M9302 Mathematical Models in Economics

HOMEWORK 3 – due to 14.05.2010, 10 a.m. (at the lecture)

Problem 1 Asymmetric Bertrand Duopoly with discrete payoffs (5 points)

Consider a market leader and a follower who compete in prices. The leader has zero unit production cost while the follower has positive unit cost of either $c_H = 4$ or $c_L = 1$. The follower knows its cost but the leader does not know it for sure. Based on its experience, the leader sustains the belief that the two values c_H and c_L are equally probable i.e. could occur with probability ½. Firms pick prices from the discrete-choice action set {2,4,6}. The consumer demand function is D(p)=8-p, where p is the lowest price in the market. If both firms choose the same price, they share the demand equally. Otherwise, the firm with lower price serves all consumers, while the firm with higher price leaves the market, produces nothing and receives zero profit.

Find all pure strategy Bayesian Nash equilibria of this game.

Problem 2 First-bid Auction (5 points)

Two bidders take part in a first-bid auction. Each of them knows that the other bidder's valuation for the object is a uniformly distributed random variable on an interval $\left[\underline{v},\bar{v}\right]$. Bidder 1 has private information about its bidding function. That is, bidder 1 knows that the bidding function of bidder 2 is $b_2(v_2) = (v_2 - \underline{v})^2 + \underline{v}$, but bidder 2 does not know the bidding function of bidder 1.

Find the best response bidding function $b_1(v_1)$ of bidder 1.

Hint: $\Pr ob\left(b_1 > (v_2 - \underline{v})^2 + \underline{v}\right) = \dfrac{\sqrt{b_1 - \underline{v}}}{\bar{v} - \underline{v}}$

(BONUS) Problem 3 Signalling (+5 BONUS points)

A married couple wants to maximize their family welfare. The current welfare of the family is 10. With probability ½ they could face an event which would endanger their relationship by reducing the welfare to 5 if no suitable preventive measure is taken.

Only one of the partners, say partner 1, knows whether the family is in danger or not. She wants to inform the other partner, say partner 2, who could only take a precautionary action. Partner 1 has two signals which she might send to partner 2: Signal *S* and signal *σ*. Sending signal S reduces welfare by 2, while *σ* can be sent at no cost.

In response, partner 2 can take one of two actions *A* and *α*. Action A, is the suitable action when the danger has occurred, it increases the family's welfare by 5. However, if it is taken when there is no danger it reduces the welfare by 5. Action *α* always increases the welfare by 1.

1. Write the game tree (+1 BONUS point)
2. Check if there is any separating equilibria. (+2 BONUS points)
3. Check if there is any pooling equilibria (+2 BONUS points)