# M9302 Mathematical Models in Economics

HANDOUT 2 - 12.03.2010

SLOT 1 – 10:00-10:50 a.m.

1. Change in the Schedule

–> Moving the last lecture a week earlier (May-14)?
–> Lecture Shuffle: Taking topics 6 and 7 directly after topic 4 and leaving topic 5 to the last of the course?

2. Homework 1 distributed – due to 26.03.2010, 10 a.m. (at the lecture)

3. Fast Revision on Lecture 1

-> Simultaneous-move games of complete information
-> Normal form representation: PLAYERS-STRATEGIES-PAYOFFS
-> Solution concepts

…covered so far…

- Strategic Dominance
- Nash Equilibrium (NE) in static games of complete information

…heading toward…

- Subgame Perfect Nash Equilibrium (SPNE) in dynamic games of complete information
- Bayesian Nash Equilibrium (BNE) in static games of incomplete information
- Perfect Bayesian Equilibrium (PBNE) in dynamic games of incomplete information

4. Dynamic Games

-> "NOT-simultaneous-move" games – players move sequentially (in different moments)!
Example: Student's Dilemma Problem (sequential version) when YOU move first.

-> Credibility: Central issue in all dynamic games
Example: Student's Dilemma Problem when YOU are given a non-credible threat of failure in the exam if YOU decide to go to pub but not to the library.

5. The aim of this lecture is to show:

   a. How to describe a dynamic game?
   b. How to solve the resulting game-theoretic problem in the simplest case when players have complete and PERFECT information?

6. The extensive form representation of a game specifies:

   1. Who are the PLAYERS.
   2.1. When each player has the MOVE.
   2.2. What each player KNOWS when she is on a move.
   2.3. What ACTIONS each player can take.
   3. What is the PAYOFF received by each player.

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7. Normal-form representation is not very convenient for dynamic games. Therefore, an alternative game-tree representation is usually applied.

Each decision node of the tree (except the first one) is a starting point for a different subgame which includes all the decision and terminal nodes following it in the game tree. So, **the number of the subgames is equal to the number of decision nodes in the tree minus 1.**

N.B. We will need a much stronger definition when considering imperfect games of imperfect information.

8. The sequential Student-Dilemma Game is a member of the simple class of dynamic games of complete and perfect information which have the following timing:

    1. Player 1 chooses and action $a_1$ from the feasible set $A_1$.
    2. Player 2 OBSERVES $a_1$ and then chooses an action $a_2$ from the feasible set $A_2$.
    3. Payoffs are $u_1(a_1,a_2)$ and $u_2(a_1,a_2)$.

9. Distinct from the static games of complete information, here the strategy set of the second player does not coincide with its set of feasible actions:

Strategy in a dynamic game – a complete plan of action – it specifies a feasible action for each contingency (other player's preceding move) in which given player might be called to act.

10. This class of games are solved by backwards induction:

Solve the game from the last to the first stage:

- Suppose a unique solution to the second stage payoff-maximization:

$$R_2(a_1) = \arg\max_{a_2 \in A_2} u_2(a_1, a_2)$$

- Then assume a unique solution to the first stage payoff-maximization:

$$a_1^* = \arg\max_{a_1 \in A_1} u_1(a_1, R_2(a_1))$$

- Call $(a_1^*, R_2(a_1^*))$ a *backwards-induction outcome*.

11. Backwards induction outcome does not involve non-credible threats – it corresponds to the subgame-perfect Nash equilibrium as a refinement of the pure-strategy NE concept.