M9302 Mathematical Models in Economics

HOMEWORK 2 – due to 05.05.2011, 2 p.m. (in class)

Problem 1 Infinitely repeated Bertrand competition game with homogeneous goods (2 points)

Consider the static Bertrand duopoly model with homogeneous products.

Consumer's demand is given by the function $D(p_i) = \begin{cases} a - p_i, & \text{when } p_i < p_j \\ \frac{a - p_i}{2}, & \text{when } p_i = p_j \\ 0, & \text{when } p_i > p_j \end{cases}$.

Firms name prices simultaneously and marginal costs are constant at $c < a$.

For the infinitely repeated game based on this stage game, show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if the discount factor $\delta \geq 1/2$.

Problem 2 First-bid Auction (3 points)

Two bidders take part in a first-bid auction. Each of them knows that the other bidder's valuation for the object is a uniformly distributed random variable on an interval $[\underline{v}, \bar{v}]$. Bidder 1 has private information about its bidding function. That is, bidder 1 knows that the bidding function of bidder 2 is $b_2(v_2) = (v_2 - \underline{v})^2 + \underline{v}$, but bidder 2 does not know the bidding function of bidder 1.

Find the best response bidding function $b_1(v_1)$ of bidder 1.

Hint: $\Pr ob\left(b_1 > (v_2 - \underline{v})^2 + \underline{v}\right) = \frac{\sqrt{b_1 - \underline{v}}}{\bar{v} - \underline{v}}$

Problem 3 Asymmetric Bertrand Duopoly with discrete payoffs (2 BONUS)

Consider a market leader and a follower who compete in prices. The leader has zero unit production cost while the follower has positive unit cost of either $c_H = 4$ or $c_L = 1$. The follower knows its cost but the leader does not know it for sure. Based on its experience, the leader sustains the belief that the two values c_H and c_L are equally probable i.e. could occur with probability ½. Firms pick prices from the discrete-choice action set {2,4,6}. The consumer demand function is D(p)=8-p, where p is the lowest price in the market. If both firms choose the same price, they share the demand equally. Otherwise, the firm with lower price serves all consumers, while the firm with higher price leaves the market, produces nothing and receives zero profit.

Find all pure strategy Bayesian Nash equilibria of this game.