M9302 Mathematical Models in Economics

HOMEWORK 1 – due to 24.03.2011, 2 p.m. (in class)

Problem 1 Simultaneous-move games with complete information (2 points)

(a) Represent the following games in normal form, i.e. by payoff matrix (1 point):

A. Stag-hunt Game

Description: The stag-hunt game is also generally known as the **game of assurance**. French philosopher, Jean Jacques Rousseau (28 June 1712 – 2 July 1778), presented the following situation. Two hunters can either jointly hunt a stag (an adult deer and rather large meal) or individually hunt a rabbit (tasty, but substantially less filling). Hunting stags is quite challenging and requires mutual cooperation. If either hunts a stag alone, the chance of success is minimal. Hunting stags is most beneficial for society but requires a lot of trust among its members.

Payoffs: Going for a stag alone yields 0, going together with the other +10; Going for rabbit pays +8 if you are alone, and only +7 if competing with the other.

B. Volunteer's Dilemma

Catherine Susan Genovese (July 7, 1935 – March 13, 1964), commonly known as Kitty Genovese, was a New York City woman who was stabbed to death near her home in the Kew Gardens section of Queens, New York. Her murder became notorious after being reported by Martin Gansberg in the New York Times two weeks later. What was staggering in this criminal case is that even though numerous neighbors witnessed the murder only few called the police or tried to help the victim survive. The article by Gansberg prompted investigation into the social psychological phenomenon that has become known as the bystander effect (or "Genovese syndrome") and especially diffusion of responsibility.

In economics the bystander effect is modelled by a game known as the volunteer's dilemma. Imagine you live in a widely crowded district where all of a sudden electricity goes out. So, somebody should call the electricity company to fix the problem. You are about to decide whether to call the electricity company yourself given that anybody else of your neighbours could do it as well.

Payoffs: If you call the electricity company you gain nothing (payoff 0) no matter whether somebody else from the crowd also calls or not. If you do not call but somebody else do, you gain 1 for having electricity without making an effort to call. However, if nobody calls both you and the neighbours lose -1 of having no electricity.

C. Pareto Coordination Game

Description: The game is named after Vilfredo Pareto (15 July 1848 – 19 August 1923), an Italian economist who introduced the welfare concept of socially optimal income distribution (aka Pareto efficiency). Informally, Pareto efficient situations are those in which it is impossible to make one person better off without necessarily making someone else worse off. In the game, two firms must simultaneously elect a technology to use for their compatible products. If the firms adopt different standards, few sales result. A common standard leads to higher sales. One technology is significantly preferred by consumers over the other. Thus, if the companies can standardize on the preferred technology, each obtains maximal profits.

Payoffs: Choosing any technology alone yields 0, colluding on the consumer's preferred technology yields +5 to both firms, colluding on the consumer's less preferred technology yields +3 to both firms.

D. Blotto Game

Description: Two armies are fighting each against the other on 3 distinct battlefields. On each battlefield the army that has allocated the most soldiers will win. However, neither of the armies knows how many soldiers the enemy will allocate to each battlefield. One of the armies is led by colonel Blotto who is tasked with finding the optimum distribution of his soldiers over the three battlefields so that to maximize the number of battlefields he expects to win. The same is the task of his enemy. Both armies consist of 6000 soldiers which could not be divided in smaller units than 1000.

Payoffs: The army that has two battlefields with more soldiers than the corresponding ones of the enemy wins the game and gets 1 point while the defeated army gets -1. If neither of the army has more soldiers than the other on at least two battlefields, the game is a draw and both armies get 0.

Hint: To construct the strategy space find all the possible combinations into which 6 units (by 1000 soldiers) could be divided into 3 battlefields. For simplicity, exclude the following combinations:

- with zero units deployed to a battlefield i.e. **generals must deploy at least one unit on each field**, as well as
- the combinations with increasing number of units deployed in the next field(s) i.e. **generals are allowed to deploy to the second field at most as much units as to the first one and to the third battlefield at most as much as to the second one**.

(b) For each game, identify all Nash Equilibria in pure strategies (1 point).

Problem 2 Nash Equilibrium VS. Iterated Elimination of Strictly Dominated Strategies (1 point + 0.5 BONUS)

Consider a 2-player simultaneous-move game with normal form representation depicted in the following 3x3 payoff matrix:

	b_1	b_2	b_3
a_1	2,0	1,1	4,2
a_2	3,4	1,2	2,3
a_3	1,3	0,2	3,0

(a) Try to solve the problem using the solution concept of iterated elimination of strictly dominated strategies (0.5 point)
(b) Find all Nash Equilibria (0.5 point)
(c) Explain the difference and the relation between the two solution concepts (+0.5 BONUS)

Problem 3 **Cournot Duopoly with asymmetric marginal costs (1 point + 0.5 BONUS)**

Consider the Cournot duopoly model where inverse demand is P(Q) = a – Q but firms face different fixed marginal costs: c_1 for firm 1 and c_2 for firm 2 ($0<c_i<a/2$, i =1,2).

(a) Find the equilibrium determined by prices and quantities. Calculate the optimal profit for each firm. (1 point)
(b) Construct and plot the reaction correspondences of players. (+1 BONUS)

Problem 4 **Stackelberg Oligipoly (1 point + 1 BONUS)**

Three oligopolists operate in a market with inverse demand given by $P(Q)=a-Q$, where $Q=q_1+q_2+q_3$ and q_i is the quantity produced by firm *i*. Each firm has a constant marginal cost of production, *c*, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively.

(a) What is the subgame-perfect outcome? (0.5 point)
(b) Which main property of backwards induction outcome stays behind the optimal outcome of the Stackelberg competition? (0.5 point)
(c) How the presence of a market leader (firm 1) affects the strategic choice and final payoffs of the followers (firms 2 and 3) compared to the game of simultaneous market entry (Cournot competition)? (+1 BONUS)