M9302 Mathematical Models in Economics

HOMEWORK 1 – due to 26.03.2010, 10 a.m. (at the lecture)

Problem 1 Simultaneous-move games with complete information (5 points)

(a) Represent the following games in normal form, i.e. by payoff matrix (3 points):

A. Stag-hunt Game

Description: The stag-hunt game is also generally known as the **game of assurance**. French philosopher, Jean Jacques Rousseau (28 June 1712 – 2 July 1778), presented the following situation. Two hunters can either jointly hunt a stag (an adult deer and rather large meal) or individually hunt a rabbit (tasty, but substantially less filling). Hunting stags is quite challenging and requires mutual cooperation. If either hunts a stag alone, the chance of success is minimal. Hunting stags is most beneficial for society but requires a lot of trust among its members.

Payoffs: Going for a stag alone yields 0, going together with the other +10; Going for rabbit pays +8 if you are alone, and only +7 if competing with the other.

B. Hawk-Dove Game

Description: The hawk-dove game is also commonly known as the **game of chicken**. Two hooligans (who have something to prove) drive at each other on a narrow road. The first to swerve loses faces among his peers. If neither swerves, however, a terminal fate plagues both. In the hawk-dove version of the tale, hooligans are replaced by armies considering going to war.

Payoffs: Staying alone yields +1, swerving alone yields -1; Swerving together with the other yields 0, staying together yields -100.

C. Pareto Coordination Game

Description: The game is named after Vilfredo Pareto (15 July 1848 – 19 August 1923), an Italian economist who introduced the welfare concept of socially optimal income distribution (aka Pareto efficiency). Informally, Pareto efficient situations are those in which it is impossible to make one person better off without necessarily making someone else worse off. In the game, two firms must simultaneously elect a technology to use for their compatible products. If the firms adopt different standards, few sales result. A common standard leads to higher sales. One technology is significantly preferred by consumers over the other. Thus, if the companies can standardize on the preferred technology, each obtains maximal profits.

Payoffs: Choosing any technology alone yields 0, colluding on the consumer's preferred technology yields +5 to both firms, colluding on the consumer's less preferred technology yields +3 to both firms.

D. Rock Paper Scissors Game

Description: To determine who is required to do the nightly chores, two children simultaneously make one of three symbols with their fists - a rock, paper, or scissors. Simple rules of "rock breaks scissors, scissors cut paper, and paper covers rock" dictate which symbol beats the other. If both symbols are the same, the game is a tie. The game is similar to the (two-strategy) Matching Pennies game.

Payoffs: Choosing the same symbol yields 0 to both players, choosing a stronger symbol yields +1, choosing a weaker symbol yields -1.

(b) For each game, identify all Nash Equilibria (2 points), including those in mixed strategies (+2 BONUS points).

Problem 2 Nash Equilibrium VS. Iterated Elimination of Strictly Dominated Strategies (2 points)

Consider a 2-player simultaneous-move game with normal form representation depicted in the following 3x3 payoff matrix:

	b_1	b_2	b_3	b_4
a_1	5,2	2,6	1,4	0,3
a_2	4,1	3,4	2,1	1,2
a_3	1,0	1,1	2,5	5,1
a_4	2,3	0,1	0,2	4,4

(a) Try to solve the problem using the solution concept of iterated elimination of strictly dominated strategies (1 point)
(b) Find all Nash Equilibria (1 point)
(c) Explain the difference and the relation between the two solution concepts (+1 BONUS point)

Problem 3 Cournot Duopoly with differentiated production (1 point)

Consider 2 producers of differentiated goods. Producer 1 produces good 1 at cost c_1, producer 2 produces good 2 at cost c_2. The inverse demand functions for their goods are $p_1 = M - 2q_1 - q_2$ and $p_2 = M - q_1 - 2q_2$. Producers choose their (output) quantity simultaneously.

(a) Find the equilibrium determined by prices and quantities. Calculate the optimal profit for each firm. (1 point)
(b) Construct and plot the reaction correspondences of players. (+1 BONUS point)

<u>Problem 4</u> Stackelberg Oligipoly (2 points)

Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm *i*. Each firm has a constant marginal cost of production, *c*, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively.

(a) What is the subgame-perfect outcome? (1 point)
(b) Which main property of backwards induction outcome stays behind the optimal outcome of the Stackelberg competition? (1 point)
(c) How the presence of a market leader (firm 1) affects the strategic choice and final payoffs of the followers (firms 2 and 3) compared to the game of simultaneous market entry (Cournot competition)? (+1 BONUS point)