Vertical Product Differentiation of Complementary Goods*

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Abstract

The paper introduces a model of vertical product differentiation as an extension of Shaked and Sutton (1982) that allows for representation of the consumer demand for a bundle of several complementary goods sold in separate markets. Two key differences are identified in the equilibrium outcome. First, it is shown that the condition on market size derived by Shaked and Sutton (1982) for having duopoly at equilibrium in an independent market might not be sufficient to restrict down to two the equilibrium number of the possible bundles of several complementary goods. Therefore, to have further decreased number of the salable bundles, the mutual size of the markets must satisfy a more restrictive condition at which all the complementary markets except one have a monopoly structure at equilibrium. Second, the solution of the model implies that as long as one of the related markets has a monopoly structure and its size exceeds half of its upper bound, this market would not be covered at its pricing-stage equilibrium. Since the good sold in this market is a complement to the goods sold in other related markets, however, the latter would not be covered at equilibrium, either. The described result diverges from the one supported by Shaked and Sutton (1982) for a single independent market where at the same conditions exactly two goods would have sales, gain positive profits and cover the whole market at equilibrium.

Keywords: vertical product differentiation, complementary goods, market size, market foreclosure

JEL classification: L11, L13, L15

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1. Introduction

Product differentiation is a main feature distinguishing Monopolistic Competition from the Perfect Competition market structures.

It is common knowledge in economics that if two goods in a market differ in quality their demand would be inelastic. At least a consumer would be willing to pay more for one of the two qualities and therefore would continue preferring it even if the other quality is offered at a slightly lower price. Therefore, product differentiation is generally agreed to be a source of market power for the firms and implies imperfect competition among them.

One of the forms of product differentiation recognized by economists is the so-called vertical product differentiation which is first described by Gabszewicz and Thisse (1979).

A key distinguishing feature of the models of vertical product differentiation is that all consumers agree on which good is the best in the market, respectively which is second best and so on, thus forming a common ranking of the goods based on their qualities. So, when product qualities are vertically differentiated, they differ not by the specific consumer taste each suits but by the extent to which any given quality is able to satisfy the consumer tastes as a whole. All consumers prefer the product which serves them better to the one that serves them worse even if the latter is offered at a slightly lower price.

What is specific about the markets with vertical product differentiation is that their equilibrium outcome differs from the optimal result in markets with classical Chamberlinian monopolistic competition structure. Even when barriers to entry are not present in a market with vertical product differentiation, only finite number of potential market entrants could make sales in equilibrium. They all earn positive profit and cover the market. Markets with vertical product differentiation, minimal barriers to entry and still endogenously bounded number of entrants that could coexist in equilibrium are known in industrial economics as natural oligopolies due to Shaked and Sutton (1983).

This paper studies vertical product differentiation of goods which are complementary though sold in different markets. More particularly, it makes a revision of a special market equilibrium described by Shaked and Sutton (1982) at which as long as the market is sufficiently narrow[1], exactly two entrants will cover the market and still make positive profits from selling their vertically differentiated products. The aim is to explore how this outcome would differ if the goods offered by the two entrants are complementary to other goods offered in a separate market.

In real world, there are a lot of examples of goods whose demand is affected by the price of their separately sold complements.

A classical example of complementary goods from the basic economics textbooks[2] is washing machines and laundry detergents. As the price of the liquid detergent for automatic laundry machines decreases, standard washing powder gets eventually obsolete. As a result, non-automatic laundry machines might get also out of use because some low-income consumers might still find the higher prices of the only available liquid washing detergents unaffordable.

Similarly, not all toothpastes are best to use with an electric toothbrush. Even though, dental studies[3] show that if used properly, electric brushes have higher efficiency in plaque and stain removal with any toothpaste, there are experts who warn against power brushing with certain types of toothpastes. Because of the larger number of brush strokes per minute use, electric toothbrushes have the risk to wear down teeth when used in combination with high-abrasive toothpastes which are otherwise less harmless if applied by manual brushes. Also, electric brushes tend to generate more foam if applied to standard toothpastes which implies that less foamy dry pastes are more suitable for power brushing. Respectively, as the price of dry low-abrasive toothpastes gets more affordable to larger share of population, they might completely remove the old foamy high-abrasive pastes from the market. Afterwards, however, some low-income consumers might find the higher price of the only available low-abrasive toothpastes unaffordable and therefore stop brushing their teeth at all.

[1] In the original work of Shaked and Sutton (1982), the size of the market is given by the variance of the consumer incomes which are assumed to be uniformly distributed in that range. Later on, however, Tirole (1988) shows that income could be considered just as an approximation of the consumer tastes for quality (see Tirole, 1988, p.96). Respectively, in the model presented here, a preference is given to the utility functional form suggested by Mussa and Rosen (1978) based on taste instead of on income parameter because it makes computations simpler without loss of generality.

[2] See Anderton, 2006, p.47.

[3] Complete list of the scientific publications on the topic is available at the following web page: http://eu.broxo.com/en/clinical_studies/complete_list.aspx

Another relevant example is the market for illuminants. With the appearance of the energy-saving light bulbs, many households diverted their demand from the lamps with standard bulbs to economize on the bill for electricity. However, for the households with low bills for electricity, buying the more expensive light bulbs might not be reasonable. So, if the energy-saving bulbs turn to be the only available in the market, households with low energy consumption might be better-off if they drop electric lighting at all.

Last, it is also worth mentioning an example from the modern telecommunications industry. The lower the price of mobile telecom services are, the larger becomes the number of cell-phone users. However, the price of cell phones could still be too high for the low-income consumers who would eventually stop using telecom services as standard fixed-line phones get obsolete.

What is common for the examples above is that competition between an innovated (high-quality) product and its standard (low-quality) precursor leads to decrease in their prices. Respectively, if the market size is small enough the sales of the non-innovated product might get efficiently foreclosed. This is a standard well-known outcome in the markets with vertical product differentiation.

What is unusual in the examples given here, however, is that markets might remain uncovered. This is because the described markets are not independent but instead the goods sold in one market are complements to the goods sold in the other market. A product purchased in a market cannot bring positive utility to the consumer if not combined with a product from the other market. For instance, consumer gains little from buying a washing machine if she has no detergent for it. Similarly, buying liquid detergent for automatic laundry machine cannot help the consumer much if she has no automatic laundry machine. Therefore, the size of both markets for washing powder and for laundry machines must be the same. Nevertheless, the critical value of the size beyond which at most two goods could have positive share differs among the markets because of the different weights the qualities of the powder and the laundry machine have in the utility of their mutual consumption.

In other words, the range of incomes among the population of consumers buying both washing detergents and laundry machines could imply such a market size which is large enough for at most two types of laundry machines to be sold – automatic and non-automatic, respectively. At the same time, however, this same market size could be too small for other than liquid detergent to be sold in the market for washing detergents. As a result, liquid detergent will be charged monopoly price which is above the reservation price of the low-income consumers. Respectively, the latter will purchase neither a washing machine nor any washing powder. Thus, both markets will remain uncovered.

The present paper suggests an extension of the model used by Shaked and Sutton (1982) which allows for representation of the consumer demand for a bundle of several complementary goods sold in separate markets. The solution of the extended model implies that at particular market conditions an equilibrium outcome would exist where the bundle of two innovated complementary products could monopolize their markets without covering them. It has two evidential differences from the equilibrium solution of the single-product model of Shaked and Sutton (1982).

First, since there are several markets, a good from one market could form a bundle with any of the goods available in another market. So, the number of possible bundles in which a good takes part increases both in the number of markets where its complements are sold and in the number of complements sold in each market. Therefore, the restriction on the market size which is derived by Shaked and Sutton (1982) as a sufficient condition for at most two goods to have positive demand in a single market still allows for all the complements available in another market to be sold as long as they could form bundles with the two goods in the first market. In order to have not more than two bundles of complementary goods sold in equilibrium, the condition on market size needs to be increasingly restrictive as the number of goods in a bundle grows. Nevertheless, the restriction has a limit and does not increase incessantly as the number of the markets for complementary goods tends to infinity.

Second, the buying decision of the consumers with low willingness to pay for quality depends not only on the price of the low-quality good sold in a single market but is also determined by the prices of all the goods that enter together with it in the low-quality bundle. To illustrate this, in the example with the market for washing detergents, let the related market for laundry machines is considered as independent and its size is in a particular narrow range which in compliance with Shaked and Sutton (1982) ensures exactly two goods with positive market shares at equilibrium. Then, the expectations would indeed be two qualities of washing machines, say automatic and non-automatic, to be sold at equilibrium and the market to be covered. In the market for detergents, however, provided that the qualities of its goods have higher weight in consumer's utility, the same market size would imply that only the best good (e.g. liquid detergent) could have a positive demand. Respectively, the liquid detergent would be optimal to be charged the monopoly price which would make not only it but also the bundles in which it takes part prohibitively expensive for the low-taste consumers. The latter would not be willing to pay the monopoly price of the only detergent available even if it is of the best (i.e. liquid) quality possible. This would make pointless for them also the purchase of a washing machine. In turn, this would drive down to zero the demand for the low-quality good in the market for washing machines. So, only the best washing machine will be sold and counter to the single-market expectations no market would be covered at equilibrium, but still all markets would have monopoly structure even without barriers to entry.

The paper is organized as follows. Section 2 introduces the extended model of vertical product differentiation adjusted to reflect the consumer demand for bundles of complementary goods. Section 3 states the general condition for at most two bundles to have sales in equilibrium and discusses the resulting equilibrium outcome in case of two related markets. Section 4 concludes.

2. The Model

The model that is going to be introduced for the purpose of the analysis in the present paper is an extension of the model suggested by Shaked and Sutton (1982). It is based on the same three-stage non-cooperative game in which first firms make an entry decision, then entrants choose quality and finally they compete in prices. The payoff of the firms that decide to enter the market in the first stage is given by their profit less an infinitesimally small cost of entry $\varepsilon > 0$. For simplicity, it is assumed that firms have no production costs. Non-entrants earn zero payoffs.

What is new in the present model is that it considers not just a single market but several markets for complementary goods. Each firm is assumed to sell only one good. So, it is identified not only by the quality of the good it produces but also by the market in which this good is offered.

For example, let's have two markets and denote them by A and B. There are *m* entrants in market A and *n* entrants in market B. In each market let firms be identified by the market rank of the quality of the good that each of them offers, as shown below:

$$f(A_1) > f(A_2) > .... > f(A_m) \qquad (1)$$

where $f(A_i)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the product of type *A* based on which it is ranked at *i*-th place by quality; $\underline{F} \leq f(A_i) \leq \overline{F}$, $i = 1,...,m$;

$$g(B_1) > g(B_2) > .... > g(B_n) \qquad (2)$$

where $g(B_j)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the product of type *B* based on which it is ranked at *j*-th place by quality; $\underline{G} \leq g(B_j) \leq \overline{G}$, $j = 1,...,n$.

Respectively, firms' profits (revenues) are represented by the following expressions:

$$\Pi_1^A = p_1^A D_1^A \left(p_1^A ,..., p_m^A \right) \tag{3}$$

………………………………..

$$\Pi_i^A = p_i^A D_i^A \left(p_1^A ,..., p_i^A ,..., p_m^A \right)$$

……………………………….

$$\Pi_m^A = p_m^A D_m^A \left(p_1^A ,..., p_m^A \right)$$

where:

Π_i^A - profit (revenue) of good ranked *i*-th by quality in market A, $i = 1,..., m$

D_i^A - the demand for good ranked *i*-th by quality in market A, $i = 1,..., m$

p_i^A - the price of good ranked *i*-th by quality in market A, $i = 1,..., m$

$$\Pi_1^B = p_1^B D_1^B \left(p_1^B ,..., p_n^B \right) \tag{4}$$

………………………………..

$$\Pi_j^B = p_j^B D_j^B \left(p_1^B ,..., p_j^B ,..., p_n^B \right)$$

……………………………….

$$\Pi_m^B = p_m^B D_m^B \left(p_1^B ,..., p_n^B \right)$$

where:

Π_j^B - profit (revenue) of good ranked *j*-th by quality in market B, $j = 1,..., n$

D_j^B - the demand for good ranked *j*-th by quality in market B, $j = 1,..., n$

p_j^B - the price of good ranked *j*-th by quality in market B, $j = 1,..., n$

The demand for a good in each of the two markets is given by the sum of the market shares of the bundles in which the good takes part:

$$D_i^A = \sum_{j=1}^{n} D_{ij} \text{ , } i = 1,...,m \tag{5}$$

$$D_j^B = \sum_{i=1}^{m} D_{ij} \text{ , } j = 1,...,n \tag{6}$$

where D_{ij} is used to denote the demand for bundle AiBj consisting of the i-th best quality good in market A and the j-th best quality good in market B.

The demand for bundles D_{ij} is derived from the consumers' optimal choices. Each consumer chooses to purchase the bundle which maximizes her individual consumer surplus given by the standard utility function introduced by Mussa and Rosen (1978):

$$U(\chi_{ij}, \theta) = \theta\chi_{ij} - p(\chi_{ij}) = \theta\chi_{ij} - p_i[f(A_i)] - p_j[g(B_j)] \tag{7}$$

where:

θ - taste parameter[4] by which consumers are identified; θ is assumed to be uniformly distributed on an interval $\underline{\theta} \le \theta \le \overline{\theta}$

χ_{ij} - quality of bundle A_iB_j

$p_{ij} = p[f(A_i), g(B_j)]$ - price of bundle A_iB_j as a function of the qualities of the goods in it

$p_i^A = p_i[f(A_i)]$ - price of good A_i as a function of its quality[5]

$p_j^B = p_j[g(B_j)]$ - price of good B_j as a function of its quality

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[4] θ could be considered as a measure of the marginal utility of quality (Mussa and Rosen, 1978) or consumer's income (Shaked and Sutton, 1982). It is straightforward to show that the two are equivalent (see Tirole, 1988, p.96).

[5] To simplify the notation, the prices of goods are denoted by p_i^A $(i = 1,...,m)$ if they are of type A and by p_j^B $(j = 1,...,n)$ if they are of type B.

In addition to the bundles in the market, consumers could choose also an outside option with quality net of price assumed to be conditional on consumer taste[6]:

$$u_O(\theta) = \chi_O - \frac{p_O}{\theta} \tag{8}$$

where:

$u_O(\theta)$ - composite operator for the quality of the outside option net of price standardly assumed in the literature for the technical purpose of avoiding overparametrization.

χ_O - quality of the outside option

p_O - price of the outside option

So, expression (7) could be rewritten for the utility of the outside option as follows:

$$U(\chi_O, \theta) = u_O \theta \tag{9}$$

A marginal taste parameter $\theta_{ij/i'j'}$ could be defined with which a consumer would be indifferent between bundle AiBj at prices p_i^A and p_j^B and bundle Ai'Bj' at prices $p_{i'}^A$ and $p_{j'}^B$. The general expression for $\theta_{ij/i'j'}$ could be derived from (7) as follows:

$$\theta_{ij/i'j'} = \frac{p_i^A + p_j^B - p_{i'}^A - p_{j'}^B}{d_{ij/i'j'}} \tag{10}$$

where:

$\theta_{ij/i'j'}$ - marginal taste parameter at which a consumer is indifferent between purchasing bundles $A_i B_j$ and $A_{i'} B_{j'}$

$d_{ij/i'j'}$ - the quality differential between bundles $A_i B_j$ and $A_{i'} B_{j'}$, i.e. $d_{ij/i'j'} = \chi_{ij} - \chi_{i'j'}$

The marginal taste parameters divide the range of consumer tastes $(\underline{\theta}, \overline{\theta})$ into subintervals corresponding to the market shares of the bundles formed by the goods available in the two markets. For example, the market share of bundle A1B1 is given by the difference

[6] The outside option is introduced to represent the consumer choice of not buying a good available in the market. The analysis however does not aim to explain how the price and the quality of the outside option are formed, so they are assumed to be exogenously given.

between the upper bound of the taste parameters' range and the marginal taste parameter between the best and second best bundle. Let the latter be A1B2. Then, the expression for best bundle's market share is as follows:

$$D_{11} = \bar{\theta} - \theta_{11/12} \qquad (11)$$

The expression for the market share of the worst bundle differs dependent on whether the market is covered or not. For instance, let the worst bundle available be A1B2. Then, its market share is as follows.

$$D_{12} = \begin{cases} \theta_{11/12} - \underline{\theta}, \text{given covered market i.e. } \underline{\theta} \geq \theta_{12/O} \\ \theta_{11/12} - \theta_{12/O}, \text{given non-covered market i.e. } \underline{\theta} \leq \theta_{12/O} \end{cases} \qquad (12)$$

where:

$\theta_{12/O}$ - taste parameter with which a consumer would be indifferent between the lowest quality bundle A1B2 and the outside option i.e. between buying and not buying in the market.

Finally, it only remains to define how good qualities, $f(A_i)$ and $g(B_j)$, determine the quality χ_{ij} of the bundle AiBj they form. Here, a standard CES (aka Dixit-Stiglitz[7]) aggregation functional form is assumed:

$$\chi_{ij} = \left[f(A_i)^r + g(B_j)^r\right]^{\frac{1}{r}} \qquad (13)$$

where $r = \frac{(s-1)}{s}$ given that the elasticity of substitution is denoted by s.

In what follows, the model is solved in compliance with the perfect equilibrium concept of Selten (1975) by using a backward induction that is starting from the pricing stage and heading to the entry stage of the game.

[77] For detailed description of the properties of CES utility function, see Dixit and Stiglitz (1977).

3. Solution for the optimal outcome at market size allowing for at most two bundles with positive sales at equilibrium

The analysis of the subgame equilibrium in the pricing stage follows closely the methodology of Shaked and Sutton (1982).

First, $(m \times n)$ potential bundles are assumed to co-exist in equilibrium. However, this is not sufficient for unambiguous explicit-form representation of the profit-maximization problems in (3) and (4). As shown in (5) and (6), distinct from the case with a single market for independent goods, when two markets for complementarily bundled goods are considered, each good could be sold as a part of several bundles. Therefore, the ranking of the bundles could vary dependent on how the qualities of the goods that form them relate to each other. Nevertheless, a general condition on the quality differences between the bundles could be set such that independent on their ranking, a system of first-order optimality equations for profit maximization could be derived which is compatible with the solution results of Shaked and Sutton (1982). The very condition and its direct implication are stated in proposition 1 below.

To facilitate the statement of proposition 1, let a subset of bundles based on A1 be better than A2B1 and denote their number by *k*, $k \in (1, n)$. Similarly, let denote by *l* the number of bundles based on A2 which are better than the superior of the two bundles A*3*B*1* and A*1*B*k+1*, $l \in (1, n)$. Respectively, the optimal demand for bundle A1Bj $(j = 1, ..., k)$ is denoted by D^*_{1j}, given that no bundle A1Bj $(j = k, ..., n)$ has positive demand. Analogously, D^*_{2j} denotes the optimal demand for bundle A2Bj $(j = 1, ..., k)$, given that no bundle A2Bj $(j = l, ..., n)$ has positive demand.

Proposition 1. Let the price-elasticity of demand for the bundles based on goods A1 and A2 fulfill the following conditions:

$$\varepsilon_{k+} \geq \frac{\sum_{j=1}^{k} D^*_{1j}}{\sum_{j=k+1}^{n} D_{1j}} + 1 \qquad (14)$$

$$\varepsilon_{l+} \geq \frac{\sum_{j=1}^{l} D^*_{2j}}{\sum_{j=l+1}^{n} D_{2j}} + 1 \qquad (15)$$

Then, for any pricing subgame equilibrium involving $(m \times n)$ potential bundles in the market, the following relationship between marginal taste parameters will hold:

$$\bar{\theta} > 2\theta_{1k/21} > 4Max(\theta_{2l/31}, \theta_{2l/1k+1}) > 4\theta_{m-1j/m1} \text{ for any } j = 1,2,...,n \qquad (16)$$

Proof: see Appendix A

To make the sense of expressions (14) and (15) more intuitive, note that the price-elasticities of their left-hand sides are both decreasing in the quality differences $(d_{1j/i'j'})$ between the bundles A*1*B*j* for $j = k+1,...,n$ and their neighbors by quality rank:

$$\varepsilon_{k+} = \left| \frac{\partial \sum_{j=k+1}^{n} D_{1j}}{\partial p_1^A} \right| \frac{p_1^A}{\sum_{j=k+1}^{n} D_{1j}} = \left| \sum_{j=k+1}^{n} \left(-\frac{1}{d_{1j/i'j'}} \right) \right| \frac{p_1^A}{\sum_{j=k+1}^{n} D_{1j}} \qquad (17)$$

$$\varepsilon_{l+} = \frac{\partial \sum_{j=l+1}^{n} D_{2j}}{\partial p_2^A} \frac{p_2^A}{\sum_{j=l+1}^{n} D_{2j}} = \left| \sum_{j=l+1}^{n} \left(-\frac{1}{d_{2j/i'j'}} \right) \right| \frac{p_2^A}{\sum_{j=l+1}^{n} D_{2j}} \qquad (18)$$

Hence, as smaller is the quality differential between the (*n*-*k*) bundles based on A1 that are worse than A2B1, the more likely it would be condition (14) to hold. Similarly, as smaller is the quality differential between the (*n*-*l*) bundles based on A2 that are worse than the superior of A3B1 and A1Bk+1, the more likely it would be for the inequality in (15) to be valid. To generalize, conditions (14) and (15) imply that low-quality bundles must differ less distinctly each from another than high-quality bundles which is in fact a reasonable assumption. Usually, high-quality innovative goods are harder to imitate than low-quality generic products. As a result the competition between the less distinguishable bundles based on the same good of type A but containing lower-quality goods of type B is stronger and their demand is more price-sensitive.

In other words, given that conditions (14) and (15) hold, to make both top-quality and bottom-quality bundles based on a good of type A salable, the price of that good must be lower than in the case when only the top-quality bundles have positive market share.

Respectively, imagine that profit maximization requires from the producer of a good of type A to charge it a (lower) price at which not only the top-quality but also the bottom-quality bundles based on this good would have positive demand. Then, since the demand function is strictly decreasing in price, the demand for the top-quality bundles at this (low) equilibrium price cannot be smaller than the same demand at any other (high) price at which only the top-quality bundles are sold. Therefore, when conditions (14) and (15) hold, the values of the marginal taste parameters that define the borders of the demand for the *k* top-quality bundles based on good A1 and the *l* top-quality bundles based on good A2 cannot get any closer each to another than the minimal difference between them given by the inequality in (16).

Straight from the expression in (16), a restriction could be imposed on the market size so that at most goods A1 and A2 would have a positive market share in market A at equilibrium:

$$\underline{\theta} \geq \frac{\overline{\theta}}{4} \tag{19}$$

This result replicates exactly the respective condition derived for single market by Shaked and Sutton (1982). However, to restrict the equilibrium number of bundles with positive market share to maximum two, it is not sufficient just to restrict down to two the number of the goods in market A. In the case with two connected markets considered here, the restriction of the maximal equilibrium number of goods in market A to two still allows to form up to (2xn) bundles with the n goods in market B. Hence, as long as $n > 1$, more restrictive condition than (19) needs to be introduced for at most two bundles to have positive market share in equilibrium. Furthermore, the larger is the number of the connected markets, the stronger should also be the required restriction on the market size. The relationship between the number of the goods in a bundle and the degree of restrictiveness of the corresponding condition on market size from below is rigorously stated in the proposition that follows:

Proposition 2. Let X be the number of the markets for complementary goods forming a set of bundles satisfying the requirements of proposition 1 adjusted for X markets. Then, for any pricing subgame equilibrium involving all possible bundles that can be formed from the goods in the market, at most two bundles (the top two) will have a positive market share at equilibrium as long as the following condition on market size holds[8]:

$$\underline{\theta} \geq \sum_{x=1}^{X} \frac{1}{2^{x+1}} \bar{\theta} \qquad (20)$$

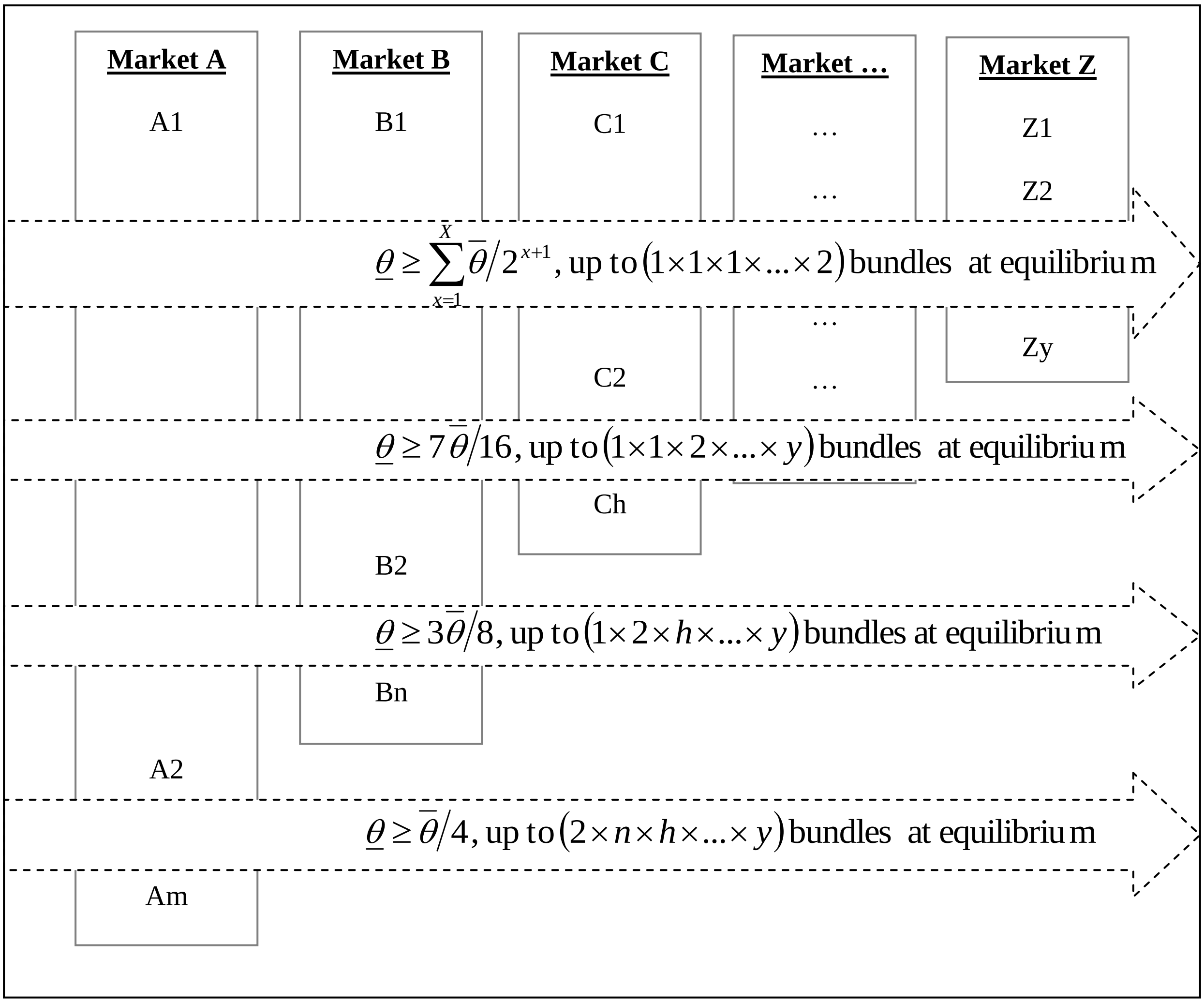
Proof: see Appendix B

The economic intuition that stands behind proposition 2 is illustrated in figure 1 below. The larger is the number of connected markets, the larger would also be the number of possible combinations in which the goods sold in these markets could be bundled. Respectively, the only way the number of salable bundles could be restricted down to two is to decrease the number of goods with positive market share in equilibrium, so that there are at most two combinations in which they could be bundled. Particularly, this implies that the restrictive condition on market size must be such that in one market at most two goods could have a positive market share while on all the rest of the markets only a single good could be sold in equilibrium. Then, the two combinations that could be sold at most will differ only by the good they contain from the former (duopoly) market while the goods from the rest of the (monopoly) markets will be the same for any of the bundles.

[8] Note that the right-hand side has a limit of $\frac{1}{2}$ as the number of goods in a bundle tends to infinity: $\lim_{X\to\infty} \sum_{x=1}^{X} \frac{1}{2^{x+1}} = \sum_{x=1}^{\infty} \frac{1}{2^{x+1}} = \frac{1}{2}$.

On figure 1, the restrictive conditions on market size are depicted by arrows. Each arrow divides the goods in each market into two groups. The goods under the arrow are the ones that have no chance to have positive demand in equilibrium if the condition by which the arrow is labeled holds. The goods above the arrow are the ones that could have positive demand in equilibrium if the condition in the arrow by which the arrow is labeled holds. The higher is an arrow, the more restrictive is the condition by which it is labeled.

Figure 1: Relationship between the number of the connected markets and the degree of restrictiveness of the condition on market size for at most two bundles to have positive markets shares in equilibrium



Note that the larger is the number of the markets the higher needs to be the arrow corresponding to the condition which must hold in order up to two bundles to be salable in equilibrium. This is because more markets are required to have only a single good i.e. monopoly at equilibrium.

For example, when there is only a single independent market A, the condition suggested by Shaked and Sutton (1982) is sufficient because it is not required for the market to have monopoly structure. Duopoly is also acceptable in market A.

When there are two connected markets, A and B, the condition on size of an independent market as derived by Shaked and Sutton (1982) is too weak and cannot reduce the equilibrium number of bundles to maximum of two. Duopoly in market A would imply up to $(2 \times n)$ possible combinations in which the goods with positive chance to be sold at equilibrium could be bundled. Ignoring market A to consider only market B as independent would imply the incorrect expectation that at most two goods could have positive market share in equilibrium. In fact, their number could be up to n but n might be larger than two. Therefore, the maximum of two bundles requires more restrictive condition. Namely, this is the condition by which is labeled the next arrow from above. It only allows for monopoly structure of market A while at most duopoly could still occur in market B.

If there are three connected markets, that is a good of market C enters a bundle together with its complements in markets A and B, also the size of market B should be further restricted within a range corresponding to a monopoly structure. Thus, with each additional connected market X, more and more markets are needed to have a monopoly structure in order maximum of two bundles to be salable at equilibrium. In turn, a more restrictive condition on market size must hold which corresponds to a higher-leveled arrow on figure 1.

In what follows, the case of two markets, A and B, will be considered with sizes assumed to satisfy the condition in (20) for X=2 i.e. $\underline{\theta} \geq \frac{3\overline{\theta}}{8}$. It remains to show how the equilibrium outcome in the pricing stage changes compared to the particular case of single independent market with vertical differentiation described by Shaked and Sutton (1982).

The critical factor for the difference in equilibrium outcomes between the case with one independent market and the case of two markets for complementary products is the implication of condition (20) for having monopoly in market A when it is connected with market B.

Given only one entrant in market A, the standard equilibrium outcome in case of monopoly suggests that the market cannot be covered as long as its size exceeds the half of its upper bound i.e. $\underline{\theta} < \frac{\overline{\theta}}{2}$. In other words, the single entrant in market A is better-off of charging its good a high price at which only the high-taste consumers are willing to purchase it than to serve all consumers at a low price.

Then, however, low-taste consumers who find the good in market A too expensive for them to afford buying it would lose incentive to do shopping in market B as well. This is because they have positive utility of good B only conditional on using it in combination with good A. So, market B would also be uncovered at equilibrium. Moreover, as stated in proposition 3 below, the resulting competitive pressure on good B2 would be so strong that it would not be able to sustain positive market share even by pricing at a cost.

Proposition 3. Let the condition of proposition 1 holds and let the common size of markets A and B satisfy the restriction $\frac{3\overline{\theta}}{8} \leq \underline{\theta} < \frac{\overline{\theta}}{2}$. Then, at equilibrium only the best good in each market, A1 in market A and B1 in market B, will have positive market shares but market will not be covered by them.

Proof: see Appendix C

As a final outcome, good B2 is driven out from the market and only B1 is sold in market B. So, both markets, A and B, would be uncovered and have a monopoly structure at equilibrium. This outcome differs from the equilibrium result in case of a single independent market. As demonstrated by Shaked and Sutton (1982) if market B is independent, both goods B1 and B2 will have positive market shares, gain positive profits and serve all consumers at equilibrium.

The subgame equilibrium of the quality-choice stage is derived explicitly in appendix D. There it is shown that the profits of both A1 and B1 are strictly increasing in their qualities. So, the optimal quality choices are given by the corner solution at which A1 and B1 are assigned the highest possible quality values, $\overline{F}$ and $\overline{G}$, respectively. All the rest

$\left(m+n-2\right)$ potential entrants gain zero revenue (i.e. negative profits of $-\varepsilon$) independent of their quality choice. Therefore, it is optimal for them not to enter the market in the first stage.

The conclusion is that at the particular perfect subgame equilibrium corresponding to the backward solution presented in the paper, only the firms offering goods A1 and B1 enter their markets, after which they assign their products the best qualities and charge them the monopoly price. In turn, the market is not covered, that is the low-taste consumers find both goods prohibitively expensive and therefore prefer to abstain from them than to pay their high monopoly prices.

4. Conclusion

In the presented paper an extended model of Shaked and Sutton (1982) is introduced as tool for analysis of the competition outcome in a market for vertically differentiated goods. What makes this market distinct from the one considered originally is that its goods are complements to goods sold in other markets. As a result, the decisions firms make in a market are not independent but influenced by the decisions of the firms in the rest of the related markets. Respectively, two key differences are identified in the equilibrium outcome.

First, since each good has one or more complements its demand depends not only on its price and quality but also on the prices and qualities of the other goods with which it could be consumed. Therefore, the condition on market size derived by Shaked and Sutton (1982), for duopoly at equilibrium might not in fact restrict down to two the equilibrium number of the bundles. For instance, in a pair of related markets with two goods in each, it is possible up to four bundles to have a positive market share at equilibrium. Therefore, to decrease their number down to two, the mutual size of the markets must satisfy a more restrictive condition at which all markets but one have a monopoly structure with just a single good sold at equilibrium.

Second, as long as at least one of the related markets has a monopoly structure and its size exceeds half of its upper bound, this market would not be covered at its pricing stage equilibrium. Since the good sold in this market is a complement to the goods sold in other related market, however, the latter cannot be covered at equilibrium, either. Furthermore, resulting competitive pressure would turn the second market to monopoly. This equilibrium outcome is very different from the one described by Shaked and Sutton (1982) for a single independent market where two goods would have positive market share, gain positive profits and cover the whole market at equilibrium.

The analysis presented in the paper could be continued in a further research on the topic. Particularly, an interesting question to be answered is whether equilibrium would exist at which two firms would enter in each market given a weaker restriction on the market size. Accordingly, it would be reasonable to check the implications of this eventually optimal outcome for the market coverage. The issue is addressed in a separate paper included as second chapter of the present thesis.

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Appendix A: Proof of proposition 1

The proof of proposition 1 follows the same logic as the proof of lemma 1 in Shaked and Sutton (1982).

To present the intuition behind conditions (14) and (15), it is useful to start with the proof of the result in (16) at a more restrictive assumption which then will be gradually relaxed to the condition of proposition 1. Particularly, it is necessary the quality ranking of bundles to be such that conditions for profit-maximization do not differ much from the profit-maximization conditions in the case with a single independent market. A straightforward way to achieve that is simply to assume that the qualities of the bundles are lexicographically ordered[9], that is:

$$\chi_{in} > \chi_{i+11}, \text{ for any } i = 1,...,m-1 \qquad \text{(i)}$$

Given the assumption for CES aggregation of good qualities, the condition in (i) could be represented in terms of good qualities as follows:

$$\left[f(A_i)^r + g(B_n)^r\right]^{\frac{1}{r}} \geq \left[f(A_{i+1})^r + g(B_1)^r\right]^{\frac{1}{r}} \qquad \text{(ii)}$$

Respectively, the profit-maximization problems of the firms selling goods of type A are:

$$\max_{p_1^A} p_1^A \sum_{j=1}^{n} D_{1j} = p_1^A\left(\overline{\theta} - \theta_{1n/21}\right) = p_1^A\left(\overline{\theta} - \frac{\left(p_1^A + p_n^B\right) - \left(p_2^A + p_1^B\right)}{d_{1n/21}}\right) \qquad \text{(iii)}$$

$$\max_{p_2^A} p_2^A \sum_{j=1}^{n} D_{2j} = p_2^A\left(\theta_{1n/21} - \theta_{2n/31}\right) = p_2^A\left(\frac{\left(p_1^A + p_n^B\right) - \left(p_2^A + p_1^B\right)}{d_{1n/21}} - \frac{\left(p_2^A + p_n^B\right) - \left(p_3^A + p_1^B\right)}{d_{2n/31}}\right)$$

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$$\max_{p_m^A} p_m^A \sum_{j=1}^{n} D_{mj} = p_m^A\left(\theta_{m-1n/m1} - \underline{\theta}\right) = p_m^A\left(\frac{\left(p_{m-1}^A + p_n^B\right) - \left(p_m^A + p_1^B\right)}{d_{m-1n/m1}} - \underline{\theta}\right)$$

[9] There are markets where this is a reasonable assumption. For instance, take the introductory example with the markets for toothpastes and toothbrushes. Let consumers have a choice between electric toothbrush bundled with high-abrasive foamy toothpaste and manual toothbrush with low-abrasive dry toothpaste, both sold at the same price. They would definitely go for the low-abrasive toothpaste because it has lower risk to harm their teeth no matter how more efficient is power brushing compared to manual one.

The corresponding first-order conditions for optimality are given by the following system of equations:

$$\bar{\theta} - \frac{2p_1^A + p_n^B - p_2^A - p_1^B}{d_{1n/21}} = 0 \qquad \text{(iv)}$$

$$\frac{p_1^A + p_n^B - 2p_2^A - p_1^B}{d_{1n/21}} - \frac{2p_2^A + p_n^B - p_3^A - p_1^B}{d_{2n/31}} = 0$$

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$$\frac{p_{m-1}^A + p_n^B - 2p_m^A - p_1^B}{d_{m-1n/m1}} - \underline{\theta} = 0$$

which after further simplification in the context of Shaked and Sutton (1982) would look as follows:

$$\bar{\theta} = 2\theta_{1n/21} + \frac{p_2^A + p_1^B - p_n^B}{d_{1n/21}} \qquad \text{(v)}$$

$$\theta_{1n/21} = 2\theta_{2n/31} + \frac{p_3^A + p_1^B - p_n^B}{d_{2n/31}} + \frac{p_2^A}{d_{1n/21}}$$

………………………………….

$$\theta_{m-2n/m-11} = 2\theta_{m-1n/m1} + \frac{p_m^A + p_1^B - p_n^B}{d_{m-1n/m1}} + \frac{p_{m-1}^A}{d_{m-2n/m-11}}$$

$$\theta_{m-1n/m1} = \underline{\theta} + \frac{p_m^A}{d_{m-1n/m1}}$$

From (v) the following relation between the marginal taste parameters can be derived:

$$\bar{\theta} > 2\theta_{1n/21} > 4\theta_{2n/31} > ... > 2^{m-1}\theta_{m-1n/m1} \qquad \text{(vi)}$$

Since $2^{m-1} > 4$ for any $m > 2$, the result in (vi) complies with the inequality in (16).

Note, however, that in order for the inequality in (16) to hold it is not necessary the qualities of all bundles to be lexicographically ordered. It is sufficient the lexicographical ordering to hold for just the bundles based on A1 and A2, as required by the following condition:

$$\chi_{1n} > \chi_{21} \text{ and } \chi_{2n} > \chi_{31} \qquad \text{(vii)}$$

As long as A1Bn is better than A2B1 and A2Bn is better than A3B1, the first two equations of (v) would still hold which means that following part of (vi) is valid:

$$\bar{\theta} > 2\theta_{1n/21} > 4\theta_{2n/31} \qquad \text{(vii)}$$

Next, note that whatever is the bundle quality ranking always the bundle of the best ranked goods of both types, A1B1, will have the top quality. Also, the best-quality bundles containing respectively A2 and A3 should always be A2B1 and A3B1.

Respectively, the lowest-quality bundle containing A1 should always be A1Bn while the worst-quality bundle containing A2 should be respectively A2Bn.

Therefore, due to the expectation all bundles to have demand at unrestricted market, the following inequality must hold at equilibrium:

$$\theta_{2n/31} > \theta_{3j/4j'} > ... > \theta_{m-1j/mj'}, \; j = 1,...,n; \; j' = 1,...,n; \; j \neq j' \qquad \text{(viii)}$$

Taken together, conditions (vii) and (viii) imply directly the inequality in (16).

Now, the last stage of the relaxation procedure would be to allow for even the bundles based on A1 and A2 not to have lexicographically ordered qualities and show that at the conditions of proposition 1 still the logic behind the result in (16) holds.

Note that for any possible ranking of the bundle qualities, there will always be a set of more than one bundle based on A1 which are better than A2B1 the best bundle containing A2[10]. In proposition 1, the number of the bundles in the set is given by the parameter *k*.

Similarly, for any possible ranking of the bundles by quality, there will always be a set of more than one bundle based on A2 which are better than the higher ranked between A3B1 - the best bundle containing A3, and A1Bk+1 - the best of the bundles based on A1 which are worse than A2B1. In proposition 1, the number of the bundles in the set is given by the parameter *l*.

The corresponding ranking of the top $(k+l)$ bundles is given by the following expression:

$$\chi_{11} > \chi_{12} > ... > \chi_{1k} > \chi_{21} > \chi_{22} > ... > \chi_{2l} > Max(\chi_{31}, \chi_{1k+1}) \qquad \text{(ix)}$$

Note that as long as only the best *k* bundles based on A1 are sold, the first-order optimality condition for profit maximization of A1 would not change much, just *n* in the indices of the first equation of (v) will be replaced by *k*:

[10] One might argue that even A1B2 could be worse than A2B1. However, note that in that case A and B will just change their roles in defining bundles' qualities. So, again there will be a set of more than one bundle this time based on B1 which are better than A1B2, the best bundle based on B2.

$$\sum_{j=1}^{k} D_{1j}^{*} + p_1^{A*}\frac{\partial \sum_{j=1}^{k} D_{1j}^{*}}{\partial p_1^{A*}} = \left(\bar{\theta} - 2\theta_{1k/21} - \frac{p_2^A + p_1^B - p_k^B}{d_{1k/21}}\right) = 0 \qquad \text{(x)}$$

Similarly, as long as only the best *l* bundles based on A2 are sold, *n* in the indices of the second equation of (v) will be replaced by *l*.

$$\begin{cases} \sum_{j=1}^{l} D_{2j}^{*} + p_2^{A*}\frac{\partial \sum_{j=2}^{l} D_{2j}^{*}}{\partial p_2^{A*}} = \left(\theta_{1k/21} - 2\theta_{2l/31} - \frac{p_3^A + p_1^B - p_l^B}{d_{2l/31}}\right) = 0, \text{given } \chi_{31} > \chi_{1k+1} \\ \sum_{j=1}^{l} D_{2j}^{*} + p_2^{A*}\frac{\partial \sum_{j=2}^{l} D_{1j}^{*}}{\partial p_2^{A*}} = \left(\theta_{1k/21} - 2\theta_{2l/1k+1} - \frac{p_1^A + p_{k+1}^B - p_l^B}{d_{2l/1k+1}}\right) = 0, \text{given } \chi_{31} < \chi_{1k+1} \end{cases} \qquad \text{(xi)}$$

In the expressions above, demands and prices are asterisked to distinguish them from the case when all bundles based on A1 and A2 are sold.

Note also that after simplification both (x) and (xi) imply that the price elasticity of demand (labeled below by $\varepsilon_I, I = k, l$) must be unit at equilibrium:

$$\varepsilon_k = \frac{p_1^{A*}}{\sum_{j=1}^{k} D_{1j}^{*}}\left|\frac{\partial \sum_{j=1}^{k} D_{1j}^{*}}{\partial p_1^{A*}}\right| = 1 \qquad \text{(xii)}$$

$$\varepsilon_l = \frac{p_2^{A*}}{\sum_{j=1}^{l} D_{2j}^{*}}\left|\frac{\partial \sum_{j=1}^{l} D_{2j}^{*}}{\partial p_2^{A*}}\right| = 1 \qquad \text{(xiii)}$$

Now, let's consider how the first-order optimality conditions change when all *n* bundles based on A1 and A2 are sold. The sum of the demands for the bundles that are not among the best *k*, respectively *l*, needs to be added to the profit functions of A1 and A2. Since its first derivative gives the right-hand side of (x) and (xi), these equations now should change as follows:

$$\sum_{j=1}^{k} D_{1j} + \sum_{j=k+1}^{n} D_{1j} + p_1^A\left(\frac{\partial \sum_{j=1}^{k} D_{1j}}{\partial p_1^A} + \frac{\partial \sum_{j=k+1}^{n} D_{1j}}{\partial p_1^A}\right) = 0 \qquad \text{(xiv)}$$

$$\sum_{j=1}^{l} D_{2j} + \sum_{j=l+1}^{n} D_{2j} + p_2^A \left(\frac{\partial \sum_{j=1}^{l} D_{2j}}{\partial p_1^A} + \frac{\partial \sum_{j=k+1}^{n} D_{2j}}{\partial p_1^A} \right) = 0 \qquad \text{(xv)}$$

Next, note that the first derivative of the sum of demand functions for the top *k* bundles based on A1, with respect to price does not change when more goods are sold:

$$\frac{\partial \sum_{j=1}^{k} D_{1j}}{\partial p_1^A} = \frac{\partial \sum_{j=1}^{k} D_{1j}^*}{\partial p_1^{A*}} = -\frac{1}{d_{1k/21}} \qquad \text{(xvi)}$$

The same hold for the first derivative of the sum of demand functions for the top *l* bundles based on A2 with respect to price when more goods are sold:

$$\begin{cases} \dfrac{\partial \sum_{j=1}^{l} D_{2j}}{\partial p_2^A} = \dfrac{\partial \sum_{j=1}^{l} D_{2j}^*}{\partial p_2^{A*}} = -\dfrac{1}{d_{1k/21}} - \dfrac{1}{d_{2l/31}}, \text{given } \chi_{31} > \chi_{1k+1} \\ \dfrac{\partial \sum_{j=1}^{l} D_{2j}}{\partial p_2^A} = \dfrac{\partial \sum_{j=1}^{l} D_{2j}^*}{\partial p_2^{A*}} = -\dfrac{1}{d_{1k/21}} - \dfrac{1}{d_{2l/1k+1}}, \text{given } \chi_{31} < \chi_{1k+1} \end{cases} \qquad \text{(xvii)}$$

After taking all the observations from above into account, (xiv) can be simplified to an expression for the ratio between the sums of demands for the top *k* bundles based on A1 when all goods have positive demand versus the case when only the *k* best of them are sold at equilibrium:

$$\sum_{j=1}^{k} D_{1j} \Big/ \sum_{j=1}^{k} D_{1j}^* = \frac{p_1^A}{p_1^{A*}} + \sum_{j=k+1}^{n} D_{1j}\varepsilon_{k+} \Big/ \sum_{j=1}^{k} D_{1j}^*\varepsilon_k - \sum_{j=k+1}^{n} D_{1j} \Big/ \sum_{j=1}^{k} D_{1j}^* \qquad \text{(xviii)}$$

Similar expression could be derived from (xv) for the corresponding ratio of the sums of demands for the top *l* bundles based on A2:

$$\sum_{j=1}^{l} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^* = \begin{cases} \dfrac{p_2^A}{p_2^{A*}} + \sum_{j=l+1}^{n} D_{2j}\varepsilon_{l+} \Big/ \sum_{j=1}^{l} D_{2j}^*\varepsilon_l - \sum_{j=l+1}^{n} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^*, \text{given } \chi_{31} > \chi_{1k+1} \\ \dfrac{p_2^A}{p_2^{A*}} \dfrac{\left| \dfrac{\partial \sum_{j=1}^{l} D_{2j}}{\partial p_2^A} \right|}{\left| \dfrac{\partial \sum_{j=1}^{l} D_{2j}^*}{\partial p_2^{A*}} \right|} + \sum_{j=l+1}^{n} D_{2j}\varepsilon_{l+} \Big/ \sum_{j=1}^{l} D_{2j}^*\varepsilon_l - \sum_{j=l+1}^{n} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^*, \text{given } \chi_{31} < \chi_{1k+1} \end{cases} \qquad \text{(xix)}$$

Substituting for the expressions of elasticities as they are assumed in (14) and (15), yields the following inequalities:

$$\sum_{j=1}^{k} D_{1j} \Big/ \sum_{j=1}^{k} D_{1j}^{*} \geq \frac{p_1^A}{p_1^{A*}} + 1 + \sum_{j=k+1}^{n} D_{1j} \Big/ \sum_{j=1}^{k} D_{1j}^{*} - \sum_{j=k+1}^{n} D_{1j} \Big/ \sum_{j=1}^{k} D_{1j}^{*} > 1 \qquad \text{(xx)}$$

$$\sum_{j=1}^{l} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^{*} \geq \begin{cases} \dfrac{p_2^A}{p_2^{A*}} + 1 + \sum_{j=l+1}^{n} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^{*} - \sum_{j=l+1}^{n} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^{*} > 1, \text{given } \chi_{31} > \chi_{1k+1} \\ \dfrac{p_2^A}{p_2^{A*}} \dfrac{\left| \dfrac{\partial \sum_{j=1}^{l} D_{2j}}{\partial p_2^A} \right|}{\left| \dfrac{\partial \sum_{j=1}^{l} D_{2j}^{*}}{\partial p_2^{A*}} \right|} + 1 + \sum_{j=l+1}^{n} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^{*} - \sum_{j=l+1}^{n} D_{2j} \Big/ \sum_{j=1}^{l} D_{2j}^{*} > 1, \text{given } \chi_{31} < \chi_{1k+1} \end{cases} \qquad \text{(xxi)}$$

After representing the demands in explicit form, the above results look as follows:

$$\bar{\theta} - \theta_{1k/21} > \bar{\theta} - \theta_{1k/21}^{*} > \theta_{1k/21}^{*} > \theta_{1k/21} \text{ i.e. } \bar{\theta} > 2\theta_{1k/21} \qquad \text{(xxii)}$$

$$\begin{cases} \theta_{1k/21} - \theta_{2l/31} > \theta_{1k/21}^{*} - \theta_{2l/31}^{*} > \theta_{2l/31}^{*} > \theta_{2l/31} \text{ i.e. } \theta_{1k/21} > 2\theta_{2l/31}, \text{given } \chi_{31} > \chi_{1k+1} \\ \theta_{1k/21} - \theta_{2l/1k+1} > \theta_{1k/21}^{*} - \theta_{2l/1k+1}^{*} > \theta_{2l/2l/1k+1}^{*} > \theta_{2l/2l/1k+1} \text{ i.e. } \theta_{1k/21} > 2\theta_{2l/1k+1}, \text{given } \chi_{31} < \chi_{1k+1} \end{cases} \qquad \text{(xxiii)}$$

Combined (xxii) and (xxiii) imply directly the relationship in (16). So, the proof of proposition 1 is complete.

Appendix B: Proof of proposition 2

Proposition 2 can be proven by induction. In what follows, the proof is presented for the case when the qualities of bundles are assumed to be lexicographically ordered. However, it is straightforward to show that it also holds for the relaxed assumption of proposition 1.

Let's introduce a third good $C_z, z = 1,...,h$. Given the bundle quality ranking of (i), the profit-maximization problems of the firms producing goods of type A would look as follows:

$$\underset{p_1^A}{Max}\, p_1^A\left(\bar{\theta} - \frac{p_1^A + p_n^B + p_h^C - p_2^A - p_1^B - p_1^C}{d_{1nh/211}}\right) \qquad \text{(xxiv)}$$

$$\underset{p_2^A}{Max}\, p_2^A\left(\frac{p_1^A + p_n^B + p_h^C - p_2^A - p_1^B - p_1^C}{d_{1nh/211}} - \frac{p_2^A + p_n^B + p_h^C - p_3^A - p_1^B - p_1^C}{d_{2nh/311}}\right)$$

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$$\underset{p_{m-1}^A}{Max}\, p_{m-1}^A\left(\frac{p_{m-2}^A + p_n^B + p_h^C - p_{m-1}^A - p_1^B - p_1^C}{d_{m-2nh/m-111}} - \frac{p_{m-1}^A + p_n^B + p_h^C - p_m^A - p_1^B - p_1^C}{d_{m-1nh/m11}}\right)$$

$$\begin{cases} \underset{p_m^A}{Max}\, p_m^A\left(\dfrac{p_{m-1}^A + p_n^B + p_h^C - p_m^A - p_1^B - p_1^C}{d_{m-1nh/m11}} - \underline{\theta}\right), \text{if } \underline{\theta} \geq \theta_{mnh/O} \\ \underset{p_m^A}{Max}\, p_m^A\left(\dfrac{p_{m-1}^A + p_n^B + p_h^C - p_m^A - p_1^B - p_1^C}{d_{m-1nh/m11}} - \dfrac{p_m^A + p_n^B + p_h^C}{d_{mnh/O}}\right), \text{if } \underline{\theta} \leq \theta_{mnh/O} \end{cases}$$

Respectively, the first-order optimality conditions are given by the system of equations (xxv) below:

$$\bar{\theta} = 2\theta_{1nh/211} + \frac{p_2^A + p_1^B + p_1^C - p_n^B - p_h^C}{d_{1nh/211}} \qquad \text{(xxv)}$$

$$\theta_{1nh/211} = 2\theta_{2nh/311} + \frac{p_3^A + p_1^B + p_1^C - p_n^B - p_h^C}{d_{2nh/311}} + \frac{p_2^A}{d_{1nh/211}}$$

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$$\theta_{m-2nh/m-111} = 2\theta_{m-1nh/m11} + \frac{p_m^A + p_1^B + p_1^C - p_n^B - p_h^C}{d_{m-1nh/m11}} + \frac{p_{m-1}^A}{d_{m-2nh/m-111}}$$

Hence, the following expression for the relation between the corresponding taste parameters could be derived:

$$\bar{\theta} > 2\theta_{1nh/211} > 4\theta_{2nh/311} > ... > 2^{m-1}\theta_{m-1nh/m11} \qquad \text{(xxvi)}$$

Note that the same condition which is sufficient $\left(\underline{\theta} \geq \frac{\bar{\theta}}{4}\right)$ for at most two goods to have positive demand at equilibrium in a single-good market, allows for up to $(2 \times n)$ bundles to be sold in a market for two-good bundles and for up to $(2 \times n \times h)$ bundles in a market for three-good bundles - $2 \times n \times h > 2 \times n > 2, \text{for } n, h > 1$. As the number of goods in a bundle rises, the number of bundles in which a good could take part is also increasing. This implies a more restrictive condition on the range of consumer tastes to hold for ensuring positive market share of at most 2 bundles.

Given that at most A1 and A2 could be sold in equilibrium, the profit-maximization problems of the firms selling goods of type B in three-good market would look as follows:

$$\underset{p_1^B}{Max}\, p_1^B\left[\left(\bar{\theta} - \frac{p_1^B + p_h^C - p_2^B - p_1^C}{d_{11h/121}}\right) + \left(\frac{p_1^A + p_n^B + p_h^C - p_2^A - p_1^B - p_h^C}{d_{1nh/211}} - \frac{p_1^B + p_h^C - p_2^B - p_1^C}{d_{21h/221}}\right)\right] \quad \text{(xxvii)}$$

$$\underset{p_2^B}{Max}\, p_2^B\left[\left(\frac{p_1^A + p_1^B + p_h^C - p_1^A - p_2^B - p_1^C}{d_{11h/121}} - \frac{p_1^A + p_2^B + p_h^C - p_1^A - p_3^B - p_1^C}{d_{12h/131}}\right) + \right.$$

$$\left. + \left(\frac{p_2^A + p_1^B + p_h^C - p_2^A - p_2^B - p_1^C}{d_{21h/221}} - \frac{p_2^A + p_2^B + p_h^C - p_2^A - p_3^B - p_1^C}{d_{22h/231}}\right)\right]$$

……………………………………………………………………………………………

$$\underset{p_{n-1}^B}{Max}\, p_{n-1}^B\left[\left(\frac{p_{n-2}^A + p_h^B - p_{n-1}^B - p_1^C}{d_{1n-2h/1n-11}} - \frac{p_{n-1}^A + p_h^B - p_n^B - p_1^C}{d_{1n-1h/1n1}}\right) + \left(\frac{p_{n-2}^A + p_h^B - p_{n-1}^B - p_1^C}{d_{2n-2h/2n-11}} - \frac{p_{n-1}^A + p_h^B - p_n^B - p_1^C}{d_{2n-1h/2n1}}\right)\right]$$

$$\underset{p_n^B}{Max}\, p_n^B\left[\left(\frac{p_{n-1}^B + p_h^C - p_n^B - p_1^C}{d_{1n-1h/1n1}} - \frac{p_n^B + p_h^C - p_1^B - p_1^C}{d_{1nh/211}}\right) + \left(\frac{p_{n-1}^B + p_h^C - p_n^B - p_1^C}{d_{2n-1h/2n1}} - \frac{p_2^A + p_n^B + p_h^C - p_3^A - p_1^B - p_1^C}{d_{2nh/311}}\right)\right]$$

The corresponding system of the first-order optimality conditions looks as follows:

$$\bar{\theta} + \theta_{1nh/211} = 2\theta_{11h/121} + 2\theta_{21h/221} + \left(p_2^B + p_1^C - p_h^C\right)\left(\frac{1}{d_{11h/121}} + \frac{1}{d_{21h/221}}\right) + \frac{p_1^B}{d_{1nh/211}} \quad \text{(xxviii)}$$

$$\theta_{11h/121} + \theta_{21h/221} = 2\theta_{12h/131} + 2\theta_{22h/231} + \left(p_3^B + p_1^C - p_h^C\right)\left(\frac{1}{d_{12h/131}} + \frac{1}{d_{22h/231}}\right) + p_2^B\left(\frac{1}{d_{11h/121}} + \frac{1}{d_{21h/221}}\right)$$

…………………………………………………………………………………………..

$$\theta_{1n-2h/1n-11} + \theta_{2n-2h/2n-11} = 2\theta_{1n-1h/1n1} + 2\theta_{2n-1h/2n1} + \left(p_n^B + p_1^C - p_h^C\right)\left(\frac{1}{d_{1n-1h/1n1}} + \frac{1}{d_{2n-1h/2n1}}\right) + p_{n-1}^B\left(\frac{1}{d_{1n-2h/1n-11}} + \frac{1}{d_{2n-2h/2n-11}}\right)$$

The optimality results in (xxvi) and (xxviii) imply the following relationship between taste parameters:

$$\frac{3}{2}\bar{\theta} > 2\theta_{11h/121} + 2\theta_{21h/221} > 4\theta_{12h/131} + 4\theta_{22h/231} > ... > 2^{n-1}\theta_{1n-1h/1n1} + 2^{n-1}\theta_{2n-1h/2n1} \quad \text{(xxix)}$$

Apparently, even the condition $\left(\underline{\theta} \geq \frac{3}{8}\bar{\theta}\right)$ which in a market for two-good bundles is sufficient for at most two bundles to have positive demand in equilibrium allows for up to $(2\times h)$ bundles to be sold in a market for three-good bundles. Therefore, for at most two bundles to be demanded at the equilibrium prices, stronger restriction is required as shown in the analysis of profit-maximization of the firms offering goods of type C. Their respective optimization problems look as follows:

$$\underset{p_1^C}{Max}\, p_1^C\left[\left(\bar{\theta} - \frac{p_1^C - p_2^C}{d_{111/112}}\right) + \left(\frac{p_2^B + p_h^C - p_2^B - p_1^C}{d_{11h/121}} - \frac{p_1^C - p_2^C}{d_{121/122}}\right)\right] \quad \text{(xxx)}$$

$$\underset{p_2^C}{Max}\, p_2^C\left[\left(\frac{p_1^C - p_2^C}{d_{111/112}} - \frac{p_2^C - p_3^C}{d_{112/113}}\right) + \left(\frac{p_1^C - p_2^C}{d_{121/122}} - \frac{p_2^C - p_3^C}{d_{122/123}}\right)\right]$$

……………………………………………………………………..

$$\underset{p_{h-1}^C}{Max}\, p_{h-1}^C\left[\left(\frac{p_{h-2}^C - p_{h-1}^C}{d_{11h-2/11h-1}} - \frac{p_{h-1}^C - p_h^C}{d_{11h-1/11h}}\right) + \left(\frac{p_{h-2}^C - p_{h-1}^C}{d_{12h-2/12h-1}} - \frac{p_{h-1}^C - p_h^C}{d_{12h-1/12h}}\right)\right]$$

$$\underset{p_h^C}{Max}\, p_h^C\left[\left(\frac{p_{h-1}^C - p_h^C}{d_{11h-1/11h}} - \frac{p_1^B + p_h^C - p_2^B - p_1^C}{d_{11h/121}}\right) + \left(\frac{p_{h-1}^C - p_h^C}{d_{12h-1/12h}} - \frac{p_2^B + p_h^C - p_3^B - p_1^C}{d_{12h-1/12h}}\right)\right]$$

The first-order optimality conditions could be represented by the following system of equations:

$$\bar{\theta} + \theta_{11h/121} = 2\theta_{111/112} + 2\theta_{121/122} + p_2^C\left(\frac{1}{d_{111/112}} + \frac{1}{d_{121/122}}\right) + \frac{p_1^C}{d_{11h/121}} \quad \text{(xxxi)}$$

$$\theta_{111/112} + \theta_{121/122} = 2\theta_{112/113} + 2\theta_{122/123} + p_3^C\left(\frac{1}{d_{112/113}} + \frac{1}{d_{122/123}}\right) + p_2^C\left(\frac{1}{d_{111/112}} + \frac{1}{d_{121/122}}\right)$$

……………………………………………………………………………………………..

$$\theta_{11h-2/11h-1} + \theta_{12h-2/12h-1} = 2\theta_{11h-1/11h} + 2\theta_{12h-1/12h} + p_h^C\left(\frac{1}{d_{11h-1/11h}} + \frac{1}{d_{12h-1/12h}}\right) + p_{h-1}^C\left(\frac{1}{d_{11h-2/11h-1}} + \frac{1}{d_{12h-2/12h-1}}\right)$$

Hence, the respective relation between the consumer taste parameters would have the following form:

$$\frac{7}{4}\overline{\theta} > 2\theta_{111/112} + 2\theta_{121/122} > 4\theta_{112/113} + 4\theta_{122/123} > ... > 2^{h-1}\theta_{11n-1/11h} + 2^{n-1}\theta_{12h-1/12h} \quad \text{(xxxii)}$$

Thus, in a market for three-good bundles the condition for at most two bundles to be demanded at the optimal prices would be as follows:

$$\underline{\theta} \geq \frac{7}{16}\overline{\theta} \quad \text{(xxxiii)}$$

which implies $\underline{\theta} > \theta_{112/113}$.

By reconsidering the derivation of the conditions in (xxvi), (xxix) and (xxxii), a general pattern can be identified how these relations are generated as function of the number of the goods in a bundle. It is apparent from the expression in (xxvi) that to have the number of the A-type goods in the market restricted to maximum two, it is sufficient to have the lower bound $\underline{\theta}$ of the range of the consumer taste parameters larger than a forth of the upper bound $\overline{\theta}$ i.e. condition (19) must hold.

Similarly, in compliance with (xxix), for the number of B-type goods with positive demand to be two at most, the lower bound $\underline{\theta}$ on the range of the consumer taste parameters must be larger or equal to a forth of the sum of the upper bound $\overline{\theta}$ and $\theta_{1nh/211}$. Note, however, that the result in (xxvi) implies that $\theta_{1nh/211}$ is at most equal to half of the upper bound $\overline{\theta}$. So, the sum of the two is at most equal to $\left(\frac{1}{2}+1\right)\overline{\theta}$. In other words, to reduce the number of the B-type goods to maximum two, it is necessary to restrict further the corresponding condition (19) for analogous reduction in the number of the A-type goods by adding $\frac{1}{4}\frac{1}{2}\overline{\theta}$ to its right-hand side:

$$\underline{\theta} \geq \frac{1}{4}\overline{\theta} + \frac{1}{4}\frac{1}{2}\overline{\theta} \quad \text{(xxxiv)}$$

Finally, the expression in (xxxiii) implies that to reduce the number of the C-type goods to maximum two, the lower bound $\underline{\theta}$ must be at least equal to a forth of the sum of the upper bound $\overline{\theta}$ and $\theta_{11h/121}$. However, according to (xxix), $\theta_{11h/121}$ is at most equal to half of the sum of the upper bound $\overline{\theta}$ and $\theta_{1nh/211}$. Respectively, this sum was already shown in the previous paragraph to be smaller or equal to $\left(\frac{1}{2}+1\right)\overline{\theta}$. Therefore, the sum of the two parameters $\overline{\theta}$ and $\theta_{11h/121}$ cannot be larger than $\left[1+\frac{1}{2}\left(\frac{1}{2}+1\right)\right]\overline{\theta}$. In other words, to reduce the number of the C-type goods to maximum two, it is necessary to restrict further the condition in (xxxiv) for analogous reduction in the number of the B-type goods by adding $\frac{1}{4}\frac{1}{2}\left(\frac{1}{2}+1\right)\overline{\theta}$ to its right-hand side:

$$\underline{\theta} \geq \frac{1}{4}\overline{\theta} + \frac{1}{4}\frac{1}{2}\overline{\theta} + \frac{1}{4}\frac{1}{2}\left(\frac{1}{2}+1\right)\overline{\theta} \tag{xxxv}$$

From the comparison of (19), (xxxiv) and (xxxv) it is trivial that each additional unit added to the number of goods in a bundle increases by half the restriction from below on the lower bound $\underline{\theta}$ for at most two bundles to be sold at the equilibrium prices. Hence, the general form of this condition would look as shown below:

$$\underline{\theta} \geq \sum_{x=1}^{X} \frac{1}{4}\left(\frac{1}{2}\right)^{x-1}\overline{\theta} = \sum_{x=1}^{X} \frac{1}{2^{x-1}}\overline{\theta} \tag{xxxvi}$$

which completes the proof of proposition 2 for the case when assumption (i) holds.

Appendix C: Proof of proposition 3

The proof of proposition 3 will start by verifying the condition on market size at which market would be covered by the two bundles in equilibrium. The purpose is to show that this condition is not ensured by the validity of the assumed condition (20) for at most two bundles to have positive market share at equilibrium as it was derived in appendix B. The coverage of the market demand will be verified by following the exact procedure applied by Shaked and Sutton (1982).

First, note that when the condition of proposition 1 holds, the range of market sizes assumed by proposition 3 corresponds to the requirement of proposition 2 for at most two bundles A1B1 and A1B2 to have positive market share at equilibrium. The demands for these two bundles are determined by two marginal taste parameters, $\theta_{11/12}$ and $\theta_{12/O}$. The latter can be presented in explicit form by using the general expression in (10):

$$\theta_{11/12} = \frac{p_1^B - p_2^B}{d_{11/12}} \qquad \text{(xxxvii)}$$

$$\theta_{12/O} = \frac{p_1^A - p_2^B}{d_{12/O}} \qquad \text{(xxxviii)}$$

For the sake of a more concise representation, the following simplifying notations will be adopted:

$$C_{11/12} = \frac{1}{\chi_{11} - \chi_{12}} = \frac{1}{d_{11/12}} \qquad \text{(xxxix)}$$

$$C_{12/O} = \frac{1}{\chi_{12} - \chi_O} = \frac{1}{d_{12/O}} \qquad \text{(xl)}$$

$$V = \frac{\chi_{11} - \chi_O}{\chi_{11} - \chi_{12}} = \frac{C_{11/12}}{C_{12/O}} + 1 \qquad \text{(xli)}$$

Then, expressions (xxxvii) and (xxxviii) are modified to present the prices of B-goods in terms of the marginal taste parameters $\theta_{11/12}$ and $\theta_{12/O}$:

$$p_1^B = \frac{\theta_{11/12}}{C_{11/12}} + \frac{\theta_{12/O}}{C_{12/O}} - p_1^A \qquad \text{(xlii)}$$

$$p_2^B = \frac{\theta_{12/O}}{C_{12/O}} - p_1^A \qquad \text{(xliii)}$$

Now, expressions (xxxix) and (xl) could be substituted for the prices of B-goods in the optimality conditions for profit-maximization to derive $\theta_{11/12}$ as a best response to $\theta_{12/O}$:

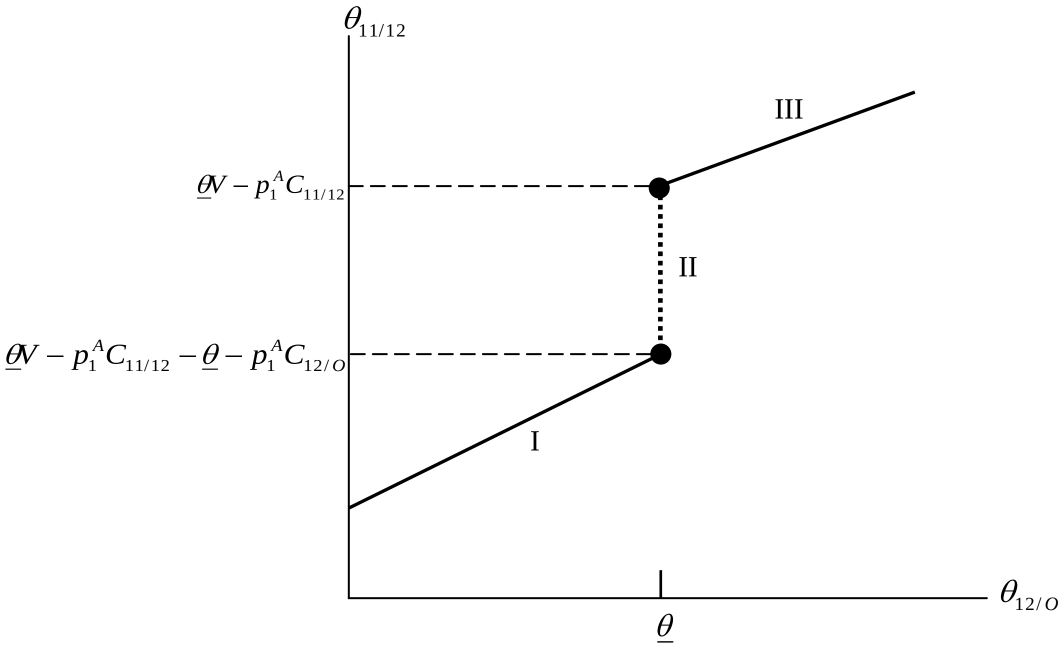
$$\theta_{11/12} = \frac{\bar{\theta} - \theta_{12/O}(V-1) + p_1^A C_{11/12}}{2} \qquad \text{(xliv)}$$

$$\theta_{11/12} = \underline{\theta} + \theta_{12/O}(V-1) - p_1^A C_{11/12}\text{, if market is covered i.e. } \theta_{12/O} \le \underline{\theta} \qquad \text{(xliv)}$$

$$\theta_{11/12} = \theta_{12/O}(V-1) - p_1^A\left(C_{11/12} + C_{12/O}\right)\text{, if market is not covered i.e. } \theta_{12/O} \ge \underline{\theta} \qquad \text{(xlvi)}$$

The best response functions in (xlii) and (xliii) could be graphically represented as shown on figure 2[11] below:

Figure 2: First-order conditions for profit-maximization by firm 2



Since the profit function of B2 is discontinuous at $\theta_{12/O} = \underline{\theta}$, at this value the best response function for $\theta_{11/12}$ could take any value in the range

[11] Note that figure 2 is the two-market counterpart of figure 1 in (Shaked and Sutton, 1982, p. 6).

$\left(\underline{\theta}(V-1)-p_1^A\left(C_{11/12}+C_{12/O}\right),\underline{\theta}V-p_1^A C_{11/12}\right)$. Respectively, three segments of its graph could be distinguished on figure 2. Whether the optimal solution of the B-good profit maximization lies in segment I, II or III depends on where the vertical $\theta_{12/O}=\underline{\theta}$ is cut by the strictly decreasing best-response function of (xliv):

$$\text{segment I if} \qquad V \geq \frac{\overline{\theta}+3\underline{\theta}+p_1^A\left(3C_{11/12}+2C_{12/O}\right)}{3\underline{\theta}}$$

$$\text{segment II if} \quad \frac{\overline{\theta}+\underline{\theta}+3p_1^A C_{11/12}}{3\underline{\theta}} \leq V \leq \frac{\overline{\theta}+3\underline{\theta}+p_1^A\left(3C_{11/12}+2C_{12/O}\right)}{3\underline{\theta}}$$

$$\text{segment III if} \qquad V \leq \frac{\overline{\theta}+\underline{\theta}+3p_1^A C_{11/12}}{3\underline{\theta}}$$

As in Shaked and Sutton (1982), the expression in (xli) implies that V is larger than 1. Hence, the solution cannot be in segment III, as long as the right-hand side of the respective inequality above is at most 1 which after simplification corresponds to the following condition on the market size:

$$\underline{\theta} \geq \frac{\overline{\theta}}{2}$$

Apparently, distinct from Shaked and Sutton (1982), the above condition is not ensured by the restriction on market size which is imposed by (20) for at most two bundles to have positive market shares at equilibrium. Therefore, it is possible in the range of the market size $\left(\frac{3\overline{\theta}}{8} \leq \underline{\theta} < \frac{\overline{\theta}}{2}\right)$ assumed by proposition 3, the market not to be covered at equilibrium.

In what follows, the solution of the profit-maximization problem will be consecutively derived for covered and non-covered market, respectively. The solution for the pricing subgame equilibrium will be given by comparison between the two results.

The profit-maximization problems of the three firms in the case of covered market (denoted by adding *c* in the upper index) would look as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^{A-c} = p_1^A\left(\overline{\theta}-\underline{\theta}\right) \qquad \text{(xlvii)}$$

$$\underset{p_1^B}{Max}\,\Pi_1^{B-c} = p_1^B\left(\overline{\theta}-\theta_{11/12}\right)= p_1^B\left(\overline{\theta}-\frac{p_1^B - p_2^B}{d_{11/12}}\right)$$

$$\underset{p_2^B}{Max}\,\Pi_2^{B-c} = p_2^B\left(\theta_{11/12}-\underline{\theta}\right)= p_2^B\left(\frac{p_1^B - p_2^B}{d_{11/12}}-\underline{\theta}\right)$$

To derive the optimal prices for B1 and B2, it is enough to consider just the last two problems whose solution is given by the following system of first-order optimality conditions:

$$2p_1^B - p_2^B - \overline{\theta}d_{11/12} = 0 \qquad \text{(xlviii)}$$

$$p_1^B - 2p_2^B - \underline{\theta}d_{11/12} = 0$$

The respective solution for the prices is as follows:

$$p_1^{B-c} = \frac{1}{3}\left(2\overline{\theta}-\underline{\theta}\right)d_{11/12} \qquad \text{(xlix)}$$

$$p_2^{B-c} = \frac{1}{3}\left(\overline{\theta}-2\underline{\theta}\right)d_{11/12}$$

The optimal price of good A1 is given as a corner solution at which the market is covered:

$$p_1^{A-c} = \underline{\theta}\chi_{12} - p_2^B = \underline{\theta}\chi_{12} - \frac{1}{3}\left(\overline{\theta}-2\underline{\theta}\right)d_{11/12} = \frac{1}{3}\left[\left(2\underline{\theta}-\overline{\theta}\right)\chi_{11}+\left(\overline{\theta}+\underline{\theta}\right)\chi_{12}\right] \qquad \text{(l)}$$

Now, the equilibrium marginal taste parameter $\theta_{11/12}$ could be derived as follows:

$$\theta_{11/12} = \frac{p_1^{B-c} - p_2^{B-c}}{d_{11/12}} = \frac{\left(\overline{\theta}+\underline{\theta}\right)}{3} \qquad \text{(li)}$$

which implies that for at least the two bundles A1B1 and A1B2 to have positive market share, the following condition must hold:

$$\theta_{11/12} - \underline{\theta} = \frac{\overline{\theta}-2\underline{\theta}}{3} \geq 0 \text{ i.e. } \underline{\theta} \leq \frac{\overline{\theta}}{2} \qquad \text{(lii)}$$

So, in the market size range assumed by proposition 3, B2 has a chance to make positive sales as long as A1 is charged a price at which the market would be covered.

In case of non-covered market (denoted by adding *nc* in the upper index), the respective profit-maximization problems for the three firms would look as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^{A-nc} = p_1^A\left(\bar{\theta}-\theta_{12/O}\right)= p_1^A\left(\bar{\theta}-\frac{p_1^A + p_2^B}{d_{12/O}}\right) \quad \text{(liv)}$$

$$\underset{p_1^B}{Max}\,\Pi_1^{B-nc} = p_1^B\left(\bar{\theta}-\theta_{11/12}\right)= p_1^B\left(\bar{\theta}-\frac{p_1^B - p_2^B}{d_{11/12}}\right)$$

$$\underset{p_2^B}{Max}\,\Pi_2^{B-nc} = p_2^B\left(\theta_{11/12}-\theta_{12/O}\right)= p_2^B\left(\frac{p_1^B - p_2^B}{d_{11/12}}-\frac{p_1^A + p_2^B}{d_{12/O}}\right)$$

The optimal prices are given by the solution of the following system of first-order optimality conditions:

$$2p_1^A + p_2^B - \bar{\theta}d_{12/O} = 0 \quad \text{(lv)}$$

$$2p_1^B - p_2^B - \bar{\theta}d_{11/12} = 0$$

$$p_1^A d_{11/12} - p_1^B d_{12/O} + 2p_2^B\left(d_{12/O} + d_{11/12}\right) = 0$$

The respective solution for the prices is as follows:

$$p_1^{A-nc} = \frac{\bar{\theta}d_{12/O}}{2} \quad \text{(lvi)}$$

$$p_1^{B-nc} = \frac{\bar{\theta}d_{11/12}}{2}$$

$$p_2^{B-nc} = 0$$

Note, however, that in this case even pricing at zero would not ensure positive demand for B2 at equilibrium because the optimal response of A1 and B1 would drive it out from the market. To show that, substitute (lvi) for the prices in the expressions for the marginal taste parameters $\theta_{11/12}$ and $\theta_{12/O}$:

$$\theta_{11/12} = \theta_{12/O} = \frac{\bar{\theta}}{2}$$

Hence, the respective demand for B2 is zero, as shown below:

$$D_{12} = \theta_{11/12} - \theta_{12/O} = 0 \quad \text{(lvii)}$$

So, for B2 there will not be an incentive to enter if A1 is going to set price at which the market would not be covered. Given no entry decision by B2, the profit maximization problems (denoted by adding *m* for 'monopoly' in the upper index) of A1 and B1 as only entrants in the first stage would look as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^{A\_ncm} = p_1^A\left(\bar{\theta}-\theta_{11/O}\right)= p_1^A\left(\bar{\theta}-\frac{p_1^A + p_1^B}{d_{11/O}}\right) \quad \text{(lviii)}$$

$$\underset{p_1^B}{Max}\,\Pi_1^{B\_ncm} = p_1^B\left(\bar{\theta}-\theta_{11/O}\right)= p_1^B\left(\bar{\theta}-\frac{p_1^A + p_1^B}{d_{11/O}}\right)$$

which implies that both entrants will charge the same prices as shown below:

$$p_1^{A\_ncm} = \frac{\bar{\theta}d_{11/O}}{3} \quad \text{(lix)}$$

$$p_1^{B\_ncm} = \frac{\bar{\theta}d_{11/O}}{3}$$

Substituting for the prices from (lix) in the expression for the marginal taste parameter $\theta_{11/O}$ yields the following result:

$$\theta_{11/O} = \frac{2\bar{\theta}}{3} \quad \text{(lx)}$$

That is, at equilibrium the market would not be covered as long as $\underline{\theta} < \frac{\bar{\theta}}{2} < \frac{2\bar{\theta}}{3}$.

Finally, it remains only to compare the optimal profits of A1 which drive its decision to set a price at which the market is covered or not after B2 enters the market in the first stage.

If B2 is in the market and A1 chooses to set a price at which the market would be covered, its optimal profit is given by substituting for the prices from (l) in its profit function:

$$\Pi_1^{A-c} = \left(\bar{\theta}-\underline{\theta}\right)\left[\underline{\theta}d_{12/O} - \frac{\left(\bar{\theta}-2\underline{\theta}\right)}{3}d_{11/12}\right] \quad \text{(lxi)}$$

Alternatively, if A1 chooses to set a price at which the market would not be covered, its optimal profit is given by substituting for the prices from (lvi) in its profit function:

$$\Pi_1^{A\_nc} = \frac{\bar{\theta}^2 d_{12/O}}{4} \quad \text{(lxii)}$$

Subtracting (lxi) from (lxii) yields the following expression:

$$\Pi_1^{A\_nc} - \Pi_1^{A\_c} = \frac{1}{12}\left(\bar{\theta} - 2\underline{\theta}\right)\left[3\left(\bar{\theta} - 2\underline{\theta}\right)d_{12/O} + 4\left(\bar{\theta} - \underline{\theta}\right)d_{11/12}\right] \qquad \text{(lxiii)}$$

which is strictly positive for $\underline{\theta} < \frac{1}{2}$. That is, given entry by B2, the entrant offering good A1 would be strictly better-off from charging price $p_1^A = \frac{\bar{\theta} d_{12/O}}{2}$ at which the sales of B2 would be foreclosed and the market would not be covered.

The last result implies that entry-deterrence threat would be credible for the assumed range of market sizes in proposition 3. So, B2 will stay out of the market and only A1 and B1 will enter at equilibrium. Furthermore, in compliance with the result in (lx), the market will not be covered. This completes the proof of proposition 3.

Appendix D: Quality-choice subgame equilibrium

Substituting for the prices from (lix) in the profit functions of the firms entering with goods A1 and B1, respectively, allows to derive the equilibrium profits in terms of quality difference $d_{11/O}$:

$$\Pi_1^A = \Pi_1^B = \frac{\bar{\theta}^2 d_{11/O}}{9} \tag{lxiv}$$

Substituting for the explicit expression of $d_{11/O}$:

$$d_{11/O} = \chi_{11} - u_O = \left[f(A_1)^r + g(B_1)^r\right]^{\frac{1}{r}} - u_O$$

from (8) and (13) yields the following expressions for the profits in terms of product qualities:

$$\Pi_1^A = \Pi_1^B = \frac{\bar{\theta}^2\left(\left[f(A_1)^r + g(B_1)^r\right]^{\frac{1}{r}} - u_O\right)}{9} \tag{lxv}$$

Obviously, (lxv) is strictly increasing in both $f(A_1)$ and $g(B_1)$ which implies that profit-maximizing quality choice for both A1 and B1 is given by the corner solution where they are assigned the upper bounds of their quality intervals:

$$f(A_1) = \bar{F} \tag{lxvi}$$

$$g(B_1) = \bar{G}$$

This completes the solution for the subgame equilibrium in the quality-choice stage.