The Social Welfare Effect of Tying in a Vertically Differentiated Market for System Goods: The Case of Microsoft*

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Abstract

The present paper is concerned with welfare analysis of the equilibrium outcome from tying a system platform to an application in the market for PC software systems. The issue has gained popularity in the last decade around the anti-trust cases against Microsoft. What is new in the present paper is that the market for PC software systems is assumed to be vertically differentiated. That is, there are well-defined leaders in quality whose brands dominate the market. First, the paper aims to show that the tying arrangement of Microsoft does not need to be driven by market-power-leverage incentives. In fact, the solution of the model shows that at the subgame-perfect equilibrium Microsoft would not impose tying and its good will be foreclosed in favor of a lower-quality one. So, the tying decision of Microsoft must rather be driven by market-share-saving incentives due to positive network effect on its good's quality. Second, the objective of the paper is to measure the effect of Microsoft's possible tying strategies on social welfare in vertically-differentiated market setting. The results show that the non-tying subgame equilibrium could be Pareto-dominated by two tying subgame equilibria. Particularly, tying-to-application subgame equilibrium will be socially optimal if a competitive system-platform supplier enters the market. If there is no entry the tying-to-platform subgame equilibrium outcome will be Pareto optimal instead.

Keywords: product tying, vertical differentiation, market foreclosure, system goods, social welfare, anti-trust policy

JEL classification: L11, L13, L15

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I. Introduction

There is a continuous debate among economists whether really merging two firms producing different components of a final good is always motivated by innocent incentives.

The debate gained popularity in the last decade around the anti-trust cases against Microsoft on allegations that Microsoft abused monopoly power by tying its operating system with other software applications. With few exceptions all the models of system-good markets used both by the opponents and the proponents of Mocrosoft in the debate assume a standard Chamberlinian market structure where goods are horizontally differentiated. However, there are authors (Etro (2006b)) who recognize a clear pattern of quality leadership in the market for PC software systems which implies that the system goods sold on these markets are rather vertically differentiated[1].

The present paper steps on the assumption that PC software systems are vertically differentiated in which case, tying behavior of Microsoft could be motivated by own-sales-saving rather than predatory (entry-deterrence) incentives. However, this possibility itself is not sufficient to claim that product tying would be a socially desirable practice under the assumed market conditions. Even when the existing level of market competition is not threatened, product tying could still hurt the society. The main objective of the paper is to clarify that issue by measuring the welfare effect of product tying in the case when it could be considered as a defensive strategy by the multi-product firm.

To the extent to which the setup of the model used in the paper is chosen to correspond closely to the structure of the market for PC software systems in the late 1990s, the implications of the paper are meant also to contribute to the economic debate on the anti-trust cases against Microsoft. Therefore, in the remainder of the introductory section, a description will be provided of the antitrust cases against Microsoft, existing literature on the problem, research objectives and suggested methodology for their fulfilment.

In 1998 the U.S. Department of Justice starts a trial against Microsoft on allegations that Microsoft abused monopoly power on Intel-based personal computers in its handling of operating system sales and web browser sales (DOJ Complaint 98-12320). In 2000, the

[1] For the difference between horizontal and vertical differentiation see (Shaked and Sutton, 1983, p. 1469).

findings of fact are issued in favor of the plaintiffs on almost all points including use of monopoly power to exclude rivals and harm competitors. The adopted legal remedies include splitting Microsoft into two companies, and imposing severe business conduct restrictions. Microsoft appealed and in 2011 the case is settled with agreement which does not prevent Microsoft from tying its operating system with other software applications.

However, independently from U.S. authorities, in 1998 the European Commission starts an investigation on how Microsoft streaming media application is integrated into its operating system. In 2003 Microsoft is ordered to disclose part of the code of its operating system as well as to offer a version of it without media player. In 2004 after a year of nonfeasance of the order, Microsoft is penalized to pay the largest fine ever handed out by the EU at the time (Commission Decision of 24.03.2004 relating to a proceeding under Article 82 of the EC Treaty (Case COMP/C-3/37.792 Microsoft)). As a result, in 2009 Microsoft executes the order of the European Commission and provides the latest version of its operating system with an option to choose with which media player to be installed.

The anti-trust cases against Microsoft have raised serious debate among economists. There are different arguments for and against Microsoft's tying arrangement but consensus is missing between the two sides in the dispute.

The proponents of Microsoft use three arguments why tying its operating system to an application would have a positive impact on the social welfare. First, Davis, MacCrisken and Murphy (1999) claim that the expansion of the system software functionality to include additional features of standalone applications is inevitable result of the evolution of the PC operating systems. So, the tying of Microsoft's products must be considered as a technological tying which increases the value of the operating system for the end user and thus increases the consumer surplus. Second, Economides (2000) shows that operating system is a network good i.e. its value for the end consumers increases in their number. Therefore, it is beneficial for the end users if they are served by a product with larger market share. Third, Etro (2006b) applies the theory of aggressive market leaders suggesting that Microsoft stays on top of the market because as a market leader it has the greatest incentives to innovate and keep its leadership position. Hence, it acts more competitively and thus serves society better as a monopolist than if there were not a dominant player in the market.

The opponents of Microsoft rather rely on the classical definition of tying as a market foreclosure practice. Carlton and Waldman (1998) state that by tying its operating system to different software applications Microsoft could leverage its market power at the operation system market to the respective market for standalone software applications. This would have a negative effect on social welfare if in this adjacent market Microsoft is a follower rather than a leader. Fudenberg and Tirole (2000) apply an overlapping generation model to show that even with network externalities there could be a negative social welfare effect of market foreclosure when the new users are relatively more than old ones. That is, the welfare loss by the presence of monopoly might not be fully compensated by the network externalities of expanding the pool of consumers served by the monopolist.

Neither the proponents, nor the opponents of Microsoft, however, have so far studied the possibility that Microsoft might have decided to tie its operating system with its software applications as a response to an entry in the monopoly market for personal-computer operating systems.

Although not in the context of the antitrust cases against Microsoft, the potential existence of such an incentive for tying is suggested by Kovac (2007). He shows that when multi-product goods are assumed to be vertically differentiated, market foreclosure incentive to bundle changes. The multi-product firm might be interested to apply tying not because in this way it would become a monopolist on both integrated markets. Rather it would tie its product as a response to a foreclosure threat from a potential lower quality entrant.

What makes the conjecture of Kovac (2007) interesting for reconsideration in the context of the anti-trust cases against Microsoft is that it seems consistent with the market facts at the time when Microsoft has taken the decision to tie its products. With the development of Internet in the beginning of 1990s, a commercial version of UNIX, the dominant brand in the server operating system market, enters the personal-computer system software market under the brand Linux. This allows consumers to use both Microsoft's browsing application Internet Explorer and its single competitor Netscape Navigator not only on Microsoft's operation system Windows but also on Linux. The resulting market situation of 3 potential entrants (Microsoft, Linux, Netscape), 2 types of components (operation system and browser), and 4 possible goods (WindowsNetscape, WindowsExplorer, LinuxNetscape, LinuxExplorer) matches exactly the market setting suggested by Kovac (2007).

Particularly, the conjecture made by Kovac (2007) when applied to the system-software market of 1990s implies that if Microsoft's browsing application Internet Explorer were not tied to its operating system Windows, consumers would not be willing to buy Internet Explorer based on Windows. They would either use it on Linux or purchase Netscape Navigator based on Windows instead. So, Microsoft would have no option to save the sales of WindowsExplorer other than tying the browser to the operating system. Tying itself would result in foreclosure of the sales of Linux which would benefit the both quality leaders, Microsoft and Netscape.

There are two inconsistencies between the described market environment and the original model of Kovac (2007) which make it inappropriate for direct application to the Microsoft's case.

The first inconsistency is an assumption in Kovac's (2007) model that does not conform completely to the standard definition of a system good. If not bundled the multi-product firm's products are assumed to be sold in separate markets. As a result the assumed initial market conditions for exactly two goods to cover the market correspond to a single-product market and differ from the ones that would be valid for a pure system-good market.

In order to improve the general validity of the main implications of Kovac (2007), Burlakov (2011b) applies an alternative model of a vertically differentiated market. The model is originally developed by Burlakov (2011a) to fit better the standard definition of a system good. In Burlakov's (2011a) setup the different types of components of the system goods are assumed to be sold together no matter whether tying arrangement is imposed or not. Respectively, by solving the new model Burlakov (2011b) shows that the non-bundling subgame equilibrium outcome described by Kovac (2007) could still hold in a pure system-good market but at a narrower spread of consumer tastes. This difference is important because it makes the non-bundling strategy but not the tying one an optimal choice for the multi-product firm. Moreover, in order the multi-product firm's good to be effectively excluded in the non-tying equilibrium, an additional condition needs to be introduced. Namely, the quality of the multi-product firm's good must be relatively closer to the quality of the best good than to the one of the worst good available in the market.

In the present paper, the equilibrium prices and profits are derived based on the application of the model of Burlakov (2011b).

The second inconsistency between Microsoft's case and Kovac (2007) is that the tying strategy suggested as an effective market-foreclosure device does not correspond to the one imposed by Microsoft. Kovac (2007) considers a classical tying strategy where the sales of the product which is competitively supplied by the multi-product firm are tied to the sales of the product which the multi-product firm supplies as a monopolist. Microsoft, however, sells its leading-quality system platform with embedded lower-quality software applications. That is, it ties the sales of the monopoly product to the competitive one. As it will be demonstrated latter in the paper, both tying strategies could be considered as motivated by own-sales-saving rather than by market-power leverage incentive. Under the specified initial conditions their application leads to effective foreclosure of competitor's sales, though.

The present paper aims to apply a modified version of the model of Burlakov (2011b) which is designed to fit closely the structure of the system-software market in the late 1990s. Based on the equilibrium solution of the model, the social welfare will be measured in case of tying[2] and non-tying. The comparison of the two measures allows checking whether society would have gained if Microsoft had abstained from tying and let instead its browser be sold separately from its system platform. It also allows for policy implications to be derived how adequate to the considered market situation are the measures taken by the U.S. and the EU.

The paper is organized as follows. In Section 2 the modified version of the Burlakov's (2011b) model is presented in the context of the system-software market in which Microsoft imposed tying arrangement during the late 1990s. Section 3 presents the solution of the model and specifies the

[2] In the case of Microsoft, the system platform is tied to the internet browser but not the other way around as suggested by Kováč (2007). However, as it is shown in the present paper the implications of Kováč (2007) still hold with the actual tying strategy of Microsoft if consumers are allowed to install the Netscape's browser together with Microsoft's one on the Windows system platform. In fact, in reality they could never be prevented to do so. Therefore, taking that into account in the current analysis not only does not restrict the validity of its results but makes them even more consistent with the real-world case being analyzed.

equilibrium outcomes with and without tying arrangement, respectively. The welfare analysis is given in Section 4. Section 5 concludes.

II. The Model

2.1. Consumer Side

As it is standard for the models of product differentiation, here it is assumed that consumers make indivisible and mutually exclusive purchases[3] in a sense that they either buy a unit of a good or do not buy at all. Another standard assumption for the models of vertical product differentiation is that consumers are characterized by their taste parameter for quality θ which is assumed to be uniformly distributed on the interval $[\underline{\theta}, \overline{\theta}]$.

The utility function measures the individual surplus of consumer θ from purchasing given good *k* and takes the following form:

$$U_{\theta,k} = s_k \cdot \theta - p_k \tag{1}$$

where:

s_k – quality of system good *k*

p_k – price of system good *k*

The bounds of the market shares of the goods are marked by the taste parameters of the so-called "marginal consumers". Each marginal consumer $\theta_{k/k'}$ is indifferent between given distinct pair of available qualities, k and k'. If feasible $\left(\theta_{k/k'} \in \left(\underline{\theta}, \overline{\theta}\right)\right)$, the value of the marginal taste parameter divides the market into two groups of consumers. The group of

[3] In the context of the software-system market, by purchasing a good, here it is meant paying the license fees for the operating system and the application software which the consumer runs on it. Since software license fees are paid per workplace and a PC user is not expected to use the same application on two computers simultaneously, her utility is defined on a unit purchase. Respectively, since the ordinary software applications (excl. client-server systems) are not meant to be used simultaneously by two users on the same computer, the purchase is considered exclusive.

consumers characterized by taste parameter that is lower than $\theta_{k/k'}$ who strictly prefer good k' and the group of consumers with taste higher than $\theta_{k/k'}$ who strictly prefer good k. Let the goods be ranked in decreasing order of their quality. Then, the demand share of any good k would be bounded from below by the marginal taste parameter $\theta_{k-1/k}$ which separates its consumers from the consumers of its neighbor by rank *k-1*. Respectively, the marginal taste parameter $\theta_{k/k+1}$ bounds the demand share of good k from below.

The expression for the marginal consumer's taste parameter could be directly derived from (1) and looks as follows:

$$\theta_{k/k'} = \frac{p_k - p_{k'}}{u_k - u_{k'}} = \frac{p_k - p_{k'}}{d_{k/k'}} \tag{2}$$

where:

k, k' – the indices of a pair of goods available in the market which yield the same magnitude of utility to a consumer with taste parameter $\theta_{k/k'}$; $k < k'$

$d_{k/k'}$ – the difference in qualities between goods k and k'

2.2. Producer Side

Here, the market structure of the software-system market will be parameterized according to how it looked in 1990s.

There is only one multi-product firm producing both system and application software. That is Microsoft. For simplicity, here the attention will be restricted only on one software application. Namely, the application for internet browsing. So, each system good on the market consists of two elements – system platform and internet browser. The system platform produced by Microsoft is called Windows while its internet browser is called Internet Explorer. Here, the former is denoted by WIN while the latter - by IE.

The rest are single-product firms. For example Linux supplies only a system platform with the same name, while Netscape produces an internet browser called Netscape Navigator. In the paper, the abbreviated notation for the former is LIN while the latter is denoted by NET.

In 1990s Microsoft has almost the whole market for PC system platforms. In conformity with the assumption that system software market is vertically differentiated, its platform quality is considered to be the highest in the market. Analogously, the best browser application is considered to be the one of Netscape. The system platform of Linux and the Microsoft's browser (IE) are respectively second-best in their category. There are no other firms producing these two types of software products in 1990s. Therefore, in the next section such market conditions are chosen at which the rest of the firms would not find it optimal to enter the market in equilibrium. Respectively, they are labeled not by name but by numerical index decreasing in their quality. Without loss of generality, it is assumed that the number of firms supplying a system platform (denoted by *m*) is at most equal to the number of firms supplying an internet browser (denoted by *n*), i.e. $m \leq n$.

Since the quality of a PC system platform is more significant for the quality of its combination with a given browsing application, it is assumed that system combinations' qualities follow a lexicographic order as shown in table 1 below:

	Platform A firm i	Browser B firm j	System Combination k=[ij]
	A_1 (WIN)	B_1 (NET)	[11]
	A_1 (WIN)	B_2 (IE)	[12]
	A_1 (WIN)	B_3	[13]
	…	…	…
	A_1 (WIN)	B_n	[1n]
	A_2 (LIN)	B_1 (NET)	[21]
	A_2 (LIN)	B_2 (IE)	[22]
	…	…	…
	A_m	B_n	[mn]
Total number:	***m***	***n***	**(*m*·*n*)**

Table 1: System combinations ranked in decreasing order of quality, the best quality has the lowest rank

The profit functions of firms A*i*, B*j* and M (for Microsoft) take the following forms:

$$\Pi_{Ai} = p_{Ai}\sum_{j=1}^{n}\left(D_{[ij]}(p_{Ai}, p_{Bj})\right), \text{ for } i = 2,\ldots,m \tag{3}$$

[4]

$$\Pi_{Bj} = p_{Bj}\sum_{i=1}^{m}\left(D_{[ij]}(p_{Ai}, p_{Bj})\right), \text{ for } j = 1,3,\ldots,n \tag{4}$$

$$\Pi_{M} = \begin{cases} p_{A1}\sum_{j\neq 2}\left(D_{[1j]}(p_{A1}, p_{Bj})\right) + p_{B2}\sum_{i=2}^{m}\left(D_{[i2]}(p_{Ai}, p_{B2})\right) + p_M D_{[12]}(p_M), \text{ if tying is imposed} \\ p_{A1}\sum_{j=1}^{n}\left(D_{[1j]}(p_{A1}, p_{Bj})\right) + p_{B2}\sum_{i=2}^{m}\left(D_{[i2]}(p_{Ai}, p_{B2})\right), \text{ otherwise} \end{cases} \tag{5}$$

where:

p_{Ai} - the price of platform A_i

p_{Bj} - the price of browser B_j

[4] For simplicity, it is assumed that all firms have zero production cost.

p_M - the price of the multi-product good given that tying is imposed

D_{ij} - the market share of good A*i*B*j*

2.3. Equilibrium Concept

The market competition is modeled according to a three-stage non-cooperative game of complete but imperfect information. First, firms make simultaneously an entry decision, to enter the market or not. Second, the multi-product firm (Microsoft) observes who is in the market and decides which tying strategy to apply – to tie the sales of B2 (IE) to A1 (WIN), to embed B2 (IE) into A1 (WIN) or not to tie. In the last stage the firms in the market compete in prices. The optimal solution of the model is derived based on the perfect equilibrium concept of Selten (1975).

To hold in equilibrium each price $p^*_{Ai/Bj}$ must solve the following profit maximization problem:

$$\max_{p_{Ai/Bj}\geq 0} \Pi_{Ai/Bj}\left(p^*_{A1},..., p^*_{Ai-1}, p^*_{Bj/Ai}, p^*_{Ai+1},..., p^*_{Am}, p^*_{B1},..., p^*_{Bj-1}, p^*_{Bj+1},..., p^*_{Bn}\right) \qquad (6)$$

If driven by rent-seeking incentives, the multi-product firm would choose its tying strategy dependent on which of the three respective price-subgame equilibria brings it the highest payoff.

Respectively, if the product of firm A*i*/B*j* has a positive profit at its equilibrium price given the tying choice in the second stage, this would imply that for firm A*i/Bj* the optimal (first-stage equilibrium) choice is to enter the market in the first stage. Otherwise, the firm cannot be better-off of entering the market, so its first-stage equilibrium choice will be not to enter the market.

The perfect equilibrium outcome will be given by the set of the firms' entry and price strategies together with the multi-product firm's tying strategy which constitute equilibrium in each of the three stages of the game given the initial values of the goods' quality differences $\left(d_{k/k'} : k < k' \in \left[[11],[mn]\right]\right)$ and the bounds ($\underline{\theta}$ and $\overline{\theta}$) of the consumer taste distribution.

III. Equilibrium Analysis

In this section the model is solved to show that such initial market conditions could exist at which there is a credible threat for foreclosure of the Microsoft's system combination in the pricing equilibrium of the non-tying subgame. Respectively, at the same conditions it would be demonstrated that the market could be still sufficiently narrow for the sales of Linux to be foreclosed in the pricing equilibrium of two alterative tying subgames. At that setting imposing a tying arrangement in equilibrium might be interpreted not only as a predatory-pricing strategy aiming market foreclosure from the point of view of Linux but also as a preventive strategy against market foreclosure from the point of view of Microsoft. The latter hypothesis is studied in more details in the end of the section. It is demonstrated that given positive (network) effect of market share of system goods on their qualities, the tying-to-browser strategy chosen by Microsoft would be optimal in the subgame perfect equilibrium solution of the model. Otherwise, the non-tying strategy would strictly dominate the two possible tying strategies and Microsoft would not have a profit-maximizing incentive to impose tying-to-browser arrangement.

3.1. Price Equilibrium

3.1.1. Non-tying Subgame

In this subsection will be established the non-tying (NoT) subgame equilibrium in prices.

Beforehand, however, the initial market condition needs to be specified for exactly the two best suppliers of each product category (i.e. Microsoft, Netscape and Linux) to have positive market shares in the equilibrium. This condition is presented by lemma 1 below.

Lemma 1. Let $\frac{3}{2}\underline{\theta} < \overline{\theta} \leq 2\underline{\theta}$. Then, of any *m* products of type A and any *n* products of type B, exactly two of each type will have positive market shares in equilibrium.

Proof: see Appendix A.

Next market condition is necessary to ensure foreclosure of the sales of the Microsoft's good A1B2 (WINIE). It is presented by lemma 2 below.

Lemma 2. Let $d_{[11]/[12]} \geq \frac{3}{2} d_{[12]/[22]}$. Then of all the four possible system combinations of the products supplied by Microsoft, Linux and Netscape only A1B1 (WINNET) and A2B2 (LINIE) will have positive market shares in equilibrium.

Proof: see Appendix B.

The basic intuition behind lemma 2 is that if the multi-product firm's good is not sufficiently differentiated from its neighbors by quality in table 1, profit maximization requires from the multi-product firm to set prices which are higher than the ones at which its good will have positive demand[5].

Based on lemma 1 and lemma 2 the third-stage price-equilibrium of the non-tying subgame is established in the following proposition:

Proposition 1. Given the conditions of lemma 1 and lemma 2:

(i) there exists a price set $\left\{ p_{A1}^{NoT} = p_{B1}^{NoT} = \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{4} d_{[11]/[22]}, p_{A2}^{NoT} = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)}{4} d_{[11]/[22]}, p_{B2}^{NoT} = 0 \right\}$ which forms a Nash equilibrium in prices of the non-tying subgame. Proof: see Appendix A.

(ii) the equilibrium outcome implies positive market shares only for two goods, A1B1 and A2B2. So, only the four firms A1, A2, B1 and B2 whose products take part in these goods will have positive market shares. The rest (m·n-2) goods (incl. the multi-product firm's good A1B2) will have zero sales. Proof: Direct implication of lemma 1 and lemma 2.

There is an important implication of the non-tying subgame equilibrium outcome at the initial conditions established in lemma 1 and lemma 2. Namely, the system A1B2 consisting of the two products supplied by Microsoft would not have a positive demand unless Microsoft has chosen to impose a tying arrangement in the second stage. The browsing application B2 (IE) would have positive sales only in combination with the competitive system platform A2 (LIN). Furthermore, it would be sold at a cost in equilibrium.

[5] For more details on the economic mechanism in which the sales of a middle-quality good could be foreclosed in favor of its lower-quality neighbor, see Burlakov (2011a).

3.1.2. Tying-to-platform Subgame

Here, the pricing subgame will be derived that would follow if Microsoft has chosen to set a classical tying strategy (TieA) at the second stage – selling the browser B2 (IE) only conditional on the purchase of the platform A1 (WIN). From technical point of view, it is a repetition of the solution for the price equilibrium of the tying subgame as presented in Burlakov (2011b). The result is given in proposition 2.

Proposition 2. Given the conditions of lemma 1 and lemma 2:

(i) there exists a price set $\left\{ p_{A1}^{TieA} = p_{B1}^{TieA} = \frac{\bar{\theta}}{3} d_{[11]/[12]}, p_{M}^{TieA} = 0 \right\}$ which forms a Nash equilibrium in prices of the classical tying subgame. Proof: see Appendix C.

(ii) the equilibrium outcome implies positive market shares only for two goods, A1B1 and A1B2. So, only the two firms M and B1 whose products take part in these goods will have positive market shares. The rest (m·n-2) goods (incl. all the goods based on A2) will have zero sales. Proof: Provided that the multi-product firm's good A1B2 is priced at a cost according to the price equilibrium derived in (i), no lower-quality good with the same cost could have a positive demand.

The economic intuition behind the equilibrium outcome described in proposition 2 is that by tying its browser to the platform, Microsoft is able to price-discriminate. It could charge its good an independent wholesale price which is small enough (p_m=0) for lower-taste consumers to prefer it to the worse-quality goods. At the same time, this does not prevent Microsoft from pricing its platform higher when it is sold in combination with the better browser of Netscape. So, Microsoft does not need to sacrifice the sales of its good in order to acquire a positive rent from the higher-taste consumers.

3.1.3. Microsoft Tying-to-browser Subgame

The third pricing subgame corresponds to the choice of tying strategy (TieB) which is actually made by Microsoft. Namely, embedding the browser B2 (IE) into the platform A1 (WIN) and selling the latter only conditional on the purchase of the former. In order the outcome of this tying arrangement to be relevant to the real market competition being analyzed, however, an additional assumption should be made. In reality, embedding the browser into the platform of Microsoft does not prevent consumers from installing additional browser together with it. Therefore, here it is assumed, that the supplier of the competitive browser A1 (NET) could still sell it in system combination with the multi-product firm's good A1B2. To simplify the notation, the resulting system good A1B2B1 is labeled by W. In addition, at this point, it is also relevant to assume that the embodiment of the browser of Microsoft to the best good does not change its quality ($s_W = s_{11}$)[6].

Distinct from the previous two subgames, here the price equilibrium changes dependent on the entry decision of the competitive system-platform supplier(s) (e.g. Linux). Propositions 3 and 4 below define the price-subgame equilibrium with and without entry, respectively. The label of the tying-to-browser subgame without entry is marked by asterisk to distinguish it from the label of the subgame with entry.

[6] One might argue that the better-quality good in the non-tying subgame increases its quality due to the embodiment of the Microsoft's browser to it in the tying-to-browser subgame. However, the counter-argument could be used that if this is really the case, nothing prevents consumers from installing Microsoft's browser to the best system which would bring the same additional quality to the better-quality good in the non-tying subgame. So, the two qualities would be equal again.

Proposition 3. Given the conditions of lemma 1 and lemma 2 and market entry of at least one more system-platform supplier except Microsoft in the first stage:

(i) there exists a price set $\left\{p_{B1}^{TieB} = p_{M}^{TieB} = \frac{\theta}{2} d_{[W]/[21]}\right\}$ which forms a Nash equilibrium in prices of the tying-to-browser subgame. Proof: see Appendix D.

(ii) the equilibrium outcome implies positive market shares only for the Microsoft's good A1B2 (WINIE) and the best browser B1 (NET) which are sold together in one system combination. So, only the two firms Microsoft and Netscape whose products take part in this combination will have positive market shares. The rest (m·n-1) feasible goods (incl. all the goods based on A2) will have zero sales. Proof: see Appendix D.

The economic intuition behind this subgame-equilibrium outcome is that by tying its platform to the browser, Microsoft is able to sell its good as a part of the highest-quality system combination in the market. So, it is not forced to sell it at a cost anymore. Respectively, the supplier of the competitive browser (Netscape) gets better-off from selling at a price which forecloses the independent sales of Microsoft's system combination. As a result the highest quality combination with two browsers forecloses the sales of all the lower quality goods including the ones based on competitive platform(s).

Proposition 4. Given the conditions of lemma 1 and lemma 2 and no market entry of other system-platform supplier than Microsoft in the first stage:

(i) there exists a price set $\left\{p_{B1}^{TieB*} = p_{M}^{TieB*} = \frac{\overline{\theta}}{3} d_{[W]/[O]}\right\}$ which forms a Nash equilibrium in prices of the tying-to-browser subgame. Proof: see Appendix D.

(ii) the market will be undercovered in the specified subgame equilibrium. Proof: see Appendix D.

The market width established with lemma 1 allows for exactly two goods to have positive market share in equilibrium. Given no entry and tying-to-browser strategy chosen by Microsoft, the two firms in the market still offer only the system combination of their products but do not need to serve the whole market in order to maximize their profits. As a result the market would remain undercovered in the tying-to-browser subgame without entry.

3.2. Tying strategies

The optimal choice of a tying strategy in the second stage would depend on the comparison of the payoffs of the multi-product firm at the equilibrium prices of the three tying subgames. In real, Microsoft chooses the tying-to-browser strategy. The payoff of it, however, could exceed the payoffs in the non-tying and classical tying subgames only if the competitive system-platform supplier(s) decides not to enter in the first stage. The condition for that is given by lemma 3.

Lemma 3. Let $d_{[W]/[O]} > \frac{9}{4} d_{[11]/[22]}$ and there is no entry of a competitive system-platform supplier at the first stage. Then, Microsoft would find it optimal to choose the tying-to-browser strategy. Given entry, however, the non-tying strategy would make Microsoft be strictly better-off.

Proof: see Appendix E.

The economic intuition behind the condition of lemma 3 follows from the positive relationship between quality differences and prices. Microsoft's equilibrium payoff will be highest in the subgame equilibrium in which the quality differential between its good and the other good in the market is the biggest. The larger quality differentiation implies higher prices of these goods and better profits for their suppliers. So, when lemma 1, lemma 2 and lemma 3 hold, the tying choice of Microsoft is also its profit-maximizing strategy but only given no entry. Because of the assumption that $s_W = s_{11}$, when Linux decides to enter at the first stage, this decreases the differential between the qualities in the market $\left(d_{W/21} < d_{11/22} < d_{W/O}\right)$. Respectively, the tying-to-browser strategy yields lower payoff to Microsoft than in the non-tying subgame equilibrium[7]. Hence, in the context of the current setup of the model any eventual threat of Microsoft to impose tying as a response to entry by Linux seems non-credible.

3.3. Entry strategies

After the equilibrium pricing and tying strategies in the third and second stage are defined, the last step of the solution analysis is to specify the optimal entry strategies of the firms in the first stage. Only the firms that supply product whose sales would not be foreclosed under the conditions of lemma 1, lemma 2 and lemma 3 will find it optimal to enter the market in equilibrium. The rest would be better-off of not entering as stated in proposition 5 below.

Proposition 5. Assume any infinitesimally small entry cost $\varepsilon \to 0$ and any number of *m-1* potential entrants supplying a system platform A and *n-1* potential entrants supplying an internet browser B plus a multi-product entrant (Microsoft) supplying the best system-platform A1 (WIN) and the second-best browser B2 (IE). Given that consumer heterogeneity

[7] Similarly, the tying-to-platform strategy is also strictly dominated by the non-tying one ($\Pi_M^{NoT} > \Pi_M^{TieA}$). The comparison between the payoffs of the two is presented in Appendix E.

and quality differentiation of goods satisfy the conditions of lemma 1 and lemma 2, respectively:

(i) there exists a subgame perfect equilibrium in which the multi-product firm (Microsoft), and the firms supplying the second-best product of type A (Linux) and the best product of type B (Netscape) enter. The multi-product firm would choose not to tie its products. The equilibrium price set is $\left\{ p_{A1}^{NoT} = p_{B1}^{NoT} = \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{4} d_{[11]/[22]}, p_{A2}^{NoT} = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)}{4} d_{[11]/[22]}, p_{B2}^{NoT} = 0 \right\}$. Proof: Direct result of proposition 1 and lemma 3.

(ii) no perfect subgame equilibrium exists in which more than one other supplier of system platform A than the multiproduct firm and/or more of two suppliers of internet browser B (including Microsoft) enter the market.

Proof. According to proposition 1 when the tying-to-browser strategy is chosen only three firms, Microsoft, Netscape and Linux, would have positive revenues to cover the entry cost and to have positive net payoff of entry. The entry payoff of all the rest would be -ε while they earn zero of no entry. So, entry cannot be an equilibrium strategy for the other suppliers of system platforms A and internet browsers B. This establishes the proof of (ii).

According to the equilibrium solution specified in proposition 5 the tying-to-browser arrangement chosen by Microsoft would be strictly dominated by the non-tying strategy at which the sales of the Microsoft’s good are foreclosed in equilibrium. Hence, the tying arrangement chosen by Microsoft cannot be motivated by maximization of its sales revenue[8]. However, this does not imply automatically that tying arrangement was imposed as a tool for market foreclosure. Given that tying-to-browser arrangement could save the sales of the multi-product firm's good at the market setting of the present model, it could be argued that Microsoft's tying decision was rather driven by incentives to save the sales of its system good.

The argument that market share might be more relevant objective than sales revenue is supported also by the recent development of the system-software market. Nowadays, except by profit-maximizing firms like Microsoft the PC users are served also by several non-profit organizations who supply open-source freeware products. A possible explanation for this market trend is the presence of network externalities which make quality of a good to increase in the number of its users. Next subsection shows how the assumption of network externality effect on the quality of the good of Microsoft could turn its tying-to-browser strategy into dominant one in the subgame perfect equilibrium.

3.4. Equilibrium solution with network externalities

Here, it is assumed that the quality of good's components in the system-software market is positively related to the number of consumers who use it. As a result, when the sales of the Microsoft's good are foreclosed in the non-tying subgame, the quality of the best system combination A1B1 based on Microsoft platform is lower than in the tying-to-platform $(s_{11} < s_{11^*})$[9] and tying-to-browser subgames $(s_{11} < s_W)$ where the Microsoft's good is not foreclosed. Furthermore, the quality of the goods based on Linux system platform (A2B1 and

[8] The same conclusion would hold if Microsoft has chosen to impose a classical tying-to-platform arrangement. For more detailed discussion on the tying incentives of a multi-product firm in such a market setting, see Burlakov (2011b).

[9] From now on, to distinguish the quality of a good in the non-tying subgame from its quality in the tying subgames where its market share is different, the rank index in the latter case will be marked by asterisk.

A2B2) would be lower when their sales are foreclosed in the tying-to-platform and tying-to-browser subgames compared to the non-tying subgame $\left(s_{22*} < s_{22}\right)$ where they are not foreclosed. Hence, the quality differential in the tying-to-browser subgame given market entry of Linux could exceed the quality differential of the non-tying subgame $\left(d_{[11]/[22]} < d_{[W]/[22*]}\right)$ which would yield a higher payoff to Microsoft. The exact condition is stated by lemma 4 below.

Lemma 4. Let $d_{[W]/[22*]} > \frac{9}{8} d_{[11]/[22]}$ due to the network effect on the quality of the system goods. Then, Microsoft would find it optimal to choose the tying-to-browser strategy with or without entry of a competitive system-platform supplier.

Proof: see Appendix E.

The condition of lemma 4 implies different subgame perfect equilibrium outcome from the one suggested by proposition 5. Given network externality effect on the quality of the system goods, the tying-to-browser strategy would be the profit maximizing choice of Microsoft. So, it would dominate the non-tying strategy in the subgame perfect equilibrium as stated in proposition 6.

Proposition 6. Assume any infinitesimally small entry cost $\varepsilon \to 0$ and any number of *m-1* potential entrants supplying a system platform A and *n-1* potential entrants supplying an internet browser B plus a multi-product entrant (Microsoft) supplying the best system-platform A1 (WIN) and the second-best browser B2 (IE). Given that consumer heterogeneity and quality differentiation of goods satisfy the conditions of lemma 1, lemma 2 and lemma 4, respectively:

(i) there exists a perfect subgame equilibrium in which only the multi-product firm (Microsoft) and the firm supplying the best product of type B (Netscape) enter, charge their products prices $\left\{ p_{B1}^{TieB*} = p_{M}^{TieB*} = \frac{\bar{\theta}}{3} d_{[W]/[O]} \right\}$, while the multi-product firm chooses to tie its system platform A1 (WIN) to the internet browser B2 (IE). So, in the end only the system combination of the multi-product firm's good (A1B2) and best browser (B1) has positive sales and yields positive revenues. Proof: direct result of proposition 4 and lemma 4.

(ii) no perfect subgame equilibrium exists in which another supplier of system platform A than the multiproduct firm and/or more of two suppliers of internet browser B (including firm M) enter the market. Proof: analogous to the proof of part (ii) of proposition 5.

Proposition 6 fulfills the objective of the section. Namely, the existence of a perfect subgame equilibrium is proven at which the tying-to-browser arrangement chosen by Microsoft would be optimal because it could save the sales of the multi-product firm's good. However, this still does not imply that the tying strategy chosen by Microsoft is socially admissible at the established market conditions. Next section aims to measure how this strategy would affect social welfare relative to the benchmark cases when the classical tying or non-tying arrangements are imposed instead. The aim is to show that the non-tying subgame equilibrium outcome does not need to be socially optimal. Furthermore, there are conditions at which the subgame perfect equilibrium in the setup with network externalities would Pareto-dominate the subgame equilibrium in the setup without network externalities. That is, the tying strategy chosen by Microsoft could be socially optimal.

IV. Social Welfare Evaluation

This section analyzes the social-welfare effect of the tying strategy chosen by Microsoft in market setting with vertically differentiated system goods. It consists of three subsections. The first subsection compares the equilibrium social surplus at each of the two tying arrangements with the social surplus that would be if the multi-product firm did not impose a tying arrangement. Respectively, the comparison is made for the cases with and without market entry of a competitive platform supplier as well as for the model setups with and without network externalities. The second subsection compares the equilibrium social surpluses between the two alternative tying arrangements – tying to platform versus tying to browser with and without competitive entry. The third section discusses the policy implications of the final results.

4.1. Measuring the social welfare effect of product tying

The social welfare analysis starts with measuring the difference between social surpluses given different tying arrangements versus the case of non-tying in the basic setup without network externalities. The respective results for the signs of the surplus differences[10] are presented in table 2 below.

	TieA vs. NoT	**TieB vs NoT**	**TieB* vs NoT**
	$d_{[11^*]/[12^*]} = d_{[11]/[12]}$ $\frac{3}{5} d_{[11]/[22]} < d_{[11]/[12]} < d_{[11]/[22]}$	$d_{[W]/[11]} = 0$	$d_{[W]/[11]} = 0$ $d_{[W]/[O]} > \frac{9}{4} d_{[11]/[22]}$
PS	-	-	+
CS	+	+	-
SW	+	+	-

Table 2: Model without network externalities: Comparison of equilibrium social surpluses at different tying arrangements versus the case of non-tying

[10] For the explicit measures see Appendix F.

The left column of table 2 presents a comparison of the social surpluses between the tying-to-platform subgame and the non-tying subgame. In compliance with proposition 2, at the conditions of lemma 1 and 2, the price equilibrium of the tying-to-platform subgame implies foreclosure of the sales of the goods based on the worse-quality platform in favor of the goods based on the better-quality one. This is related to smaller quality differentiation ($d_{[11]/[12]} < d_{[11]/[22]}$), lower average price and higher average quality[11] in the market. Respectively, there is lower producer surplus and higher social welfare in the tying-to-platform subgame.

The middle column of table 2 presents a comparison of the social surpluses between the tying-to-browser subgame with entry (of a competitive platform supplier) and the non-tying subgame. In compliance with proposition 3, at the conditions of lemma 1 and 2, the price equilibrium of the tying-to-platform subgame implies foreclosure of the sales of the goods based on the worse-quality platform in favor of the best good with embedded browser from Microsoft. Given the assumption of the basic model that $s_W = s_{11}$, this refers to identical quality differentiation ($d_{[W]/[22]} = d_{[11]/[22]}$) but lower average price and larger market share of the better good, that is higher average quality in the market. Respectively, there is lower producer surplus and higher social welfare in the tying-to-platform subgame with entry.

The right column of table 2 presents a comparison of the social surpluses between the tying-to-browser subgame without entry (of a competitive platform supplier) and the non-tying subgame. In the tying-to-browser subgame without entry, the best quality good with embedded browser of Microsoft serves the market alone again but this time the ‘foreclosure’

[11] For more detailed analysis how vertical quality differentiation and market width affect social welfare and producer surplus see Burlakov (2011b).

outcome is set by construction. So, the best good competes only with the outside option which implies larger quality differentiation ($d_{[W]/[O]} > d_{[11]/[22]}$), higher average price and lower average quality in the (undercovered) market. Respectively, there is higher producer surplus and lower social welfare in the tying-to-platform subgame without entry.

Table 3 presents the same social comparisons as in table 2 but measured in the setup of the model with network externalities.

	TieA vs. NoT			**TieB vs NoT**	**TieB* vs NoT**			
	small* $d_{[11^*]/[12^*]}$	middle** $d_{[11^*]/[12^*]}$	big*** $d_{[11^*]/[12^*]}$	$d_{[W]/[11]} > \frac{3}{2} d_{[11]/[22]}$	small $d_{[W]/[11]}{}^{+}$	middle $d_{[W]/[11]}{}^{++}$	big $d_{[W]/[11]}{}^{+++}$	very big $d_{[W]/[11]}$
PS	-	-/+ $\bar{\theta} \to 2\underline{\theta}$	+	+	+	+	+	+
CS	+	+	+	+	-	-	+/- $\bar{\theta} \to 2\underline{\theta}$	+
SW	+	+	+	+	+/- $\bar{\theta} \to 2\underline{\theta}$	+	+	+

$* \frac{3}{5} d_{[11]/[22]} < d_{[11^*]/[12^*]} < d_{[11]/[22]}$; $** \, d_{[11]/[22]} < d_{[11^*]/[12^*]} < \frac{171}{128} d_{[11]/[22]}$; $*** \frac{171}{128} d_{[11]/[22]} < d_{[11^*]/[12^*]}$;

$^{+} \, 0 < d_{[W]/[11]} < \frac{112 d_{[11]/[O]} - 81 d_{[11]/[22]}}{320}$; $^{++} \, \frac{112 d_{[11]/[O]} - 81 d_{[11]/[22]}}{320} < d_{[W]/[11]} < 4 d_{[11]/[O]} - 4 d_{[11]/[22]}$; $^{+++} \, 4 d_{[11]/[O]} - 4 d_{[11]/[22]} < d_{[W]/[11]} < \frac{368 d_{[11]/[O]} - 423 d_{[11]/[22]}}{64}$;

Table 3: Model with network externalities: Comparison of equilibrium social surpluses at different tying arrangements versus the case of non-tying

The main left column presents the difference in social surpluses between the tying-to-platform subgame and the non-tying subgame. The best quality good A1B1 has larger market share which implies that the positive network effect on its quality should be stronger. Therefore, the quality differentiation in the tying-to-platform subgame with network externalities is at least equal to the one in the setup without network externalities $\left(d_{[11^*]/[12^*]} \geq d_{[11]/[12]}\right)$. So, if larger enough it implies higher average price and producer surplus.

However, the positive network effect on quality of both goods refers to strictly higher average quality in the market. Therefore, the plus-signed difference in social welfare between the tying-to-platform subgame and the non-tying subgame is even higher.

The main middle column of table 3 presents comparison in the social surpluses between the tying-to-browser subgame with entry and the non-tying subgame. The condition of lemma 4 $\left(d_{[W]/[22*]} > \frac{9}{8} d_{[11]/[22]}\right)$ ensures higher quality differentiation in the tying-to-browser subgame with entry. This implies strictly higher average price and product surplus. The best quality good with embedded browser of Microsoft is the only good that serves the whole market. Therefore, the positive network effect on its quality implies plus-signed difference in social welfare between the tying-to-browser subgame and the non-tying subgame.

The main right column of table 3 presents comparison in the social surpluses between the tying-to-browser subgame without entry and the non-tying subgame. The positive network effect on the quality of the best good with embedded browser of Microsoft implies even higher differentiation than in the basic setup without network externalities. If the market is narrow $\left(\overline{\theta} \to \frac{3}{2}\underline{\theta}\right)$ and/or the higher quality s_W is large enough $\left(d_{[W]/[11]} > \frac{368 d_{[11]/[O]} - 423 d_{[11]/[22]}}{64}\right)$, the latter would compensate for the market share of the consumers who prefer the outside option. So, the social welfare effect would be higher than in the non-tying subgame in spite of the undercovered market in the tying-to-browser subgame without entry[12].

[12] It should be kept in mind that the assumption for fixed zero price of the outside option is artificial simplification which is introduced to allow for partial equilibrium analysis. In reality, it would rather be positive and interrelated to the prices of the other qualities in the market. So, the claim that consumers would gain from not purchasing these qualities in the tying subgame cannot be plausible if based only on the current analysis. The effect of tying-to-browser on social welfare in the undercovered market given no entry should also be quoted cautiously.

Table 2 and table 3 provide evidence that the non-tying subgame outcome does not need to be socially optimal. However, they do not show which tying subgame outcome would Pareto-dominate the others in such a case. For the purpose, next subsection compares the social surpluses between the tying subgames.

4.2. Comparison of the social effects of product tying

In this subsection an explicit comparative analysis of the social surpluses between the tying-to-platform subgame and the tying-to-browser subgame with and without entry will be provided. For simplicity it is assumed that the network effect on the best quality good varies insignificantly between the compared tying subgames $s_W = s_{11^*}$. So, the presented results are independent on whether a model setup with or without network externalities is applied for their derivation. The respective signs of the surplus differences[13] are presented in table 4 below.

[13] For the explicit measures see Appendix F.

	TieB vs TieA	**TieB* vs TieA**	**TieB* vs TieB**	
	$d_{[W]/[21]} > d_{[11]/[12]}$	$d_{[W]/[O]} > d_{[11]/[12]}$	$d_{[W]/[O]} < \frac{9}{8} d_{[W]/[21]}$	$d_{[W]/[O]} > \frac{9}{8} d_{[W]/[21]}$
PS	+	+	+/- $\bar{\theta} \to 2\underline{\theta}$	+
CS	-	-	-	-
SW	+	-	-	-

Table 4: Comparison of equilibrium social surpluses in the tying-to-platform subgame (TieA) and the tying-to-browser subgame with entry (TieB) and without entry (TieB*)

The difference in social surpluses between the tying-to-platform subgame and the tying-to-browser subgame with entry and without entry follows from the relation of the quality differentials between the better and worse good in each subgame - $d_{[11^*]/[12]} < d_{[W]/[22^*]} < d_{[W]/[O]}$. The tying-to-browser subgame without entry has the second-largest differentiation but in it the best good serves the whole market. Therefore, it yields the highest social welfare. The tying-to-platform subgame has the smallest differentiation. Therefore it refers to the lowest prices and producer surplus. The tying-to-browser subgame without entry has the largest differentiation. Respectively, it is related to the highest price and producer surplus given narrow enough market $\left(\bar{\theta} \to \frac{3}{2}\underline{\theta}\right)$. However, since the differentiation is in favor of the outside option, it implies the lowest social welfare from the three tying subgames.

The last subsection summarizes the main implications of the social welfare comparisons.

4.2. Policy implications

As it was described in the introduction, two separate anti-trust cases on the same allegations were carried out against Microsoft in the U.S. and in the EU. In the U.S. the case was settled with an agreement and no policy measures were imposed against Microsoft. In the EU, however, Microsoft was convicted and casted to unbundle the sales of its software applications from the sales of its system platform. In addition, it had to pay a penalty fee for delay in the execution of the unbundling measure. In this subsection, the relevance of imposing an unbundling measure will be discussed in the context of the market setting considered in the paper.

The main issues are:

- what is the outcome of unbundling and how it relates to the outcome of tying chosen by Microsoft
- whether the unbundling measure maximizes social welfare
- what are the conditions for the tying strategies to be socially admissible

First, the outcome of unbundling corresponds to the one in the non-tying subgame. It is profit-maximizing for Microsoft in the setup without network externalities where market share does not affect quality. In such a case the choice of tying-to-browser strategy by Microsoft cannot be driven by profit or market share maximization incentives. It could only be driven by incentive to foreclose the sales of the competitive system-platform supplier. However, given vertical quality differentiation, no tying strategy could lead to foreclosure of the sales of the best-quality application software because it takes part in the top-ranked good A1B1 in the market. This means that the market-power leverage hypothesis is irrelevant in a vertically differentiated market where the multi-product firm offers a lower quality product. So, the allegations against Microsoft in using its market power in the system-platform market segment in order to eliminate the competition in some of the application software segments cannot be credible given that the software market is vertically differentiated and Microsoft is not quality leader in the application software segments.

Second, the social welfare analysis in table 2 shows that the non-tying subgame outcome would be Pareto-dominated by the tying-to-platform subgame outcome. So, even though the unbundling measure appears to be relevant response to the market foreclosure of the sales of a potential entrant in the system-platform market segment, it is still not the best policy available. Social welfare would be higher if instead of charging Microsoft to unbundle its products the EU anti-trust authorities had obligated it to offer a version of its browser for Linux. Then, the outcome would have corresponded to the one of the tying-to-platform subgame.

Third, the results in table 2 and table 3 show that there are two market situations in which, the tying-to-browser strategy chosen by Microsoft would Pareto-dominate the non-tying subgame outcome. The first situation would occur if there is a positive network effect of market share on quality of goods and the market is narrow enough. Then, tying-to-browser strategy appears as a profit maximizing strategy for Microsoft. So, its choice of a tying strategy appears as a rent-seeking but not entry-deterrence behavior. By tying its products Microsoft saves the sales of its good which increases its quality and yields both higher profit and social welfare than in the non-bundling subgame (see the right column of table 3). The second market situation would appear if there is a market entry of a competitive system-platform supplier. In compliance with proposition 5, given no network effect on the quality of

Microsoft's good, its threat to use tying would not be credible. So, it would be optimal for Linux to enter. Even though Microsoft would be strictly worse-off than in the case of no entry, however, the outcome of the tying-to-browser subgame would Pareto-dominate the tying-to-platform outcome and will be socially optimal (see the left column of table 4).

According to the discussion above, the U.S. settlement of the case against Microsoft seems as more relevant in the context of the market setup analyzed in the present paper. It must be taken into account, however, that the market situation modeled in the present paper corresponds better to the real case being judged in the U.S. trial against Microsoft. Even though, the U.S. and EU trials are based on the same allegation against Microsoft they differ in time and particular markets being considered. The U.S. trial refers to the U.S. market for system software in the mid1990s while the EU trial is concerned with the European market in the 2000s when the number of both PC users (market width) and software application suppliers is much larger than it is assumed in the current model.

Nevertheless, the very presence of vertical quality differentiation in the system software market implies that the sales of a higher quality application could not be effectively foreclosed by tying. Therefore, the market-power leverage hypothesis cannot be an issue in any vertically differentiated market independent on its width and number of players. Respectively, requiring from Microsoft to provide a version of its application software for the competitive system platform of Linux emerges as a stronger competition-enhancing policy measure than unbundling.

V. Conclusion

The present paper is concerned with welfare analysis of a specific equilibrium outcome that could occur as a result of tying arrangement in a vertically differentiated market for system goods. Correspondingly, the market for system software during the 1990s is chosen as illustrative example of such a market. The evaluation of the social welfare effect of tying is therefore presented in the context of the anti-trust cases against Microsoft.

Under initially stated market conditions the paper establishes three distinct equilibrium outcomes – non-tying, tying-to-platform and tying-to-browser, respectively. What makes them special is that in the particular market situation being considered, it could be argued that the multi-product firm chooses to tie its products not driven by predatory pricing incentives but just to save the sales of its system combination which otherwise would be foreclosed. The question is whether the tying behavior of the multi-product firm should be socially admissible provided that it is not strictly anti-competitive at the particular market conditions.

The main findings have the following implications:

- at the established special market conditions there exists a subgame-perfect equilibrium in which Microsoft would choose to tie its platform to the browser and sell them at a single wholesale price as a part of the best system combination. This tying arrangement maximizes Microsoft's equilibrium profit but leaves the market undercovered which could have a negative effect on the social welfare.
- both tying strategies, tying to platform and tying to browser, have a potential to improve the social welfare by increasing the average quality offered in the market. However,

this outcome is related to foreclosure of the sales of the second-best system platform supplied by Linux.

- the non-tying subgame equilibrium imposed by the unbundling measure of the European Commission could emerge as a perfect equilibrium but only given that market share of goods does not have a positive network effect on their qualities. In the model setup with network externalities, the tying-to-browser strategy chosen by Microsoft maximizes its profit and therefore is optimal in the perfect equilibrium. Moreover, the tying-to-browser subgame outcome could be socially optimal given market entry of a competitive system-platform supplier.
- independent on the model setup, with or without network externalities, tying-to-platform outcome Pareto-dominates the non-tying subgame outcome. So, making Microsoft to supply a Linux version of its application software emerges as a stronger competition-enhancing policy measure than unbundling.

Finally, it needs to summarize the main restrictions on the general validity of the paper's implications. The model in the paper relies on several assumptions without which its results would not hold.

First, it takes as an example the system-software market from the 1990s when all suppliers were profit-maximizing firms. This is not the case nowadays when many freeware products are offered by non-profit suppliers which use voluntary labor and finance their production activities by charity.

Second, it is assumed that the spread of consumer tastes is restricted and the market is sufficiently narrow to accommodate exactly two goods in equilibrium. In the last decade, the demographics of PC users expanded significantly and as a result there is a greater variety of software products available in the market.

Third, it is assumed that the system-software market is vertically differentiated and goods in it consist of only two products. In fact, though Microsoft's operation platform Windows dominates the market, there is a small segment of IT professionals who use Linux not because it is cheaper but because they find it more valuable. The model used in the current paper rather ignores them. Moreover, the decision what operation system to buy depends not only on the quality of the browsing application that goes with it but also on the quality of a number of other applications that could be run on it.

In compliance with the above, the following suggestions could be made for future research on the topic. A non-profit version of the model of a vertically differentiated market for system goods could be developed and used to check how competitive outcome and social welfare change when entrants are not profit-maximizers. Respectively, a market setting with wider spread of consumer tastes needs to be considered. Finally, it would be worth if the implications for two-product system goods could be shown to hold generally for combinations of larger number of products.

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Appendix A: Proof of Lemma 1

Let except the multi-product firm in the market be also (*m*-1) firms producing a product of type A and (*n*-1) firms producing a product of type B. Then, to be consistent with the searched price equilibrium a self-selection equilibrium outcome is assumed where only the goods with equally ranked components have positive market share in the non-tying subgame equilibrium. As shown in the proof of lemma 2 (Appendix B), there is an explicit condition on the quality differential from the neighbor by rank from below which ensures the existence of such a self-selection equilibrium in prices.

The proof of lemma 1 starts with establishing the condition for at most two suppliers of each type to have positive market shares in equilibrium. In technical terms, this implies that the taste of the marginal consumer who is indifferent between the goods based on the second-best and third-best products should not be feasible, i.e. $\theta_{[22]/[33]} \notin [\underline{\theta}, \bar{\theta}]$.

To show that this condition holds when $\bar{\theta} < 2\underline{\theta}$, consider the following profit maximization problems of the firms supplying the components of the three best goods based on equally-ranked products (A1B1, A2B2, A3B3):

$$\underset{p_{A1}, p_{B2}}{Max}\, \Pi_M = p_{A1}\left(\bar{\theta} - \theta_{[11]/[22]}\right) + p_{B2}\left(\theta_{[11]/[22]} - \theta_{[22]/[33]}\right) =$$

$$= p_{A1}\left(\bar{\theta} - \frac{(p_{A1} + p_{B1}) - (p_{A2} + p_{B2})}{d_{[11]/[22]}}\right) + p_{B2}\left(\frac{(p_{A1} + p_{B1}) - (p_{A2} + p_{B2})}{d_{[11]/[22]}} - \frac{(p_{A2} + p_{B2}) - (p_{A3} + p_{B3})}{d_{[22]/[33]}}\right)$$

$$\underset{p_{B1}}{Max}\, \Pi_{B1} = p_{B1}\left(\bar{\theta} - \theta_{[11]/[22]}\right) = p_{B1}\left(\bar{\theta} - \frac{(p_{A1} + p_{B1}) - (p_{A2} + p_{B2})}{d_{[11]/[22]}}\right)$$

$$\underset{p_{A2}}{Max}\,\Pi_{A2} = p_{A2}\left(\theta_{[11]/[22]} - \theta_{[22]/[33]}\right) = p_{B2}\left(\frac{(p_{A1} + p_{B1}) - (p_{A2} + p_{B2})}{d_{[11]/[22]}} - \frac{(p_{A2} + p_{B2}) - (p_{A3} + p_{B3})}{d_{[22]/[33]}}\right)$$

$$\underset{p_{A3}}{Max}\,\Pi_{A3} = p_{A3}\left(\theta_{[22]/[33]} - \theta_{[33]/[44]}\right) = p_{A3}\left(\frac{(p_{A2} + p_{B2}) - (p_{A3} + p_{B3})}{d_{[22]/[33]}} - \frac{(p_{A3} + p_{B3}) - (p_{A4} + p_{B4})}{d_{[33]/[44]}}\right)$$

$$\underset{p_{B3}}{Max}\,\Pi_{B3} = p_{B3}\left(\theta_{[22]/[33]} - \theta_{[33]/[44]}\right) = p_{B3}\left(\frac{(p_{A2} + p_{B2}) - (p_{A3} + p_{B3})}{d_{[22]/[33]}} - \frac{(p_{A3} + p_{B3}) - (p_{A4} + p_{B4})}{d_{[33]/[44]}}\right)$$

The corresponding first-order necessary conditions for profit maximization are as follows:

$$\bar{\theta} - \frac{\left(2p_{A1} + p_{B1}\right) - \left(p_{A2} + 2p_{B2}\right)}{d_{[11]/[22]}} = 0$$

$$\frac{(2p_{A1} + p_{B1}) - (p_{A2} + 2p_{B2})}{d_{[11]/[22]}} - \frac{(p_{A2} + 2p_{B2}) - (p_{A3} + p_{B3})}{d_{[22]/[33]}} = 0$$

$$\bar{\theta} - \frac{\left(p_{A1} + 2p_{B1}\right) - \left(p_{A2} + p_{B2}\right)}{d_{[11]/[22]}} = 0$$

$$\frac{(p_{A1} + p_{B1}) - (2p_{A2} + p_{B2})}{d_{[11]/[22]}} - \frac{(2p_{A2} + p_{B2}) - (p_{A3} + p_{B3})}{d_{[22]/[33]}} = 0$$

$$\frac{(p_{A2} + p_{B2}) - (2p_{A3} + p_{B3})}{d_{[22]/[33]}} - \frac{(2p_{A3} + p_{B3}) - (p_{A4} + p_{B4})}{d_{[33]/[44]}} = 0$$

$$\frac{(p_{A2} + p_{B2}) - (p_{A3} + 2p_{B3})}{d_{[22]/[33]}} - \frac{(p_{A3} + 2p_{B3}) - (p_{A4} + p_{B4})}{d_{[33]/[44]}} = 0$$

There are three key relations between prices that could be derived from the above first-order optimality equations:

- from the first and third equation the following relation could be shown to hold between the price of the best product of type A and the prices of the two best products of type B:

$$p_{A1} = p_{B1} + p_{B2} \qquad (*)$$

- from the second and forth equation the following relation could be shown to hold between the price of the second-best product of type B and two best products of type A:

$$p_{A1} = \left(\frac{d_{[11]/[22]} + d_{[22]/[33]}}{d_{[22]/[33]}} \right) (p_{B2} - p_{A2}) \text{ which implies that } p_{B2} \geq p_{A2} \qquad (**)$$

- from the fifth and sixth equation it is straightforward to see that equality must hold in equilibrium between the prices of the third best goods of each type:

$$p^*_{A3} = p^*_{B3} = p^*_{33} \qquad (***)$$

After rearrangement of the third first-order optimality equation, it looks as follows:

$$\bar{\theta} = 2\theta_{[11]/[22]} + \frac{(p_{A2} + p_{B2}) - p_{A1}}{d_{[11]/[22]}}$$

Using (*) and (**) it could be shown that:

$$(p_{A2} + p_{B2}) - p_{A1} = 2p_{B2} + \frac{d_{[11]/[22]}}{d_{[22]/[33]}} (p_{B2} - p_{A2}) > 0$$

Hence, the inequality relation follows:

$$\bar{\theta} > 2\theta_{[11]/[22]} \qquad (+)$$

Analogously, after rearrangement of the second first-order optimality equation, it looks as follows:

$$\theta_{[11]/[22]} = \theta_{[22]/[33]} + \left(\frac{d_{[22]/[33]} + d_{[11]/[22]}}{d_{[11]/[22]} d_{[22]/[33]}} \right) p_{A2}$$

Hence, the inequality relation follows:

$$\theta_{[11]/[22]} > \theta_{[22]/[33]} \tag{++}$$

Finally, from (+) and (++) it follows that:

$$\overline{\theta} > 2\theta_{[22]/[33]}$$

which implies that the $\theta_{[22]/[33]} \notin [\underline{\theta}, \overline{\theta}]$ holds for any $\overline{\theta} < 2\underline{\theta}$. This completes the proof of the first part of lemma 1.

The second part of lemma 1 specifies the condition for internal solution i.e. $\theta_{[11]/[22]} \in (\underline{\theta}, \overline{\theta})$. The profit maximization problems of the two top quality firms of each type given that they cover the market are given by the following expressions:

$$\underset{p_{A2}, p_{B2}}{Max} \Pi_M = p_{A2}\left(\overline{\theta} - \theta_{[22]/[11]}\right) + p_{B2}\left(\theta_{[22]/[11]} - \underline{\theta}\right) =$$

$$= p_{A2}\left(\overline{\theta} - \frac{(p_{A2} + p_{B2}) - (p_{A1} + p_{B1})}{d_{[22]/[11]}} \right) + p_{B2}\left(\frac{(p_{A2} + p_{B2}) - (p_{A1} + p_{B1})}{d_{[22]/[11]}} - \underline{\theta} \right)$$

$$\underset{p_{B1}}{Max} \Pi_{B1} = p_{B1}\left(\overline{\theta} - \theta_{[22]/[11]}\right) = p_{B1}\left(\overline{\theta} - \frac{(p_{A2} + p_{B2}) - (p_{A1} + p_{B1})}{d_{[22]/[11]}} \right)$$

$$\underset{p_{A2}}{Max} \Pi_{A2} = p_{A2}\left(\theta_{[22]/[11]} - \underline{\theta}\right) = p_{A2}\left(\frac{(p_{A2} + p_{B2}) - (p_{A1} + p_{B1})}{d_{[22]/[11]}} - \underline{\theta} \right)$$

The respective first-order optimality conditions could be reduced to the following system of equations:

$$\begin{cases} p_{A1} = 2p_{B2} + p_{B1} \\ p_{A1} = p_{B2} - p_{A2} \end{cases}$$

which implies a corner solution for the equilibrium price of B2:

$$\begin{cases} p_{A1}^{NoT} = p_{B1}^{NoT} = \dfrac{\left(2\overline{\theta} - \underline{\theta}\right)}{4} d_{[11]/[22]} \\ p_{A2}^{NoT} = \dfrac{\left(2\overline{\theta} - 3\underline{\theta}\right)}{4} d_{[11]/[22]} \\ p_{B2}^{NoT} = 0 \end{cases}$$

The corresponding expression for the marginal taste parameter is as follows:

$$\theta_{[11]/[22]} = \frac{\left(2\overline{\theta} + \underline{\theta}\right)}{4}$$

Then, the condition $\frac{3}{2}\underline{\theta} < \overline{\theta}$ implies that $\theta_{[11]/[22]} > \underline{\theta}$ which together with (+) completes the proof of the second part of lemma 1.

To conclude, $\frac{3}{2}\underline{\theta} < \overline{\theta} < 2\underline{\theta}$ implies that exactly the system goods (A1B1, A2B2) consisting of the equally-indexed best and second-best products of each type will have positive market shares in equilibrium. Q.E.D.

Appendix B: Proof of Lemma 2

Now, it needs to proof that the assumption for self-selection equilibrium outcome holds at the derived equilibrium prices of the non-tying subgame.

Note that the equilibrium prices of the two products of type A are identical. Furthermore, the equilibrium price of the second-best product of type B is zero. Hence, the price of the multi-product firm's good (A1B2) would be strictly lower in equilibrium than the price of its neighbor by rank (A2B1) from below. Therefore, if consumers are unwilling to buy the former good at its equilibrium price, they would be even more reluctant to buy the latter lower-quality good at its higher equilibrium price.

It remains to show that at the condition of lemma 2 the multi-product firm's good would be foreclosed in equilibrium i.e. the following relation holds:

$$\theta_{[11]/[12]} \leq \theta_{[12]/[22]}$$

In explicit terms it looks as follows:

$$\frac{p_{B1}^{NoT} - p_{B2}^{NoT}}{d_{[11]/[12]}} \leq \frac{p_{A1}^{NoT} - p_{A2}^{NoT}}{d_{[12]/[22]}} \text{ or } \frac{\left(2\overline{\theta} - \underline{\theta}\right) d_{[11]/[22]}}{4 d_{[11]/[12]}} \leq \frac{\underline{\theta} d_{[11]/[22]}}{2 d_{[12]/[22]}}$$

which holds for any $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ given the condition $d_{[11]/[12]} \geq \frac{3}{2} d_{[12]/[22]}$ which completes the proof of lemma 2.

Appendix C: Proof of Proposition 2

The proof of proposition 2 is straightforward. The multi-product firm sells its product of type B only conditional on the purchase of its product of type A. The system good (A1B2) consisting of the two multi-product firm's products is sold at a wholesale price. The profit maximization problems of the two entrants look as follows:

$$\underset{p_{A1},p_M}{Max}\,\Pi_M = p_{A1}\left(\overline{\theta}-\theta_{[11]/[12]}\right)+p_M\left(\theta_{[11]/[12]}-\underline{\theta}\right)=$$

$$= p_{A1}\left(\overline{\theta}-\frac{(p_{A1}+p_{B1})-p_M)}{d_{[11]/[12]}}\right)+p_M\left(\frac{(p_{A1}+p_{B1})-p_M}{d_{[11]/[12]}}-\underline{\theta}\right)$$

$$\underset{p_{B1}}{Max}\,\Pi_{B1} = p_{B1}\left(\overline{\theta}-\theta_{[11]/[M]}\right)=p_{B1}\left(\overline{\theta}-\frac{(p_{A1}+p_{B1})-p_M}{d_{[11]/[12]}}\right)$$

The respective first-order optimality conditions could be reduced to the system of equations:

$$\begin{cases} \overline{\theta}d_{[11]/[12]}-p_{A1}-2p_{B1}+p_M=0 \\ p_{A1}=p_{B1}+p_M \\ p_{B1}=\dfrac{\overline{\theta}d_{[11]/[12]}}{3} \end{cases}$$

which implies a corner solution for the equilibrium price of the multi-product firm's good M:

$$\begin{cases} p_{A1}^{TieA}=p_{B1}^{TieA}=\dfrac{\overline{\theta}}{3}d_{[11]/[12]} \\ p_M^{TieA}=0 \end{cases}$$

Apparently, if the second-best good is priced zero no lower-quality good could have a positive demand. So, for any bounds $\underline{\theta}$ and $\overline{\theta}$ of the consumer taste interval, in equilibrium at most the best and second-best good will have positive market shares. That is prices are consistent with the initial assumptions for covered market.

The corresponding expression for the marginal taste parameter is as follows:

$$\theta_{[11]/[12]} = \frac{2\overline{\theta}}{3}$$

Finally, the condition $\frac{3}{2}\underline{\theta} < \overline{\theta}$ implies an internal solution $\theta_{[11]/[12]} \in \left(\underline{\theta}, \overline{\theta}\right)$ which completes the proof of proposition 2.

Appendix D: Proof of Proposition 3 and Proposition 4

The proof of proposition 3 goes backwards to its definition.

First, it is shown that in the subgame equilibrium with potential entrant, it is optimal for both firms supplying the components of the best system combination to set sufficiently low prices to foreclose the sales of all the other goods (including the good of the multi-product firm). Second, the optimal prices are derived at which only the best good would have a positive market share at a covered market.

Assume that the good of the multi-product firm A1B2 has a positive market share and covers the market together with the best-quality good A1B2B1. Then, the profit-maximization problem of the competitive supplier of browser B1 would look as follows:

$$\underset{p_{B1}}{Max}\,\Pi_{B1} = p_{B1}\left(\overline{\theta} - \theta_{[11]/[12]}\right) = p_{B1}\left(\overline{\theta} - \frac{(p_M + p_{B1}) - p_M}{d_{[11]/[12]}}\right)$$

The respective first-order condition implies the following solution for the price:

$$p_{B1} = \frac{\overline{\theta} d_{[11]/[12]}}{2}$$

and the corresponding expression for the marginal consumer indifferent between the two goods (A1B1 and A1B2) is as follows:

$$\theta_{[11]/[12]} = \frac{\overline{\theta}}{2}$$

which is apparently not feasible $\left(\theta_{[11]/[12]} \notin \left(\underline{\theta}, \overline{\theta}\right)\right)$ given that $\overline{\theta} < 2\underline{\theta}$. So, only the best good would serve the market which completes the proof of part (ii) of proposition 3.

Given the assumption that only the best good would have a positive market share at a covered market the optimization problems of the two firms M and B1 look as follows:

$$\underset{p_M}{Max}\,\Pi_M = p_M\left(\overline{\theta} - \underline{\theta}\right)$$

$$\underset{p_{B1}}{Max}\,\Pi_{B1} = p_{B1}\left(\overline{\theta} - \underline{\theta}\right)$$

The respective equilibrium price set is given by the corner solution at which the sales of the other potential entrants (particularly good A2B1) would be foreclosed. When shared equally between firm M and B1, the subgame equilibrium price of the best good corresponds to the symmetric price equilibrium of proposition 3:

$$p_M^{TieB} = p_{B1}^{TieB} = \frac{\underline{\theta}}{2} d_{[W]/[22]}$$

which completes the proof of part (ii) of proposition 3.

After substituting in the respective expression for the marginal taste parameter between the best and second-best good it takes the form:

$$\theta_{[11]/[12]} = \frac{\left(p_M^{TieB} + p_{B1}^{TieB}\right) - (p_{A1}^{TieB} + p_{B2}^{TieB})}{d_{[W]/[12]}} = \frac{\underline{\theta} d_{[W]/[22]}}{2d_{[W]/[12]}}$$

which is smaller than $\underline{\theta}$ for $d_{[11]/[22]} < 2d_{[W]/[12]}$ but according to the condition of lemma 2 $d_{[W]/[22]} = d_{[W]/[11]} + d_{[11]/[22]} \leq d_{[W]/[11]} + \frac{5}{3} d_{[11]/[12]} < 2d_{[W]/[11]} + 2d_{[11]/[12]} = 2d_{[W]/[12]}$. This confirms that the market will be covered solely by the best good in the subgame equilibrium.

The proof of proposition 4 is analogous to the proof of proposition 3. The difference is that there are no other entrants in the market. So, the optimal price is derived in "competition" with the outside option.

First, it is shown that the multi-product firm's good alone could not have demand in the tying-to-B equilibrium. For the purpose, assume that it has a positive demand and solve the resulting profit maximization problems of firm B1:

$$\underset{p_{B1}}{Max}\,\Pi_{B1} = p_{B1}\left(\overline{\theta} - \theta_{[11]/[12]}\right) = p_{B1}\left(\overline{\theta} - \frac{(p_M + p_{B1}) - p_M}{d_{[11]/[12]}}\right)$$

The respective first-order condition implies the following solution for the price:

$$p_{B1} = \frac{\overline{\theta} d_{[11]/[12]}}{2}$$

and the corresponding expression for the marginal consumer indifferent between the two goods (A1B1 and A1B2) is as follows:

$$\theta_{[11]/[12]} = \frac{\overline{\theta}}{2}$$

which is apparently not feasible $\left(\theta_{[11]/[12]} \notin \left(\underline{\theta}, \overline{\theta}\right)\right)$ given that $\overline{\theta} < 2\underline{\theta}$. So, only the best good will serve the market which would be under-covered in equilibrium. This completes the proof of part (ii) of proposition 4.

Given the assumption for under-covered market in the tying-to-B subgame, the optimization problems of the two firms M and B1 look as follows:

$$\underset{p_M}{Max}\,\Pi_M = p_M\left(\overline{\theta} - \theta_{[11]/[12]}\right) = p_M\left(\overline{\theta} - \frac{(p_M + p_{B1})}{d_{[W]/[O]}}\right)$$

$$\underset{p_{B1}}{Max}\,\Pi_{B1} = p_{B1}\left(\bar{\theta} - \theta_{[11]/[12]}\right) = p_{B1}\left(\bar{\theta} - \frac{(p_M + p_{B1})}{d_{[W]/[O]}}\right)$$

The respective first-order conditions are given by the following system of equations:

$$\begin{cases} 2p_M + p_{B1} = \bar{\theta} d_{[W]/[O]} \\ p_M + 2p_{B1} = \bar{\theta} d_{[W]/[O]} \end{cases}$$

which corresponds to the symmetric price equilibrium of proposition 4:

$$p_M^{TieB*} = p_{B1}^{TieB*} = \frac{\bar{\theta}}{3} d_{[W]/[O]}$$

which completes the proof of part (i) of proposition 4.

The expression for the marginal consumer who is indifferent between buying the best good and not buying at all looks as follows:

$$\theta_{[W]/[O]} = \frac{2\bar{\theta}}{3}$$

which is larger than $\underline{\theta}$ for $\bar{\theta} > \frac{3}{2}\underline{\theta}$. So, the market is under-covered in equilibrium re-confirms the statement of part (ii) of proposition 4.

Appendix E: Proof of Lemma 3 and Lemma 4

Based on the results for the optimal prices derived in Appendices A, C and D, the explicit expressions for the respective profits of the multi-product firm look as follows:

$$\Pi_M^{NoT} = \frac{\left(2\overline{\theta} - \underline{\theta}\right)^2}{16} d_{[11]/[22]}$$

$$\Pi_M^{TieA} = \frac{\overline{\theta}^2}{9} d_{[11]/[12]}$$

$$\Pi_M^{TieB*} = \frac{\overline{\theta}^2}{9} d_{[W]/[O]}$$

$$\Pi_M^{TieB} = \frac{\overline{\theta}^2}{9} d_{[W]/[22]}$$

Note that the relationship between the payoffs of the multi-product firm in any pair of subgames depends on the respective quality differentials between the better and worse quality in each subgame, as follows:

- the relation between payoffs of the multi-product firm in the tying-to-A subgame (TieA) and the tying-to-B subgame with entry (TieB) depends on the relation between the quality differentials $d_{[11]/[12]}$ and $d_{[W]/[22*]}$. The difference between the two payoffs is given by the following quadratic expression in $\overline{\theta}$:

$$\left(\Pi_M^{TieA} - \Pi_M^{TieB}\right) = \frac{1}{18}\left[2d_{[11]/[12]}\overline{\theta}^2 - 9d_{[W]/[22*]}\overline{\theta}\underline{\theta} + 9d_{[W]/[22*]}\underline{\theta}^2\right]$$

which is negative in the whole interval $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ given that $d_{[W]/[22*]} \geq d_{[11]/[22]} > d_{[11]/[12]}$.

That is $\Pi_M^{TieA} < \Pi_M^{TieB}$.

- the relation between the payoffs of the multi-product firm in the tying-to-B subgame without entry (TieB*) and the non-tying subgame (NoT) depends on the relation between the quality differentials $d_{[W]/[O]}$ and $d_{[11]/[22]}$. The difference between the two payoffs is given by the following quadratic expression in $\overline{\theta}$:

$$\left(\Pi_M^{TieB*} - \Pi_M^{NoT}\right) = \frac{1}{144}\left[4\left(4d_{[W]/[O]} - 9d_{[11]/[22]}\right)\overline{\theta}^2 + 36d_{[11]/[22]}\overline{\theta}\underline{\theta} - 9d_{[11]/[22]}\underline{\theta}^2\right]$$

which is positive in the whole interval $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $d_{[W]/[O]} > \frac{9}{4}d_{[11]/[22]}$. That is, $\Pi_M^{TieB*} > \Pi_M^{NoT}$.

- the relation between the equilibrium payoffs of the multi-product firm in the tying-to-B subgame with entry (TieB) and the non-tying subgame (NoT) depends on the relation between the quality differentials $d_{[W]/[22*]}$ and $d_{[11]/[22]}$. The difference between the two payoffs is given by the following quadratic expression in $\overline{\theta}$:

$$\left(\Pi_M^{TieB} - \Pi_M^{NoT}\right) = \frac{1}{16}\left[-4d_{[11]/[22]}\overline{\theta}^2 + \left(4d_{[11]/[22]} + 8d_{[W]/[22*]}\right)\overline{\theta}\underline{\theta} - \left(d_{[11]/[22]} + 8d_{[W]/[22*]}\right)\underline{\theta}^2\right]$$

which is:

  - negative in the whole interval $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $d_{[W]/[22*]} = d_{[11]/[22]}$. That is, $\Pi_M^{TieB} < \Pi_M^{NoT}$.
  - positive $\left(\Pi_M^{TieB} > \Pi_M^{NoT}\right)$ in the subinterval $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, \frac{d_{[11]/[22]} + 2d_{[W]/[22*]} + 2\sqrt{-d_{[11]/[22]}d_{[W]/[22*]} + d_{[W]/[22*]}^2}}{2d_{[11]/[22]}}\underline{\theta}\right)$ and negative $\left(\Pi_M^{TieB} < \Pi_M^{NoT}\right)$ in the

subinterval $\bar{\theta} \in \left(\frac{d_{[11]/[22]} + 2d_{[W]/[22*]} + 2\sqrt{-d_{[11]/[22]}d_{[W]/[22*]} + d_{[W]/[22*]}^2}}{2d_{[11]/[22]}} \underline{\theta}, 2\underline{\theta} \right)$ for $d_{[11]/[22]} \leq d_{[W]/[22*]} < \frac{9}{8} d_{[11]/[22]}$.

- positive in the whole interval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta} \right)$ for $d_{[W]/[22*]} > \frac{9}{8} d_{[11]/[22]}$. That is, $\Pi_M^{TieB} > \Pi_M^{NoT}$.

- the relation between the equilibrium payoffs of the multi-product firm in the tying-to-B subgame with entry (TieB) and without entry (TieB*) depends on the relation between the quality differentials $d_{[W]/[21]}$ and $d_{[W]/[O]}$. The difference between the two payoffs is given by the following quadratic expression in $\bar{\theta}$:

$$\left(\Pi_M^{TieB} - \Pi_M^{TieB*} \right) = \frac{1}{18} \left[-2d_{[W]/[O]}\bar{\theta}^2 + 9d_{[W]/[22*]}\bar{\theta}\underline{\theta} - 9d_{[W]/[22*]}\underline{\theta}^2 \right]$$

which is negative in the whole interval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta} \right)$ for $d_{[W]/[22*]} < d_{[W]/[O]}$. That is, $\Pi_M^{TieB} < \Pi_M^{TieB*}$.

- the relation between the equilibrium payoffs of the multi-product firm in the tying-to-A subgame (TieA) and the non-tying subgame (NoT) depends on the relation between the quality differentials $d_{[11*]/[12*]}$ and $d_{[11]/[22]}$. The difference between the two payoffs is given by the following quadratic expression in $\bar{\theta}$:

$$\left(\Pi_M^{TieA} - \Pi_M^{NoT} \right) = \frac{1}{144} \left[4\left(4d_{[11*]/[12*]} - 9d_{[11]/[22]} \right)\bar{\theta}^2 + 36d_{[11]/[22]}\bar{\theta}\underline{\theta} - 9d_{[11]/[22]}\underline{\theta}^2 \right]$$

which is:

- negative in the whole interval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $d_{[11*]/[12*]} < d_{[11]/[22]}$ $\left(3/5\, d_{[11]/[22]} < d_{[11*]/[12*]} \text{ by virtue of lemma 2}\right)$. That is, $\Pi_M^{TieA} < \Pi_M^{NoT}$.

- positive $(\Pi_M^{TieA} > \Pi_M^{NoT})$ in the subinterval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, \frac{3\left(-3d_{[11]/[22]} - \sqrt{9d_{[11]/[22]}^2 - \left(9d_{[11]/[22]} - 4d_{[11]/[12]}\right)d_{[11]/[22]}}\right)\underline{\theta}}{2\left(4d_{[11]/[12]} - 9d_{[11]/[22]}\right)}\right)$, and negative $(\Pi_M^{TieA} < \Pi_M^{NoT})$ in the subinterval $\bar{\theta} \in \left(\frac{3\left(-3d_{[11]/[22]} - \sqrt{9d_{[11]/[22]}^2 - \left(9d_{[11]/[22]} - 4d_{[11]/[12]}\right)d_{[11]/[22]}}\right)\underline{\theta}}{2\left(4d_{[11]/[12]} - 9d_{[11]/[22]}\right)}, 2\underline{\theta}\right)$ for $d_{[11]/[22]} < d_{[11*]/[12*]} < \frac{81}{64}d_{[11]/[22]}$.

- positive in the whole interval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $\frac{81}{64}d_{[11]/[22]} < d_{[11*]/[12*]}$. That is, $\Pi_M^{TieA} > \Pi_M^{NoT}$.

In summary, given the basic setup of the model without network externalities and the condition of lemma 3 the payoffs of the multi-product firm in the tying subgames are related as follows: $\Pi_M^{TieA} < \Pi_M^{TieB} < \Pi_M^{NoT} < \Pi_M^{TieB*}$. Hence, given no entry of a competitive supplier of system platforms, it would be optimal for the multi-product firm to choose the tying-to-browser (TieB) strategy. Given entry, however, the non-bundling strategy (NoT) would make the multi-product firm be strictly better-off. This completes the proof of lemma 3.

Given the setup of the model with network externalities and the condition of lemma 4 the payoffs of the multi-product firm in the tying subgames are related as follows:

- $\Pi_M^{TieA} < \Pi_M^{NoT} < \Pi_M^{TieB} < \Pi_M^{TieB*}$ if $d_{[11*]/[12*]} < d_{[11]/[22]}$ or $d_{[11*]/[12*]} \in \left(d_{[11]/[22]}, \frac{81}{64} d_{[11]/[22]} \right)$

  and $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, \frac{3\left(-3d_{[11]/[22]} - \sqrt{9d_{[11]/[22]}^2 - \left(9d_{[11]/[22]} - 4d_{[11]/[12]}\right)d_{[11]/[22]}}\right)\underline{\theta}}{2\left(4d_{[11]/[12]} - 9d_{[11]/[22]}\right)} \right)$

- $\Pi_M^{NoT} < \Pi_M^{TieA} < \Pi_M^{TieB} < \Pi_M^{TieB*}$ if $d_{[11*]/[12*]} > \frac{81}{64} d_{[11]/[22]}$ or $d_{[11*]/[12*]} \in \left(d_{[11]/[22]}, \frac{81}{64} d_{[11]/[22]} \right)$

  and $\bar{\theta} \in \left(\frac{3\left(-3d_{[11]/[22]} - \sqrt{9d_{[11]/[22]}^2 - \left(9d_{[11]/[22]} - 4d_{[11]/[12]}\right)d_{[11]/[22]}}\right)\underline{\theta}}{2\left(4d_{[11]/[12]} - 9d_{[11]/[22]}\right)}, 2\underline{\theta}, \right)$

Hence, with or without entry the tying-to-browser (TieB or TieB*) strategy would be optimal for the multi-product firm. This completes the proof of lemma 4.

Appendix F: Social Welfare Comparisons

Based on the expressions for equilibrium prices derived in Appendix A, Appendix C and Appendix D, here are given the positive optimal profits and marginal taste parameters, respectively:

- in the non-tying (NoT) subgame

$$\Pi_{M}^{NoT} = \Pi_{B1}^{NoT} = \frac{\left(2\overline{\theta} - \underline{\theta}\right)^{2}}{16} d_{[11]/[22]}$$

$$\Pi_{A2}^{NoT} = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)^{2}}{16} d_{[11]/[22]}$$

$$\theta_{[11]/[22]}^{NoT} = \frac{\left(2\overline{\theta} + \underline{\theta}\right)}{4}$$

- in the tying-to-A (TieA) subgame

$$\Pi_{M}^{TieA} = \Pi_{B1}^{TieA} = \frac{\overline{\theta}^{2}}{9} d_{[11*]/[12*]}$$

$$\theta_{[11]/[12]}^{TieA} = \frac{2\overline{\theta}}{3}$$

- in the tying-to-B (TieB) subgame with entry

$$\Pi_{M}^{TieB} = \Pi_{B1}^{TieB} = \frac{\left(\overline{\theta} - \underline{\theta}\right)\underline{\theta}}{2} d_{[W]/[22*]}$$

$$\theta_{[W]/[22]}^{TieB} = \underline{\theta}$$

- in the tying-to-B (TieB*) subgame without entry

$$\Pi_{M}^{TieB} = \Pi_{B1}^{TieB} = \frac{\overline{\theta}^{2}}{9} d_{[W]/[O]}$$

$$\theta^{TieB}_{[W]/[O]} = \frac{2\bar{\theta}}{3}$$

In general terms, equilibrium social welfare (SW) could be expressed as follows:

$$SW = \begin{cases} \int_{\theta_{k/k'}}^{\bar{\theta}} s_k \theta d\theta + \int_{\underline{\theta}}^{\theta_{k/k'}} s_{k'} \theta d\theta = \frac{1}{2}\left[\left(\bar{\theta}^2 - \theta^2_{k/k'}\right)s_k + \left(\theta^2_{k/k'} - \underline{\theta}^2\right)s_{k'}\right], \text{for } \theta_{k/k'} > \underline{\theta} \\ \int_{\underline{\theta}}^{\bar{\theta}} s_k \theta d\theta = \frac{1}{2}\left(\bar{\theta}^2 - \underline{\theta}^2\right)s_k, \text{for } \theta_{k/k'} < \underline{\theta} \end{cases}$$

where $\theta_{k/k'}$ is the taste parameter of the marginal consumer who is indifferent between the two qualities in the market denoted by s_k and $s_{k'}$ ($k > k'$), respectively. Equilibrium product surplus is given by the sum of the optimal profits in each subgame. Consumer surplus is computed as difference between social welfare and product surplus.

Here are the results, respectively:

- for the non-tying (NoT) subgame

$$SW^{NoT} = \frac{1}{32}\left[\left(12\bar{\theta}^2 - 4\bar{\theta}\underline{\theta} - \underline{\theta}^2\right)s_{11} + \left(4\bar{\theta}^2 + 4\bar{\theta}\underline{\theta} - 15\underline{\theta}^2\right)s_{22}\right]$$

$$PS^{NoT} = \frac{1}{16}\left(12\bar{\theta}^2 - 20\bar{\theta}\underline{\theta} + 11\underline{\theta}^2\right)d_{[11]/[22]}$$

$$CS^{NoT} = \frac{1}{32}\left[\left(-12\bar{\theta}^2 + 36\bar{\theta}\underline{\theta} - 23\underline{\theta}^2\right)s_{11} + \left(28\bar{\theta}^2 - 36\bar{\theta}\underline{\theta} + 7\underline{\theta}^2\right)s_{22}\right]$$

- for the tying-to-A (TieA) subgame

$$SW^{TieA} = \frac{1}{18}\left[5\bar{\theta}^2 s_{11*} + \left(2\bar{\theta} - 3\underline{\theta}\right)\left(2\bar{\theta} - 3\underline{\theta}\right)s_{12*}\right]$$

$$PS^{TieA} = \frac{2}{9}\bar{\theta}^2 d_{[11*]/[12*]}$$

$$CS^{TieA} = \frac{1}{18}\left[\bar{\theta}^2 s_{11*} + \left(8\bar{\theta}^2 - 9\underline{\theta}^2\right)s_{12*}\right]$$

- for the tying-to-B (TieB) subgame with entry

$$SW^{TieB} = \frac{1}{2}\left(\overline{\theta}^2 - \underline{\theta}^2\right)s_W$$

$$PS^{TieB} = \left(\overline{\theta} - \underline{\theta}\right)\underline{\theta} d_{[W]/[22*]}$$

$$CS^{TieB} = \frac{1}{2}\left[\overline{\theta}^2 s_W - 2\overline{\theta}\underline{\theta} d_{[W]/[22*]} + \underline{\theta}^2\left(2d_{[W]/[22*]} - s_W\right)\right]$$

- for the tying-to-B (TieB*) subgame without entry

$$SW^{TieB*} = \frac{1}{18}\left[5\overline{\theta}^2 s_W + \left(2\overline{\theta} - 3\underline{\theta}\right)\left(2\overline{\theta} - 3\underline{\theta}\right)s_O\right]$$

$$PS^{TieB*} = \frac{2}{9}\overline{\theta}^2 d_{[W]/[O]}$$

$$CS^{TieB*} = \frac{1}{18}\left[\overline{\theta}^2 s_W + \left(8\overline{\theta}^2 - 9\underline{\theta}^2\right)s_O\right]$$

1. *TieA vs. NoT*

The analysis starts with comparison between tying-to-A (TieA) and non-tying subgame equilibria (NoT).

1.1. *Social Welfare Comparison* $\left(SW^{TieA} - SW^{NoT}\right)$

The difference in social welfare between the two is given by the following quadratic function in $\overline{\theta}$:

$$\left(SW^{TieA} - SW^{NoT}\right) = \frac{1}{288}\left[\left(80s_{11*} - 108s_{11} + 64s_{12*} - 36s_{22}\right)\overline{\theta}^2 + 36\left(s_{11} - s_{22}\right)\overline{\theta}\underline{\theta} + \left(9s_{11} - 144s_{12*} + 135s_{22}\right)\underline{\theta}^2\right]$$

1.1.A) Social Welfare Comparison $\left(SW^{TieA} - SW^{NoT}\right)$ *- without network externalities*

In the initial setup of the model without network externalities the quality of the best good does not differ between the two subgames, i.e. $s_{11^*} = s_{11}$. So, the above quadratic expression takes the form:

$$\left(SW^{TieA} - SW^{NoT}\right)\Big|_{s_{11^*} = s_{11}} = \frac{1}{288}\left[\left(-64d_{[11]/[12]} + 36d_{[11]/[22]}\right)\bar{\theta}^2 + 36d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(144d_{[11]/[12]} - 135d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is positive in the whole interval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $d_{[11]/[12]} < d_{[11]/[22]}$ $\left(\frac{3}{5} d_{[11]/[22]} < d_{[11]/[12]} \text{ by virtue of lemma 2}\right)$. That is, $SW^{TieA} > SW^{NoT}$.

1.1.B) Social Welfare Comparison $\left(SW^{TieA} - SW^{NoT}\right)$ *- with network externalities*

In the setup of the model with network externalities both the quality of the best good s_{11} and the quality of the second-best good s_{12} might differ in the tying-to-A subgame compared to the non-tying subgame. The difference between the two in the tying-to-A subgame however cannot be smaller than in the non-tying game, i.e. $d_{[11^*]/[12^*]} \geq d_{[11]/[12]}$. Having this in mind the difference in social welfare between the two subgames takes the form:

$$\left(SW^{TieA} - SW^{NoT}\right) = \left(SW^{TieA} - SW^{NoT}\right)\Big|_{s_{11^*} = s_{11}} + \frac{1}{288}\left[80\left(\bar{\theta}^2 d_{[11^*]/[11]} - \underline{\theta}^2 d_{[12^*]/[12]}\right) + 64\left(\bar{\theta}^2 - \underline{\theta}^2\right)d_{[12^*]/[12]}\right]$$

The right addend is positive for any $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. The left addend was shown to be positive above. Hence, the whole expression is positive in the interval $\bar{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $SW^{TieA} > SW^{NoT}$.

1.2. Producer Surplus Comparison $\left(PS^{TieA} - PS^{NoT}\right)$

The difference in producer surplus between the tying-to-A and non-tying subgame equilibria could be analyzed analogously. It is given by the following quadratic function in $\bar{\theta}$:

$$\left(PS^{TieA} - PS^{NoT}\right) = \frac{1}{144}\left[\left(32d_{[11^*]/[12^*]} - 108d_{[11]/[22]}\right)\bar{\theta}^2 + 180d_{[11]/[22]}\bar{\theta}\underline{\theta} - 99d_{[11]/[22]}\underline{\theta}^2\right]$$

1.2.A) Producer Surplus Comparison $\left(PS^{TieA} - PS^{NoT}\right)$ *- without network externalities*

In the basic setup of the model without network externalities the following relation between the two differentials holds: $d_{[11^*]/[12^*]} = d_{[11]/[12]} < d_{[11]/[22]} < \frac{27}{4}d_{[11]/[22]}$. So the quadratic expression above takes the form:

$$\left.\left(PS^{TieA} - PS^{NoT}\right)\right|_{d_{[11^*]/[12^*]} = d_{[11]/[12]}} = \frac{1}{144}\left[\left(32d_{[11]/[12]} - 108d_{[11]/[22]}\right)\bar{\theta}^2 + 180d_{[11]/[22]}\bar{\theta}\underline{\theta} - 99d_{[11]/[22]}\underline{\theta}^2\right]$$

which is negative in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $PS^{TieA} < PS^{NoT}$.

1.2.B) Producer Surplus Comparison $\left(PS^{TieA} - PS^{NoT}\right)$ *- with network externalities*

In the setup of the model with network externalities the following relation holds $d_{[11]/[22]} < d_{[11^*]/[12^*]}$. So, the above quadratic expression is:

- positive ($PS^{TieA} > PS^{NoT}$) in the subinterval $\left(\frac{3}{2}\underline{\theta}, \frac{3\left(-15d_{[11]/[22]} - \sqrt{225d_{[11]/[22]}^2 - \left(297d_{[11]/[22]} - 88d_{[11^*]/[12^*]}\right)d_{[11]/[22]}}\right)\underline{\theta}}{2\left(8d_{[11^*]/[12^*]} - 27d_{[11]/[22]}\right)}\right)$ and negative $(PS^{TieA} < PS^{NoT})$ in the subinterval

$$\left(\frac{3\left(-15d_{[11]/[22]} - \sqrt{225d_{[11]/[22]}^2 - \left(297d_{[11]/[22]} - 88d_{[11^*]/[12^*]}\right)d_{[11]/[22]}}\right)\underline{\theta}}{2\left(8d_{[11^*]/[12^*]} - 27d_{[11]/[22]}\right)}, 2\underline{\theta} \right)$$

for $d_{[11]/[22]} < d_{[11^*]/[12^*]} < \frac{171}{128} d_{[11]/[22]}$.

- positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $\frac{171}{128} d_{[11]/[22]} < d_{[11^*]/[12^*]} < \frac{3}{2} d_{[11]/[22]}$. That is, $PS^{TieA} > PS^{NoT}$.

- For $\frac{3}{2} d_{[11]/[22]} < d_{[11^*]/[12^*]}$ the value $\overline{\theta}^*$ is belongs to the interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. The function has a positive value at both $\overline{\theta} = \frac{3}{2}\underline{\theta}$ and $\overline{\theta} = 2\underline{\theta}$. This relation between the quality differentials cannot hold in the initial setup of the model without network externalities. The model setup with network externalities, however, allows for it. So, when it holds the above quadratic expression is positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $PS^{TieA} > PS^{NoT}$.

1.3. Consumer Surplus Comparison $\left(CS^{TieA} - CS^{NoT}\right)$

1.3.A) Consumer Surplus Comparison $\left(CS^{TieA} - CS^{NoT}\right)$ *- without network externalities*

In the setup of the model without network externalities the quality of the best good does not differ between the tying-to-A and non-tying subgame equilibria, i.e. $s_{11^*} = s_{11}$. Therefore, the difference in consumer surplus between the two equilibria is given by the following quadratic function in $\overline{\theta}$:

$$\left(CS^{TieA} - CS^{NoT}\right)\Big|_{s_{11^*} = s_{11}} = \frac{1}{288}\left[\left(-128d_{[11]/[12]} + 252d_{[11]/[22]}\right)\overline{\theta}^2 + 252d_{[11]/[22]}\overline{\theta}\underline{\theta} + \left(144d_{[11]/[12]} + 53d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ given that the system good qualities in the setup of the model without network externalities are set such $d_{[11]/[12]} < d_{[11]/[22]}$. That is, $CS^{TieA} > CS^{NoT}$.

1.3.B) Consumer Surplus Comparison $\left(CS^{TieA} - CS^{NoT}\right)$ *- with network externalities*

In the setup of the model with network externalities both the quality of the best good s_{11} and the quality of the second-best good s_{12} might differ in the tying-to-A subgame compared to the non-tying subgame. The difference between the two in the tying-to-A subgame however cannot be smaller than in the non-tying game, i.e. $d_{[11^*]/[12^*]} \geq d_{[11]/[12]}$. Having this in mind the difference in consumer surplus between the two subgames takes the form:

$$\left(CS^{TieA} - CS^{NoT}\right) = \left(CS^{TieA} - CS^{NoT}\right)\Big|_{s_{11^*} = s_{11}} + \frac{1}{288}\left[16\left(\overline{\theta}^2 d_{[11^*]/[11]} - \underline{\theta}^2 d_{[12^*]/[12]}\right) + 128\left(\overline{\theta}^2 - \underline{\theta}^2\right)d_{[12^*]/[12]}\right]$$

The right addend is positive for any $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. The left addend was shown to be positive above. Hence, the whole expression is positive in the interval $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieA} > CS^{NoT}$.

2. *TieB vs. NoT*

2.1. *Social Welfare Comparison* $\left(SW^{TieB} - SW^{NoT}\right)$

The difference in social welfare between the tying-to-B subgame with entry and the non-tying subgame is given by the following quadratic expression in $\bar{\theta}$:

$$\left(SW^{TieB} - SW^{NoT}\right) = \frac{1}{32}\left[\left(16s_W - 12s_{11} - 4s_{22}\right)\bar{\theta}^2 + 4\left(s_{11} - s_{22}\right)\bar{\theta}\underline{\theta} + \left(-16s_W + s_{11} + 15s_{22}\right)\underline{\theta}^2\right]$$

which if presented in terms of quality differentials would look as follows:

$$\left(SW^{TieB} - SW^{NoT}\right) = \frac{1}{32}\left[\left(16d_{[W]/[11]} + 4d_{[11]/[22]}\right)\bar{\theta}^2 + 4d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(-16d_{[W]/[11]} - 15d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

2.1.A) *Social Welfare Comparison* $\left(SW^{TieB*} - SW^{NoT}\right)$ *- without network effect*

In the setup of the model without network externalities it is relevant to assume that $d_{[W]/[11]} = 0$. So, the difference in social welfare between the tying-to-B subgame with entry and the non-tying subgame takes the form:

$$\left.\left(SW^{TieB} - SW^{NoT}\right)\right|_{d_{[W]/[11]} = 0} = \frac{1}{32}\left[4d_{[11]/[22]}\bar{\theta}^2 + 4d_{[11]/[22]}\bar{\theta}\underline{\theta} - 15d_{[11]/[22]}\underline{\theta}^2\right]$$

which is strictly positive in the interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $SW^{TieB} > SW^{NoT}$.

2.1.B) *Social Welfare Comparison* $\left(SW^{TieB} - SW^{NoT}\right)$ *- with network effect*

In the setup of the model with network externalities, the quality differential $d_{[W]/[11]}$ is defined as $d_{[W]/[11]} \geq 0$. So, the difference in social welfare between the tying-to-B subgame with entry and the non-tying subgame takes the initial form:

$$\left(SW^{TieB} - SW^{NoT}\right) = \frac{1}{32}\left[\left(16d_{[W]/[11]} + 4d_{[11]/[22]}\right)\bar{\theta}^2 + 4d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(-16d_{[W]/[11]} - 15d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which could be represented also as:

$$\left(SW^{TieB}-SW^{NoT}\right)=\left(SW^{TieB}-SW^{NoT}\right)\Big|_{d_{[W]/[11]}=0}+\frac{d_{[W]/[11]}}{2}\left(\overline{\theta}^2-\underline{\theta}^2\right)$$

The right-hand addend in the expression above is positive for any $\overline{\theta}\in\left(\frac{3}{2}\underline{\theta},2\underline{\theta}\right)$. It was shown that the difference in social welfare between the two subgames in the model without network externalities is strictly positive. So, in the model with network externalities it is also strictly positive for any $\overline{\theta}\in\left(\frac{3}{2}\underline{\theta},2\underline{\theta}\right)$. That is, $SW^{TieB}>SW^{NoT}$.

2.2. *Product Surplus Comparison* $\left(PS^{TieB}-PS^{NoT}\right)$

The difference in producer surplus between the tying-to-B subgame with entry and the non-tying subgame is given by the following quadratic expression in $\overline{\theta}$:

$$\left(PS^{TieB}-PS^{NoT}\right)=\frac{1}{16}\left[\left(-12s_{11}+12s_{22}\right)\overline{\theta}^2+\left(16s_w+20s_{11}-16s_{22*}-20s_{22}\right)\overline{\theta}\underline{\theta}+\left(-16s_W-11s_{11}+16s_{22*}+11s_{22}\right)\underline{\theta}^2\right]$$

which if presented in terms of quality differentials would look as follows:

$$\left(PS^{TieB}-PS^{NoT}\right)=\frac{1}{16}\left[-12d_{[11]/[22]}\overline{\theta}^2+\left(16d_{[W]/[11]}+16d_{[11]/[22*]}+20d_{[11]/[22]}\right)\overline{\theta}\underline{\theta}+\left(-16d_{[W]/[11]}-16d_{[11]/[22*]}-11d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

2.2.A) *Product Surplus Comparison* $\left(PS^{TieB}-PS^{NoT}\right)$ *- without network effect*

In the setup of the model without network effect, it is relevant to assume that $d_{[W]/[11]}=0$ and $d_{[11]/[22*]}=d_{[11]/[22]}$. So, the difference in producer surplus between the tying-to-B subgame with entry and the non-tying subgame takes the form:

$$\left(PS^{TieB}-PS^{NoT}\right)\Big|_{d_{[W]/[11]}=0}=-\frac{1}{64}\left(2\overline{\theta}-3\underline{\theta}\right)^2 d_{[11]/[22]}$$

which is strictly negative in the whole interval $\left(\frac{3}{2}\underline{\theta},2\underline{\theta}\right)$. That is, $PS^{TieB}<PS^{NoT}$.

2.2.B) *Product Surplus Comparison* $\left(PS^{TieB} - PS^{NoT}\right)$ - *with network effect*

In the setup of the model with network effect, the assumption holds that $d_{[W]/[11]} > 0$ and $d_{[W]/[22^*]} > \frac{9}{8} d_{[11]/[22]}$. So, the difference in producer surplus between the tying-to-B subgame with entry and the non-tying subgame takes the form:

$$\left(PS^{TieB} - PS^{NoT}\right) = \frac{1}{16}\left[-12d_{[11]/[22]}\bar{\theta}^2 + \left(16d_{[W]/[22^*]} + 20d_{[11]/[22]}\right)\bar{\theta}\underline{\theta} + \left(-16d_{[W]/[22^*]} - 11d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $PS^{TieB} > PS^{NoT}$.

2.3. *Consumer Surplus Comparison* $\left(CS^{TieB} - CS^{NoT}\right)$

The difference in consumer surplus between the tying-to-B subgame with entry and the non-tying subgame is given by the following quadratic expression in $\bar{\theta}$:

$$\left(CS^{TieB} - CS^{NoT}\right) = \frac{1}{32}\left[\left(16s_W + 12s_{11} - 28s_{22}\right)\bar{\theta}^2 + \left(-32s_w - 36s_{11} + 32s_{22^*} + 36s_{22}\right)\bar{\theta}\underline{\theta} + \left(16s_W + 23s_{11} - 32s_{22^*} - 7s_{22}\right)\underline{\theta}^2\right]$$

which if presented in terms of quality differentials would look as follows:

$$\left(CS^{TieB} - CS^{NoT}\right) = \frac{1}{32}\left[\left(16d_{[W]/[11]} + 28d_{[11]/[22]}\right)\bar{\theta}^2 + \left(-32d_{[W]/[11]} - 32d_{[11]/[22^*]} - 36d_{[11]/[22]}\right)\bar{\theta}\underline{\theta} + \left(16d_{[W]/[11]} + 32d_{[11]/[22^*]} + 7d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

2.3.A) *Consumer Surplus Comparison* $\left(CS^{TieB} - CS^{NoT}\right)$ - *without network effect*

In the setup of the model without network effect, it is relevant to assume that $d_{[W]/[11]} = 0$ and $d_{[11]/[22^*]} = d_{[11]/[22]}$. So, the difference in producer surplus between the tying-to-B subgame with entry and the non-tying subgame takes the form:

$$\left(CS^{TieB} - CS^{NoT}\right)\Big|_{d_{[W]/[11]} = 0} = \frac{1}{896}\left(14\bar{\theta} - 13\underline{\theta}\right)\left(2\bar{\theta} - 3\underline{\theta}\right)d_{[11]/[22]}$$

which is strictly positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieB} > CS^{NoT}$.

2.3.B) *Consumer Surplus Comparison* $\left(CS^{TieB} - CS^{NoT}\right)$ *- with network effect*

In the setup of the model with network effect, the assumption holds that $d_{[W]/[11]} > 0$ and $d_{[W]/[22*]} > \frac{9}{8} d_{[11]/[22]}$. So, the difference in producer surplus between the tying-to-B subgame with entry and the non-tying subgame takes the form:

$$\left(CS^{TieB} - CS^{NoT}\right) = \frac{1}{32}\left[\left(16d_{[W]/[11]} + 28d_{[11]/[22]}\right)\overline{\theta}^2 + \left(-32d_{[W]/[11]} - 32d_{[11]/[22*]} - 36d_{[11]/[22]}\right)\overline{\theta}\underline{\theta} + \left(16d_{[W]/[11]} + 32d_{[11]/[22*]} + 7d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is strictly positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieB} > CS^{NoT}$.

3. *TieB* vs. NoT*

3.1. *Social Welfare Comparison* $\left(SW^{TieB*} - SW^{NoT}\right)$

The expressions for the difference in social welfare, producer and consumer surpluses between the tying-to-B subgame without entry and the non-tying subgame have identical forms and coefficients as the ones between the tying-to-A and non-tying subgames. The difference is that in the tying-to-B subgame without entry the best good consists not of two but of three products (A1B2B1) and instead of the second-best good (A1B2) the worse quality is given by the quality of the outside option s_O:

$$\left(SW^{TieB*} - SW^{NoT}\right) = \frac{1}{288}\left[\left(80s_W - 108s_{11} + 64s_O - 36s_{22}\right)\overline{\theta}^2 + 36\left(s_{11} - s_{22}\right)\overline{\theta}\underline{\theta} + \left(9s_{11} - 144s_O + 135s_{22}\right)\underline{\theta}^2\right]$$

3.1.A) *Social Welfare Comparison* $\left(SW^{TieB*} - SW^{NoT}\right)$ *- without network effect*

In the setup of the model without network externalities it is relevant to assume that the quality of the best good does not differ between the two subgames, i.e. $s_W = s_{11}$. Then, the above quadratic expression in $\bar{\theta}$ takes the form:

$$\left(SW^{TieB*} - SW^{NoT}\right)\Big|_{s_W = s_{11}} = \frac{1}{288}\left[\left(-64d_{[11]/[O]} + 36d_{[11]/[22]}\right)\bar{\theta}^2 + 36d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(144d_{[11]/[O]} - 135d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

Which has a strictly negative value in that interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ given that system good qualities in the setup of the model without network externalities are set such $d_{[11]/[O]} > d_{[11]/[12]}$. That is, $SW^{TieB*} < SW^{NoT}$.

3.1.B) *Social Welfare Comparison* $\left(SW^{TieB*} - SW^{NoT}\right)$ *- with network effect*

In the setup of the model with network externalities the quality of the best good s_W might differ in the tying-to-B subgame without entry compared to the non-tying subgame. Respectively, the quality differential between the best good and the outside option in the tying-to-B subgame without entry cannot be smaller than in the non-tying game, i.e. $d_{[W]/[O]} \geq d_{[11]/[O]}$. Having this in mind the difference in consumer surplus between the two subgames takes the form:

$$\left(SW^{TieB*} - SW^{NoT}\right) = \frac{1}{288}\left[\left(80d_{[W]/[11]} - 64d_{[11]/[O]} + 36d_{[11]/[22]}\right)\bar{\theta}^2 + 36d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(144d_{[11]/[O]} - 135d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is:

- positive $\left(SW^{TieB*} > SW^{NoT}\right)$ in the subinterval

$$\left(\frac{3}{2}\underline{\theta},\frac{\left(-36d_{[11]/[22]}-\sqrt{1296d_{[11]/[22]}^{2}-4\left(-135d_{[11]/[22]}+144d_{[11]/[O]}\right)\left(36d_{[11]/[22]}-64d_{[11]/[O]}+80d_{[W]/[11]}\right)}\right)\underline{\theta}}{8\left(9d_{[11]/[22]}-16d_{[11]/[O]}+20d_{[W]/[11]}\right)}\right)$$ but

negative $\left(SW^{TieB*} < SW^{NoT}\right)$ in the subinterval

$$\left(\frac{\left(-36d_{[11]/[22]}-\sqrt{1296d_{[11]/[22]}^{2}-4\left(-135d_{[11]/[22]}+144d_{[11]/[O]}\right)\left(36d_{[11]/[22]}-64d_{[11]/[O]}+80d_{[W]/[11]}\right)}\right)\underline{\theta}}{8\left(9d_{[11]/[22]}-16d_{[11]/[O]}+20d_{[W]/[11]}\right)},2\underline{\theta}\right)$$

for $d_{[W]/[11]} < \dfrac{\left(112d_{[11]/[O]}-81d_{[11]/[22]}\right)}{320}$.

- positive in the whole interval $\left(\frac{3}{2}\underline{\theta},2\underline{\theta}\right)$ for $\dfrac{\left(112d_{[11]/[O]}-81d_{[11]/[22]}\right)}{320} < d_{[W]/[11]}$. That is, $SW^{TieB*} > SW^{NoT}$.

3.2. *Producer Surplus Comparison* $\left(PS^{TieB*}-PS^{NoT}\right)$

The difference in equilibrium producer surplus between the tying-to-B subgame without entry and the non-tying subgame is given by the following quadratic function in $\bar{\theta}$:

$$\left(PS^{TieB*}-PS^{NoT}\right)=\frac{1}{144}\left[\left(32d_{[W]/[O]}-108d_{[11]/[22]}\right)\bar{\theta}^{2}+180d_{[11]/[22]}\bar{\theta}\underline{\theta}-99d_{[11]/[22]}\underline{\theta}^{2}\right]$$

3.2.A) Producer Surplus Comparison $\left(PS^{TieB*}-PS^{NoT}\right)$ *- without network externalities*

In the initial setup of the model without network externalities the following relation between the quality differentials in the two subgames holds: $d_{[W]/[O]}=d_{[11]/[O]}>\frac{9}{4}d_{[11]/[22]}$.

Respectively, the above quadratic expression takes the form:

$$\left.\left(PS^{TieB*}-PS^{NoT}\right)\right|_{s_W=s_{11}}=\frac{1}{144}\left[\left(32d_{[11]/[O]}-108d_{[11]/[22]}\right)\bar{\theta}^{2}+180d_{[11]/[22]}\bar{\theta}\underline{\theta}-99d_{[11]/[22]}\underline{\theta}^{2}\right]$$

which is positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $\frac{9}{4}d_{[11]/[22]} < d_{[11]/[O]}$. That is, $PS^{TieB*} > PS^{NoT}$.

3.2.B) Producer Surplus Comparison $\left(PS^{TieB*} - PS^{NoT}\right)$ *- with network externalities*

In the setup of the model with network externalities the following relation between the differentials in the two subgames holds: $d_{[W]/[O]} > d_{[11]/[O]} > \frac{9}{4}d_{[11]/[22]}$. Otherwise, the results of the functional analysis are analogous to the case without network externalities. Just the conditions are set on $d_{[W]/[O]}$ which is not identical to $d_{[11]/[O]}$:

For $\frac{9}{4}d_{[11]/[22]} < d_{[11]/[O]} < d_{[W]/[O]}$ the function is positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $PS^{TieB*} > PS^{NoT}$.

3.3. *Consumer Surplus Comparison* $\left(CS^{TieB*} - CS^{NoT}\right)$

The difference in consumer surplus between the tying-to-B subgame without entry and the non-tying subgame equilibrium is given by the following quadratic function in $\overline{\theta}$:

$$\left(CS^{TieB*} - CS^{NoT}\right) = \frac{1}{288}\left[\left(16d_{[W]/[11]} - 128d_{[11]/[O]} + 252d_{[11]/[22]}\right)\overline{\theta}^2 - 324d_{[11]/[22]}\overline{\theta}\underline{\theta} + \left(144d_{[11]/[O]} + 63d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

3.3.A) Consumer Surplus Comparison $\left(CS^{TieB*} - CS^{NoT}\right)$ *- without network externalities*

In the setup of the model without network externalities, the system qualities are assumed to satisfy the condition $d_{[W]/[O]} = d_{[11]/[O]} > \frac{9}{4}d_{[11]/[22]}$. In compliance with it, the difference in consumer surplus between the tying-to-B subgame without entry and the non-tying subgame takes the form:

$$\left(CS^{TieB*} - CS^{NoT}\right)\Big|_{s_W = s_{11}} = \frac{1}{288}\left[\left(-128d_{[11]/[O]} + 252d_{[11]/[22]}\right)\bar{\theta}^2 - 324d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(144d_{[11]/[O]} + 63d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is negative in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieB*} < CS^{NoT}$.

3.3.B) Consumer Surplus Comparison $\left(CS^{TieB*} - CS^{NoT}\right)$ *- with network externalities*

In the setup of the model with network externalities, the system qualities are assumed to satisfy the condition $d_{[W]/[O]} > d_{[11]/[O]} > \frac{9}{4}d_{[11]/[22]}$. In compliance with it, the difference in consumer surplus between the tying-to-B subgame without entry and the non-tying subgame takes the form:

$$\left(CS^{TieB*} - CS^{NoT}\right) = \frac{1}{288}\left[\left(16d_{[W]/[11]} - 128d_{[11]/[O]} + 252d_{[11]/[22]}\right)\bar{\theta}^2 - 324d_{[11]/[22]}\bar{\theta}\underline{\theta} + \left(144d_{[11]/[O]} + 63d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is:

- negative in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $d_{[W]/[11]} < 4d_{[11]/[O]} - 4d_{[11]/[22]}$. That is, $CS^{TieB*} < CS^{NoT}$.
- positive ($CS^{TieB*} > CS^{NoT}$) in the subinterval $\left(\frac{3}{2}\underline{\theta}, \frac{\left(324d_{[11]/[22]} + \sqrt{104976d_{[11]/[22]}^2 - 4\left(63d_{[11]/[22]} + 144d_{[11]/[O]}\right)\left(252d_{[11]/[22]} - 128d_{[11]/[O]} + 16d_{[W]/[11]}\right)}\right)\underline{\theta}}{2\left(252d_{[11]/[22]} - 128d_{[11]/[O]} + 16d_{[W]/[11]}\right)}\right)$ and negative ($CS^{TieB*} < CS^{NoT}$) in the subinterval $\left(\frac{\left(324d_{[11]/[22]} + \sqrt{104976d_{[11]/[22]}^2 - 4\left(63d_{[11]/[22]} + 144d_{[11]/[O]}\right)\left(252d_{[11]/[22]} - 128d_{[11]/[O]} + 16d_{[W]/[11]}\right)}\right)\underline{\theta}}{2\left(252d_{[11]/[22]} - 128d_{[11]/[O]} + 16d_{[W]/[11]}\right)}, 2\underline{\theta}\right)$

for $4d_{[11]/[O]} - 4d_{[11]/[22]} < d_{[W]/[11]} < \frac{368d_{[11]/[O]} - 423d_{[11]/[22]}}{64}$.

- positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $\frac{368d_{[11]/[O]} - 423d_{[11]/[22]}}{64} < d_{[W]/[11]}$. That is, $CS^{TieB*} > CS^{NoT}$.

4 *TieB vs. TieA*

Here, it is relevant to assume that $s_W = s_{11^*}$ which implies that the social welfare differences would be independent on whether a setup of the model with or without network externalities is applied. Respectively, the asterisk mark is skipped from the indices below.

4.1. Social Welfare Comparison $\left(SW^{TieB} - SW^{TieA}\right)$

The difference in social welfare between the tying-to-B subgame with entry and the tying-to-A subgame is given by the following quadratic function in $\overline{\theta}^*$:

$$\left(SW^{TieB} - SW^{TieA}\right) = \frac{1}{18}\left[\left(9s_W - 5s_{11} - 4s_{12}\right)\overline{\theta}^2 - 9\left(s_W - s_{12}\right)\underline{\theta}^2\right]$$

which after taking into account the assumption that $s_W = s_{11}$ takes the form:

$$\left(SW^{TieB} - SW^{TieA}\right) = \frac{1}{18}\left(4\overline{\theta}^2 - 9\underline{\theta}^2\right)d_{[11]/[12]}$$

The above expression is strictly positive for $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $SW^{TieB} > SW^{TieA}$.

4.2. Producer Surplus Comparison $\left(PS^{TieB} - PS^{TieA}\right)$

The difference in producer surplus between the tying-to-B subgame with entry and the tying-to-A subgame is given by the following quadratic function in $\overline{\theta}^*$:

$$\left(PS^{TieB} - PS^{TieA}\right) = \frac{1}{9}\left[-2(s_{11} - s_{12})\overline{\theta}^2 + 9(s_W - s_{22})\overline{\theta}\underline{\theta} - 9(s_W - s_{22})\underline{\theta}^2\right]$$

which after taking into account the assumption that $s_W = s_{11}$ takes the form:

$$\left(PS^{TieB} - PS^{TieA}\right) = \frac{1}{9}\left[-2d_{[11]/[12]}\overline{\theta}^2 + 9d_{[11]/[22]}\overline{\theta}\underline{\theta} - 9d_{[11]/[22]}\underline{\theta}^2\right]$$

which is strictly positive in the interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $PS^{TieB} > PS^{TieA}$.

4.3. Consumer Surplus Comparison $\left(CS^{TieB} - CS^{TieA}\right)$

The difference in producer surplus between the tying-to-B subgame with entry and the tying-to-A subgame is given by the following quadratic function in $\overline{\theta}^*$:

$$\left(CS^{TieB} - CS^{TieA}\right) = \frac{1}{18}\left[(9s_W - s_{11} - 8s_{12})\overline{\theta}^2 - 18(s_W - s_{22})\overline{\theta}\underline{\theta} + 9(-s_W - s_{12} + 2s_{22})\underline{\theta}^2\right]$$

which after taking into account the assumption that $s_W = s_{11}$ takes the form:

$$\left(CS^{TieB} - CS^{TieA}\right) = \frac{1}{18}\left[8d_{[11]/[12]}\overline{\theta}^2 - 18d_{[11]/[22]}\overline{\theta}\underline{\theta} + 9\left(d_{[11]/[12]} - 2d_{[11]/[22]}\right)\underline{\theta}^2\right]$$

which is strictly negative in the interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieB} < CS^{TieA}$.

5 TieB* vs TieA

Again it is relevant to assume that $s_W = s_{11*}$ which implies that the social welfare differences would be independent on whether a setup of the model with or without network externalities is applied.

5.1. Social Welfare Comparison $\left(SW^{TieB*} - SW^{TieA}\right)$

The difference in social welfare between the tying-to-B subgame without entry and the tying-to-A subgame is given by the following quadratic expression in $\overline{\theta}^*$:

$$\left(SW^{TieB*} - SW^{TieA}\right) = \frac{1}{18}\left[\left(5s_W - 5s_{11} + 4s_O - 4s_{12}\right)\overline{\theta}^2 + 9\left(s_{12} - s_O\right)\underline{\theta}^2\right]$$

which after taking into account the assumption that $s_W = s_{11}$ takes the form:

$$\left(SW^{TieB*} - SW^{TieA}\right) = -\frac{1}{18}\left(4\overline{\theta}^2 - 9\underline{\theta}^2\right)d_{[12]/[O]}$$

The above expression is strictly negative for $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $SW^{TieB*} < SW^{TieA}$.

5.2. Producer Surplus Comparison $\left(PS^{TieB*} - PS^{TieA}\right)$

The difference in producer surplus between the tying-to-B subgame without entry and the tying-to-A subgame is given by the following quadratic expression in $\overline{\theta}^*$:

$$\left(PS^{TieB*} - PS^{TieA}\right) = \frac{2}{9}\left(s_W - s_O - s_{11} + s_{12}\right)\overline{\theta}^2$$

which after taking into account the assumption that $s_W = s_{11}$ takes the form:

$$\left(SW^{TieB*} - SW^{TieA}\right) = \frac{2}{9}\overline{\theta}^2 d_{[12]/[O]}$$

The above expression is strictly positive for $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $PS^{TieB*} > PS^{TieA}$.

5.3. Consumer Surplus Comparison $\left(CS^{TieB*} - CS^{TieA}\right)$

The difference in consumer surplus between the tying-to-B subgame without entry and the tying-to-A subgame is given by the following quadratic expression in $\overline{\theta}^*$:

$$\left(CS^{TieB*} - CS^{TieA}\right) = \frac{1}{18}\left[\left(s_W + 8s_O - s_{11} - 8s_{12}\right)\overline{\theta}^2 + 9\left(s_{12} - s_O\right)\underline{\theta}^2\right]$$

which after taking into account the assumption that $s_W = s_{11}$ takes the form:

$$\left(CS^{TieB*} - CS^{TieA}\right) = \frac{\left(-8\overline{\theta}^2 + 9\underline{\theta}\right)}{18} d_{[12]/[O]}$$

The above expression is strictly negative for $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieB*} < CS^{TieA}$.

6 TieB vs. TieB*

It is relevant to assume that $s_W = s_{11}$ which implies that the social welfare differences would be independent on whether a setup of the model with or without network externalities is applied.

6.1. Social Welfare Comparison $\left(SW^{TieB*} - SW^{TieB}\right)$

The difference in social welfare between the tying-to-B subgames without and with entry is given by the following quadratic function in in $\overline{\theta}^*$:

$$\left(SW^{TieB*} - SW^{TieB}\right) = \frac{1}{18}\left(-4\left(s_W - s_O\right)\overline{\theta}^2 + 9\left(s_W - s_O\right)\underline{\theta}^2\right)$$

which if represented in terms of the quality differential would look as follows:

$$\left(SW^{TieB*} - SW^{TieB}\right) = -\frac{1}{18}\left(4\overline{\theta}^2 - 9\underline{\theta}^2\right) d_{[W]/[O]}$$

The above expression is strictly negative for $\overline{\theta} \in \left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $SW^{TieB*} < SW^{TieB}$.

6.2. Product Surplus Comparison $\left(PS^{TieB*} - PS^{TieB}\right)$

The difference in social welfare between the tying-to-B subgames without and with entry is given by the following quadratic function in in $\overline{\theta}^*$:

$$\left(PS^{TieB*} - PS^{TieB}\right) = \frac{1}{9}\left(2(s_W - s_O)\overline{\theta}^2 - 9(s_W - s_{22})\overline{\theta}\underline{\theta} + 9(s_W - s_{22})\underline{\theta}^2\right)$$

which if represented in terms of the quality differential would look as follows:

$$\left(PS^{TieB*} - PS^{TieB}\right) = \frac{1}{9}\left(2d_{[W]/[O]}\overline{\theta}^2 - 9d_{[W]/[22]}\overline{\theta}\underline{\theta} + 9d_{[W]/[22]}\underline{\theta}^2\right).$$

The above expression is:

- positive ($PS^{TieB*} > PS^{TieB}$) in the subinterval $\left(\frac{3}{2}\underline{\theta}, \frac{3\left(3d_{[W]/[22]} - \sqrt{9d_{[W]/[22]}^2 - 8d_{[W]/[22]}d_{[W]/[O]}}\right)\underline{\theta}}{4d_{[W]/[O]}}\right)$ and negative ($PS^{TieB*} < PS^{TieB}$)in the subinterval $\left(\frac{3\left(3d_{[W]/[22]} - \sqrt{9d_{[W]/[22]}^2 - 8d_{[W]/[22]}d_{[W]/[O]}}\right)\underline{\theta}}{4d_{[W]/[O]}}, 2\underline{\theta}\right)$ for $d_{[W]/[O]} < \frac{9}{8}d_{[W]/[22]}$.
- positive in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$ for $\frac{9}{8}d_{[W]/[22]} < d_{[W]/[O]}$. That is, $PS^{TieB*} > PS^{TieB}$.

6.3. *Consumer Surplus Comparison* $\left(CS^{TieB*} - CS^{TieB}\right)$

The difference in consumer surplus between the tying-to-B subgames without and with entry is given by the following quadratic function in in $\overline{\theta}^*$:

$$\left(CS^{TieB*} - CS^{TieB}\right) = \frac{1}{18}\left(-8(s_W - s_O)\overline{\theta}^2 + 18(s_W - s_{22})\overline{\theta}\underline{\theta} + 9(-s_O - s_W + 2s_{22})\underline{\theta}^2\right)$$

which if represented in terms of the quality differential would look as follows:

$$\left(CS^{TieB*} - CS^{TieB}\right) = \frac{1}{18}\left(-8d_{[W]/[O]}\overline{\theta}^2 + 18d_{[W]/[22]}\overline{\theta}\underline{\theta} + 9\left(d_{[W]/[O]} - 2d_{[W]/[22]}\right)\underline{\theta}^2\right).$$

This expression is negative in the whole interval $\left(\frac{3}{2}\underline{\theta}, 2\underline{\theta}\right)$. That is, $CS^{TieB*} < CS^{TieB}$.