M9302 Mathematical Models in Economics

HOMEWORK 2 – due to 23.04.2010, 10 a.m. (at the lecture)

Problem 1 Dynamic game with complete information (5 points)

Consider a game with 2 players and perfect information. In the beginning, player 1 chooses one of the actions, *L* or *R*. Afterwards follows player 2 choosing *l* or *r* (in all cases). In the last stage, if at the beginning *R* was played by player 1 the game continues by again player 1 choosing *t* or *b*. If however, *L* was played in the first round, then the game ends with player's 2 move.

The payoffs are the following:
(2,3) -> if player 1 chooses *L* and player 2 plays *l*.
(1,1) -> if player 1 chooses *L* and player 2 plays *r*.
(3,1) -> if player 1 chooses *R* and *a* after player 2 plays *l*.
(4,3) -> if player 1 chooses *R* and *b* after player 2 plays *l*.
(2,0) -> if player 1 chooses *R* and *a* after player 2 plays *r*.
(0,2) -> if player 1 chooses *R* and *b* after player 2 plays *r*.

(a) Draw the game-tree. (0.5 point)
(b) How many subgames does this game have? (0.5 point)
(c) Draw a normal-form representation of this game. (1 point)
(d) Find all pure-strategy Nash equilibria. (1 point)
(e) Find all subgame-perfect equilibria. (2 points)
(f) Consider the same game but player 1 now (in his second move) cannot observe the action of player 2 – imperfect information. Repeat the exercise (a-f) for this modification. (+5 BONUS points)

Problem 2 Rubinstein's infinite-horizon bargaining game (3 points)

Reconsider the infinite-horizon sequential bargaining game solved in class (see copy handout "2.1.D Sequential Bargaining" by Gibbons (1992), pp. 68-71). Suppose now that the players have different discount factors: δ_1 for player 1 and δ_2 for player 2. Show that in the backwards-induction outcome, player 1 offers the settlement

$$\left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$

to player 2 who accepts.

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<u>Problem 3</u> Infinitely repeated Bertrand competition game with homogeneous goods (2 points)

Consider the static Bertrand duopoly model with homogeneous products.

Consumer's demand is given by the function $D(p_i) = \begin{cases} a - p_i, \text{ when } p_i < p_j \\ \frac{a - p_i}{2}, \text{ when } p_i = p_j \\ 0, \text{ when } p_i > p_j \end{cases}$.

Firms name prices simultaneously and marginal costs are constant at $c < a$.

For the infinitely repeated game based on this stage game, show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if the discount factor $\delta \geq 1/2$.