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Mathematical Models in Economics – Spring 2011

Supplementary Handout to TUTORIAL SESSION 5

Economic Application: Cournot Duopoly Competition under Asymmetric Information

Description of the Bayesian game representation:

1. 2 players – firm 1 and firm 2

2. Each firm chooses the quantity of output to produce without observing the choice of the other.

3. Payoffs are given by the profits of the two firms

3.1. Inverse demand is given by the standard functional form:

$$P(Q) = a - Q\,,$$

where $Q = q_1 + q_2$ is the aggregate quantity on the market.

3.2. Firm 1's cost function and respectively payoff is common knowledge:

$$C_1(q_1) = cq_1$$

However, firm 1 is uncertain about the cost function and payoff of firm 2 (incomplete information):

$$C_2(q_2) = \begin{cases} c_H q_2, \text{ with probability } \theta \\ c_L q_2, \text{ with probability } (1\text{-}\theta) \end{cases}$$

Economic Intuition: Firm 2 is a new market entrant with a different technology than firm 1. Firm 1 is already in the market. So, Firm 2 is aware of both its technology and the technology used by firm 1. Since, firm 2 is a new player, however, firm 1 could only guess its technology. Firm 1 **believes** that firm 2 is a **high type** (i.e. has worse technology and high marginal cost c_H) with **probability** $\boldsymbol{\theta}$, and a **low type** (i.e. has better technology and low marginal cost c_H) with **probability** ***(1- θ)***.

Strategic Interaction: Firm 2 is interested to set lower quantity $q_2^*(c_H)$ when it has higher marginal cost. Respectively, firm 1 anticipates this but is not 100% sure before choosing its optimal quantity $q_1^*(c)$. So, firm 2 would benefit from the uncertainty of firm 1. The opposite would hold if firm 2 had lower marginal cost.

Solution: We search for a Bayesian Nash equilibrium represented by the couple of quantity decision functions $\{q_1^*(c, c_L, c_H); q_2^*(c, c_L, c_H)\}$ which maximize the expected profits of both firm 1 and firm 2.

Profit Maximization Problem:

Firm 2
/if high type/: $\underset{q_2}{Max}\,\Pi_2(q_1^*, c_H) = [(a - q_1^* - q_2) - c_H] \cdot q_2$

/if low type/: $\Pi_2(q_1^*, c_L) = \underset{q_2}{Max}[(a - q_1^* - q_2) - c_L] \cdot q_2$

The first-order condition for optimality yields the following best-response functions for each type of firm 2:

$$R_{2H}(q_1^*) = q_2^*(q_1^*, c_H) = \frac{a - q_1^* - c_H}{2} \qquad (1)$$

$$R_{2L}(q_1^*) = q_2^*(q_1^*, c_L) = \frac{a - q_1^* - c_L}{2} \qquad (2)$$

Firm 1
$$\underset{q_1}{Max}\,E\Pi_1(q_1^*, c, c_H, c_L) = \theta \cdot [(a - q_1 - q_2^*(c_H)) - c] \cdot q_1 + (1 - \theta) \cdot [(a - q_1 - q_2^*(c_H)) - c] \cdot q_1$$

The first-order condition for optimality yields the following best-response function of firm 1:

$$R_1(q_2^*(c_H), q_2^*(c_L)) = q_1^*(q_2^*(c_H), q_2^*(c_L), c) = \frac{\theta \cdot (a - q_2^*(c_H) - c) + (1 - \theta) \cdot (a - q_2^*(c_L) - c)}{2} \qquad (3)$$

After substituting for $q_2^*(c_H)$ and $q_2^*(c_L)$ from equations (1) and (2) into (3), the following expression could be derived for firm 1's optimal decision function:

$$q_1^*(c, c_H, c_L) = \frac{a - 2c + \theta \cdot c_H + (1-\theta) \cdot c_L}{3}$$

Dependent on the type of firm 2, its optimal decision function is given by the following expressions:

$$q_2^*(c_H) = \frac{a - 2c_H - c}{3} + \frac{(1-\theta) \cdot (c_H + c_L)}{6} \left(= q_{2H}^* + \frac{(1-\theta) \cdot (c_H + c_L)}{6} \right) > q_{2H}^*$$

$$q_2^*(c_L) = \frac{a - 2c_L - c}{3} - \frac{\theta \cdot (c_H + c_L)}{6} \left(= q_{2L}^* - \frac{\theta \cdot (c_H + c_L)}{6} \right) < q_{2L}^*$$

Implication: Note that the first member of each expression gives exactly the optimal strategy $(q_{2H}^* \text{ or } q_{2L}^*)$ of the respective type of firm 2 under complete information. So, the uncertainty of firm 1 would benefit firm 2 if it had worse technology (Type H) and would hurt it if it had better technology (Type L).