basic economies are studied – parametric reforms, social security with immigration, and social security reform with immigration. Prior to this work the social security literature studied immigration only as a source for sustaining the existing social security policy. In contrast, the current work analyses the welfare implication of the social-security-sustaining-via-immigration policy *vis-à-vis* combination of social security reform and increased immigration policy.

As a result the paper finds that solely parametric reforms are Þareto-inferior to any of the other policies, *i.e.* policies in combination with increased immigration. Meantime, the paper shows that the two policies (sustaining and reforming) in combination with increased immigration yield identical welfare for all types of agents (with a reasonable level of immigration) across and within the generations.

Further, an alternative economy is studied where the government is constrained to balance the social security budget periodically (to match the exact definition of unfunded social security system). As opposed to the first two economies, under these restrictions the government lacks a dynamic smoothing mechanism. Thus the welfare under restrictions changes its behavior depending on the skill level of the immigrants and on the type of assimilation process the generations of immigrants undergo. However, in neither of the cases the restricted or unrestricted economy is preferable for all and each of the generations: Under each case at least one generation is worse off.

While comparing the types of assimilation it was shown that full assimilation makes the great majority of the generations worse off compared to the cases of partial- or non-assimilation. The result heavily depend on the fact that the immigrants have higher fertility rates compared to the natives and for the sake of sustaining social security higher inheritance of higher fertility is preferred. The case of skill inheritance can as well be explained in simple terms: if the immigrants are on average higher skilled then it is preferable the generations to inherit that level and *vice versa*.

The welfare in the restricted economy converges over time to a level which will sustain for infinitely many generations. However that level does not depend on the skill distribution in the economy (neither the initial skill distribution of the natives nor of the immigrants'), while in case of unrestricted economy the welfare of all the agents is directly connected to the distribution. However even in this case the welfare comparison depend on the assimilation type and there are many possible combinations of the losers and gainers from the reform, such as initial generations lose while the later generations gain, or that initial and later generations gain and some intermediate generations lose.

In conclusion, the increased immigration policy is Pareto-improving, however the social security schemes are either identical in welfare or Pareto-incomparable in case of identical immigration policy. The second message of the study is the importance of the balancing term of the social security budget as it changes the welfare distribution across the generations. The third message is the high importance of the nature of assimilation the immigrant generations undergo as it changes the welfare level and the path over time and changes the direction of the reform effect on different generations.

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Appendix

A Proof to the *Proposition 5*, *6* and *7*

Here the two propositions from the Section 2.8.2 and the proposition of the section 2.8.3 are proved:

Proof to the *Proposition 5* To proof the proposition it is necessary to show that there is a level of average skill level of immigrants which makes changes the sign of the difference of the welfares under restricted PAYG and *Laissez Faire*. From *Lemma 5* the welfare levels W_1^{RP} and W^L are given in (62) and (65). If the following notation is introduced:

$$\pi \equiv \frac{(p-1)(1+r-p)+\zeta p}{(p-1)(1+r-p)} \tag{75}$$

where $p = (1+\psi)(1+\zeta)$,it can be written

$$W_1^{RP} - W^L = \phi \frac{N_0}{N_1}\left(1 - \frac{N_0 + \Lambda\psi}{N_0(1+r)} - \pi\right) \tag{76}$$

Thus from (76) $W_1^{RP} \gtreqless W^L$ if

$$\lambda \gtreqless \theta \frac{(1+r)(1-\pi)-1}{\psi} + \frac{\varepsilon(0)}{\varepsilon(1)-\varepsilon(0)} \cdot \frac{(1+r)(1-\pi)-(1+\psi)}{\psi} \tag{77}$$

Hence,

$$\bar{\lambda} \equiv \theta \frac{(1+r)(1-\pi)-1}{\psi} + \frac{\varepsilon(0)}{\varepsilon(1)-\varepsilon(0)} \cdot \frac{(1+r)(1-\pi)-(1+\psi)}{\psi} \tag{78}$$

proves the proposition. ■

Proof to the *Proposition 6* To prove the first part of the proposition it is sufficient to show that the derivative of the welfare measure with respect to time is positive. Thus:

$$\frac{\partial}{\partial t} W_t^{RP} = \frac{\phi(p-1)\zeta p^{-(t+3)} \ln p}{\left(1-p^{-1}-\zeta p^{-2}+\zeta p^{-t}\right)^2}\left(p^2-p-\zeta\right) \tag{79}$$

The ratio on the right-hand-side of (79) is always positive, thus the sign of the expression is determined by

$$\begin{aligned} p^2 - p - \zeta &= p(p-1) - \zeta \\ &= (1+\psi)(1+\zeta)((1+\psi)(1+\zeta)-1) - \zeta \\ &= \psi(1+\zeta) + (\zeta + \psi(1+\zeta))^2 \end{aligned} \tag{80}$$

which is always positive. Thus this proves that the welfare of generations $t \geq 2$ is growing with W_2^{RP} having the lowest value and $\lim_{t\to\infty} W_t^{RP}$ from (66) being the upper bound.

The second part of the proposition claims that there is a generation for which the welfare under restricted PAYG, *Laissez Faire* and PAYG are equal, that the generations before that are worse off under restricted PAYG and the generations after are better off. Obviousely, in order to find that generation, $W_t^{RP} = W^L$ should solved for t :

$$\begin{aligned} \bar{t} &= \log_p \zeta + \log_p \left(r - \frac{N_0}{N_0 + \Lambda\psi} \pi \right) \\ &\quad - \log_p \left(p^{-1} - p^{-2} - \zeta p^{-3} \right) \\ &\quad - \log_p \left(1 + r - p - (1+r) p \frac{N_0}{N_0 + \Lambda\psi} \pi \right) \end{aligned} \tag{81}$$

where as before $p = (1+\psi)(1+\zeta)$ and π is given by (75).

However, because of complicated form of (81) it is easer to prove the claim indirectly. First it needs to be shown that there are values of λ for which $\lim_{t\to\infty} W_t^{RP} < W^L$, *i.e.* $\bar{t}$ is infinite. Then if it is shown that there are values of λ for which $\bar{t}$ is less then two, i.e. the value of λ for which the lowest level of the welfare under restricted PAYG (for generations $t > 2$) is higher than the welfare under *Laissez Faire*, then for each of the remaining values of λ there will be a finite $\bar{t}$.

Thus from (65) and (66) $\lim_{t\to\infty} W_t^{RP} < W^L$ if

$$\lambda > \theta \left(\frac{\pi p (1+r)}{\psi (1+r-p)} - \frac{1}{\psi} \right) + \frac{\varepsilon(0)}{\varepsilon(1) - \varepsilon(0)} \left(\frac{\pi p (1+r)}{\psi (1+r-p)} - \frac{1+\psi}{\psi} \right) \tag{82}$$

where as before $p = (1+\psi)(1+\zeta)$ and π is given by (75).

At the same time from (63) and (65) $W_2^{RP} > W^L$ if

$$\begin{aligned}\lambda &> \theta\frac{\pi(1+r)(1+\psi(1+\zeta))-(r-\psi(1+\zeta))}{\psi(r-\psi(1+\zeta))} \\ &+\frac{\varepsilon(0)}{\varepsilon(1)-\varepsilon(0)}\frac{\pi(1+r)(1+\psi(1+\zeta))-(1+\psi)(r-\psi(1+\zeta))}{\psi(r-\psi(1+\zeta))} \quad (83)\end{aligned}$$

where as before π is given by (75). ■

Proof to the *Proposition 7* To prove the claim it will be shown that, first, the welfare under restricted PAYG is increasing, decreasing or constant depending on the average skill level of the immigrants and to show that the initial and the limit value given in (73) are on the two different sides of the welfare under *Laissez Faire* and PAYG.

Thus

$$\frac{\partial}{\partial t}W_t^{RP} = \frac{\Lambda\psi(p-1)p^t\ln p}{p(N_0(1+p)+\Lambda\psi(1-p^t))^2}(\Lambda\psi+N_0(1-p)) \quad (84)$$

The ratio on the right hand side of (84) is always positive, so that the sign of the derivative depends on $\Lambda\psi + N_0(1-p)$ which is zero when

$$\lambda = \bar{\lambda} \equiv \theta\frac{\psi+\zeta(1+\psi)}{\psi} + \frac{\varepsilon(0)}{\varepsilon(1)-\varepsilon(0)}\frac{\zeta(1+\psi)}{\psi} \quad (85)$$

in which case the welfare is constant over the generations. Alternatively, if the average skill level $\lambda > \bar{\lambda}$ then $\Lambda\psi + N_0(1-p) > 0$ and the welfare under restricted PAYG is higher for the first generation and then gradually decreases. Inversely, from $\lambda > \bar{\lambda}$ follows $\Lambda\psi + N_0(1-p) > 0$ and that the welfare increases over the generations. In case of $\lambda < \bar{\lambda}$ the same expression is negative and thus the welfare decreases.

On the other hand, while comparing the the welfare of the first generation and the limit value of the welfare under the restricted PAYG to the welfare under Laissez Faire, from $\lambda > \bar{\lambda}$ follows that $W_1^{RP} > W^L > \lim W_t^{RP}$ and vice verca, $\lambda < \bar{\lambda}$ makes $W_1^{RP} < W^L < \lim W_t^{RP}$. Thus if W_t^{RP} is increasing or decreasing in t and it has values on the both sides of W^L then there is a unique value of $\bar{t}$ which solves $W_t^{RP} = W^L$:

$$\bar{t} = \log_p(\Lambda\psi + N_0(1-p)) + \log_p W^L - \log_p \Lambda\psi\left(1 - W^L - p^{-1}\right) \quad (86)$$

Meanwhile if $\lambda = \bar{\lambda}$ the welfare under restricted PAYG and Laissez Faire and

PAYG are equal:

$$W_t^{RS} = W^{LF} = W^P = 1 - \phi \frac{1 + r - p}{p\,(1 + r)} \tag{87}$$

for any $t \in [1, \infty)$. $\blacksquare$