Why Middle-qualities May Not Survive in Market Equilibrium? The Case of Vertical Product Differentiation with Complementary Goods*

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Abstract

In the classical literature on vertical differentiation, the goods are assumed to be single products each offered by a different firm and consumed separately one from another. This paper departs from the standard setup and explores the price competition in a vertically differentiated market where a firm's product is consumed not separately but in a bundle with another firm's complementary product. In the new market setting, firms would prefer to charge their products sufficiently high prices at which only the most differentiated bundles would have positive market shares. As a result, in equilibrium the sales of a bundle with middle quality may be foreclosed in favor of a bundle with lower quality because it is more differentiated from the best bundle in the market. What makes this equilibrium outcome special is that it cannot occur in a standard single-product market with vertical product differentiation.

Keywords: complementary goods, vertical product differentiation, market foreclosure

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1. Introduction

In the field of industrial organization, markets in which goods differ in quality in such a way that all consumers agree which of any couple of goods is better than the other are called vertically differentiated markets. When considered as independent from the others, each vertically differentiated market is characterized by the property that given small marginal cost of quality improvement, the market power of a good increases in its quality[1]. That is, no good could foreclose the sales of another good if the latter has a higher quality. So, in equilibrium middle-quality goods cannot be driven out of the market in favor of the bottom-quality goods.

In real world, however, the vertically differentiated markets are rarely independent. There are many examples of vertically differentiated goods that are complements to other vertically differentiated goods so that consumers always purchase them both in a bundle even though they are sold by detached firms, in different markets and at separately defined prices. For instance, when buying a new flat or a house from a real-estate company, consumers also buy together with it furniture from the home-furnishing shop or order it from a cabinet-maker. Also, when going on a business trip, employees should be provided not only with transportation from an airline company but also with accommodation from a hotel at the destination of the trip. Or, observing the modern high-tech industries, a user buys a computer with CPU manufactured by PC hardware firm, screen from TV producer, operation platform from system software provider, and application software from an application developer.

Some of the described industries are usually also characterized by the application of wide variety of tying arrangements or mergers between the firms from the different markets which restrict the free choice of the consumers to a narrow range of pre-selected bundles. For

[1] For detailed analysis of the conditions on production cost for this property to hold, see Shaked and Sutton (1983).

example, system software providers offer also applications that are not fully compatible with their rivals' products. In video-gaming industry every producer of a game console develops also games that are designed to be played only on its console. In the market for smart phones, some producers offer both the phone and operation platform for it which allows them to restrict the user's access to the applications developed by other firms for competitive smart phones. Respectively, some mobile-phone devices are coded to function only with the service provided by a single telecom operator. Similarly, travel agencies offer single-price package tours that include both travel and accommodation pre-selected so that their consumers cannot choose other bundles.

Nevertheless, in some countries mergers or tying arrangements fall under the jurisdiction of the anti-trust authorities which prohibit them as potential market power enhancing devices. Or, alternatively, even when these restrictive practices are allowed, firms find it technically or administratively impossible to implement them. At the markets where consumers are not restricted to buy only pre-selected bundles, however, still examples could be found of self-selection consumer behavior. That is, only the bundles of goods with similarly ranked qualities and prices are purchased by consumers while the mixed-quality[2] (resp. mixed-price) bundles are ignored.

For instance, it would be very unusual to see consumers who buy an expensive luxurious house and furnish it with cheap do-it-yourself furniture. Or vice versa, normally consumers who buy small low-price flats do not equip them with high-design furniture[3].

[2] In the present paper, "mixed quality" and "middle quality" are used interchangeably to denote the bundles that do not consist of the two top-quality or the two bottom-quality goods in a duopoly market. However, as it will be discussed later, the second term might be less precise in the particular case represented by one of the model setups when the conditions for general solution hold.

[3] For description of the low-cost-mass-taste strategy of the one of the largest world-wide furniture retailers, see Byrne, J. and Capell, K. (2005).

Likewise, businessmen who order business-jet charter flights naturally choose to be accommodated in five-star hotels which is not the case of small-firm entrepreneurs or lower-level managers whose budget only allows them to travel in the business or even economy class of the regular airline flights[4].

A relevant example from the high-tech industries is the TV and home-video markets. In principle, high-definition (HD) content could be watched on a standard- (SD) or enhanced-definition (ED) TV screen[5] by means of HDTV set-top-box for cable and/or satellite TV[6], or Blu-ray video-player. Also, standard definition (SD) content could be watched on a HDTV screen[7]. However, it is unreasonable for a consumer to pay more for HDTV set-top box without buying also a more expensive HDTV screen or vice versa, to buy the latter without paying for the former[8]. In both cases, the quality of the image will not differ significantly from SDTV just the cost of watching it will be higher[9].

The real-life observations above imply that the standard property of the vertically differentiated markets (where the sales of the middle qualities can never be foreclosed by the lowest quality) may not apply to the market for their bundles. Moreover, they suggest that

[4] For the advantages of business-jet charter flights to regular flights see Lapidos (2008).
[5] The three general standards of digital display correspond to different range of screen resolution – the minimum-performance digital TV sets are the SD ones which perform as standard analogue TV sets with maximal resolution 480i, the ED screens are of middle quality with maximal resolution of 720p, while HD sets are so far the best on the market with maximal resolution 1080p. For more information on the difference in the three standards see Katzmaier (2011); Brain (2001); Powell (2003).
[6] A general technological specification of all HD set top boxes is that they are designed to convert 1080i signal into 480i TV output which makes them 100% compatible with a standard-definition TV set. For more detailed description of the HD STB standard see Nist (2009).
[7] In fact, HD standard appeared in the TV set market long before media industry was able to broadcast an HD signal and to distribute HD content. Waggoner (2004). Therefore, in the beginning before Blu-ray and HDTV technologies were commercially implemented, the first consumers were not able to enjoy the full capacity of their HD sets and were watching mainly SD content on them, see Bertolucci (2008).
[8] "Don't skimp on your cable/satellite bill by not getting the HD channels. If you're spending all that money on a new TV, not putting HD on it would be like buying a Ferrari and replacing the V12 with a lawnmower engine." Morrison (2011).
[9] For comparative price review on SD and HD equipment, see Maxwell (2006); Kogan (2006).

firms might be able and willing to restrict consumer choice not directly but through setting prices which prevent consumers from buying mixed-quality bundles.

The aim of the present paper is to introduce a model which allows for identifying the conditions at which the middle-quality bundles of two vertically differentiated goods would be driven out from the market in equilibrium. For the purpose, three different setups are developed of a natural duopoly market for two-good bundles.

The paper is organized as follows. Section 2 describes the model, section 3 presents a particular solution and the general conditions for it to hold for all the three considered model setups, section 4 summarizes the results, comments on their implications and draws directions for further research on the topic.

2. Description of the model

The analysis starts with the introduction of a model of a market for bundles consisting of two types of goods each offered by a separate firm. The good types are denoted by A and B, respectively. All consumers agree on how goods of a type should be ranked based on their different qualities. Therefore, goods could be identified by their quality rank starting from the best-quality good being ranked by 1, the second-best by 2 and so on, as shown below:

$$f(A_1) > f(A_2) > .... > f(A_m) \tag{1}$$

where $f(A_i)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the product of type A based on which it is ranked at i-th place; $\underline{F} \leq f(A_i) \leq \overline{F}$, $i = 1,...,m$;

$$g(B_1) > g(B_2) > .... > g(B_n) \tag{2}$$

where $g(B_j)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the product of type B based on which it is ranked at j-th place; $\underline{G} \leq g(B_j) \leq \overline{G}$, $j = 1,...,n$.

Even though the quality ranking of goods of a type is unambiguously defined for both the A-type and B-type, additional specifications are required for defining the qualities of all the bundles (AiBj) that might be composed from the available A-type and B-type goods. One particular way to express the consumers' preferences over the bundle qualities is to assume particular functional form for the relationship between the quality of the bundle and the qualities of the two goods in it. Respectively, the current paper suggests three such possible representative forms – additive, multiplicative and minimum function – and explores the solution of all the three corresponding setups of the main model that result from that assumption. For simplicity, it is initially assumed that all consumers also agree on the quality ranking of the bundles which does not need to be the case even when (1) and (2) hold. The choice of each of the three functional forms is respectively motivated in the three subsections that follow.

2.1. Additive function

When the characteristics by which consumers assess the qualities of the two types of goods are independent, it is reasonable to assume that the goods' qualities enter the quality of the bundle by adding to each other. Then, the quality of a bundle could be expressed functionally as a sum of the qualities of the goods in it:

$$\chi(A_i, B_j) = f(A_i) + g(B_j) \tag{3}$$

where:

$\chi(A_i, B_j)$ - the value that all consumers assign to the quality of bundle $A_i B_j$

Real-estate industry could be considered as an example of a market where the additive functional relationship between the qualities of two goods might be suitable representation for the quality of their bundle. Flat ownership provides its lodgers with privacy and security protected by law while furniture adds comfort without interacting it with the level of privacy and security of the flat.

2.2. Multiplicative function

Alternatively, the qualities of the two goods could be interrelated and augment each other's magnitude. So, they will enter in the quality of the bundle by multiplying the qualities of each other. Then, it would be reasonable to express functionally the quality of a bundle as a product of the qualities of the goods in it:

$$\chi(A_i, B_j) = f(A_i)g(B_j), i = 1,..., m; j = 1,..., n \tag{4}$$

The representation of the bundle quality as a multiplicative function of the qualities of the goods in it might be suitable for modeling the business-travel market. The quality of business travel and accommodation aims to minimize the loss of working efficiency due to moving from one place to another. In this sense, having a good flight implies that passenger will be less tired of flying. However, if the conditions of the hotel do not allow for perfect relaxation after the flight, the good quality of the latter will be mutilated by the extent to which the quality of the hotel differs from its best possible (five-star) status. Similarly, if the flight is scheduled late in the night or requires switching from one airliner to another in the middle of the journey, even if the hotel offers perfect conditions for relaxation, its efficiency will be mitigated according to the level of discomfort during the flight.

2.3. Minimization function

In the extreme case of interaction between the qualities of two goods in a bundle, the characteristic that determines the quality of the one good might be identical to the characteristic that defines the quality of the other good. Then, the quality of the bundle will be limited down to the value of the lower of the qualities of the two goods in it. Therefore, it would be useful to express the bundle quality as a minimization function of the qualities of its goods:

$$\chi(A_i, B_j) = \mathrm{Min}[f(A_i), g(B_j)], i = 1,...,m; j = 1,...,n \tag{5}$$

All modern high-tech industries for systems could be considered as examples where minimum function would be relevant to be used to represent the relationship between the quality of a bundle and the qualities of the goods in it. Particularly, in the example with high-definition television given in the introduction, both the quality of the TV-set and the quality of the content watched on it are assessed by the definition of the picture visualized on the TV screen. Thus, even when the quality of the signal allows for high-definition picture, the actual quality for the viewers will be limited by the quality (definition) of the picture that their TV-set is technically built to visualize and vice versa. The quality (definition) of the picture on the TV screen will be restricted up to the quality (definition) of the TV signal that is transmitted to the TV-set yet if the latter allows for picture with higher definition.

From here on, $\chi_{ij} = \chi(A_i, B_j)$ replaces the quality parameter in the standard utility function for vertically differentiated markets suggested by Mussa and Rosen (1978):

$$u(\chi_{ij}, \theta) = \theta\chi_{ij} - p(\chi_{ij}) = \theta\chi_{ij} - p_i[f(A_i)] - p_j[g(B_j)] \tag{7}$$

where:

θ - taste parameter[10] by which consumers are identified; θ is assumed to be uniformly distributed on an interval $\underline{\theta} \leq \theta \leq \overline{\theta}$

χ_{ij} - quality of bundle A_iB_j

$p_{ij} = p\big(f(A_i), g(B_j)\big)$ - price of bundle A_iB_j as a function of the qualities of the goods in it

$p_i^A = p_i[f(A_i)]$ - price of good A_i as a function of its quality[11]

$p_j^B = p_j[g(B_j)]$ - price of good B_j as a function of its quality

The difference from the ordinary single-good case is that the price which consumer pays for a bundle is a sum of the prices of the goods in that bundle. The latter prices are competitively defined by distinct producers. Moreover, these prices affect not only the demand for the given bundle A_iB_j but also the demand for the other bundles in which its components A_i or B_j could individually participate.

As it is standard in the models of vertical differentiation, the range of consumer tastes $\left(\underline{\theta}, \overline{\theta}\right)$ is cut into subintervals by the marginal taste parameters $\theta_{ij/i*j*}$ which distinguish the group of consumers who would buy one bundle (A_iB_j) from the group of consumers who would buy another bundle ($A_{i*}B_{j*}$) at the given prices. That is, the marginal taste parameter $\theta_{ij/i*j*}$ identifies the consumer who experiences the same utility from any of the compared bundles and is therefore indifferent between the two:

$$u\left(\chi_{ij}, \theta_{ij/i*j*}\right) = \theta\chi_{ij} - p\left(\chi_{ij}\right) = \theta\chi_{i*j*} - p\left(\chi_{i*j*}\right) = u\left(\chi_{i*j*}, \theta_{ij/i*j*}\right)$$

i.e.

$$\theta_{ij/i*j*} = \frac{p\left(\chi_{ij}\right) - p\left(\chi_{i*j*}\right)}{\chi_{ij} - \chi_{i*j*}} = \frac{p\left(\chi_{ij}\right) - p\left(\chi_{i*j*}\right)}{d_{ij/i*j*}}, i = 1,..., m, j = 1,..., n \qquad (8)$$

[10] θ could be considered as a measure of the marginal utility of quality (Mussa and Rosen, 1978) or consumer's income (Shaked and Sutton, 1982)

[11] To simplify the notation, the prices of goods are denoted by p_i^A $(i = 1,..., m)$ if they are of type A and by p_j^B $(j = 1,..., n)$ if they are of type B.

where:

$d_{ij/i*j*}$ - quality difference between bundle $A_i B_j$ and $A_{i*} B_{j*}$.

Consumers have also an outside option, not to buy from the available bundle qualities but to purchase a good in another market which would correspond to receiving a bundle of quality χ_O for free. Thus, the condition for the market to be covered by the *m* qualities of type A and the *n* qualities of type B is as follows:

$$\underline{\theta} > \theta_{mn/O} = \frac{p(\chi_{mn}) - p(\chi_O)}{\chi_{mn} - \chi_O}$$

In a covered market, the smaller is $\theta_{ij/i*j*}$, the larger will be the share of consumers who prefer to buy A_iB_j to $A_{i*}B_{j*}$. If each of the two bundles were offered as one package good supplied by a single firm, then the firm's profit would fully depend on the consumer's demand for that bundle. Therefore, as the quality difference $d_{ij/i*j*}$ between the two shrinks, the stronger will be the competitive downward pressure on the bundle prices, $p(\chi_{ij})$and $p(\chi_{i*j*})$. No matter how close are the qualities of the two bundles, however, the producer of the better-ranked quality bundle would always have more market power which would allow it to set at least a bit higher price mark-up than its competitor and still maintain its market share.

When every bundle in the market is offered only as a package good by a multi-product firm, the situation is identical to the standard single-good case. There are two properties that always hold in the markets for vertically differentiated single goods. First, provided the assumption that the unit production cost does not vary much in quality, when a good has a positive market share, the same should also be true for the goods with higher quality. That is, a good with lower quality cannot foreclose the sales of a good with higher quality because the latter is associated with more market power and its market share is decisive for the supplier's profit. Second, if given a choice, both suppliers would prefer the quality distance between their products to be the largest possible provided that the worse-quality good is still good enough for it to have positive sales. In the economic literature on product differentiation, this property is well-known as maximal differentiation principle.

Nevertheless, when the goods in the bundles are supplied by independent firms, a bundle's market share is not that decisive for the firms' profits anymore. A firm's good could take part in several bundles. So, for the firm it is not that important to save the sales of all of them. As a result, if the quality difference between two bundles is sufficiently small, it might be optimal for the firms to keep their prices high and foreclose the sales of these bundles in favor of the other bundles in which their products participate.

In what follows, a particular example will be given how in case of unrestricted bundling the first property of the single-good markets could be violated so that a lower-quality bundle would drive out a higher-quality bundle in equilibrium. Moreover, this violation will be shown to be still in line with the second property of the single-good markets described above. That is, the resulting equilibrium outcome could be considered as a manifestation of the maximal differentiation principle.

The strategic interaction between the firms while choosing bundle qualities and prices is presented as a three-stage game. Firms first make an entry decision (entry stage), then after observing how many entrants are in the market, choose simultaneously their good qualities (quality-choice stage), and finally after the qualities are known, compete in prices (pricing stage). In the next section, the simplest case of natural duopoly is solved where the range of consumer tastes is chosen to be narrow enough so that exactly two entrants of a good type will enter in equilibrium.

3. Equilibrium solution

3.1. Solution of the model – entry stage

The precise condition for natural duopoly is given by proposition 1 below:

Proposition 1. Let $\frac{3}{8}\overline{\theta} \le \underline{\theta} < \frac{2}{3}\overline{\theta}$ and the top two quality choices of the goods of each type satisfy the condition $f(A_1) = g(B_1) > f(A_2) = g(B_2) \ge \frac{7}{9} f(A_1)$. Then, for any $i = 1,...,m$, exactly two firms of a type (namely, A1, A2, B1, B2) will enter the market in equilibrium, provided that the functional relationship between the qualities of the goods in the bundle is any of the three functional forms, resp. additive, multiplicative or minimization function.

Proof: See Appendix A

The restriction on the range of consumers' tastes $\left(\frac{3}{8}\overline{\theta} \le \underline{\theta} < \frac{2}{3}\overline{\theta}\right)$ ensures that the market is narrow enough so that exactly only the two top-quality goods of type A could have positive market shares and at most the two top-quality goods of type B could have positive sales in a bundle with good A2. [12]

The restriction on good qualities ($f(A_2) = g(B_2) \ge \frac{7}{9} f(A_1)$) guarantees that a third entrant of type B cannot have positive sales in a bundle with good A1. The latter is a generalization of the particular conditions required for each of the three functional forms but is not necessary to hold for any of them. The condition on good qualities for keeping a potential third entrant out of the market is unnecessary in case of additive function and less restrictive in case of minimization function (see conditions (xxv-xxviii) and (xxx-xxxiii) in Appendix A). [13]

[12] Note that the condition on the range of consumer tastes here is less restrictive than the one in case of standard (single-product) natural duopoly $\left(\frac{3}{8}\overline{\theta} > \frac{1}{4}\overline{\theta} \text{ and } \frac{2}{3}\overline{\theta} > \frac{1}{2}\overline{\theta}\right)$, (see Shaked and Sutton, 1983, p. 7).

[13] The difference in conditions across the model setups could be explained with the corresponding different extent of compatibility between the characteristics that define the quality of the two types of goods. When the qualitative characteristics of the two goods are incompatible and add each to another, the variance of the one could always compensate for the variance of the other. That is weaker exogenous restriction on good qualities is required for blockading a third B-type entrant in case of additive functional relationship. However, in the cases when the goods' qualities limit each other's effect on their bundle's quality, a firm is unable to restrict an entry

3.2. Solution of the model – quality choice

Since the firms gain from the differentiation, one good of each type will have the best possible quality. The other two products, respectively, will choose the worst possible quality at which the market would still be covered. Therefore, if the lower bounds on quality choices of the two types of goods are sufficiently large relative to the quality of the outside option χ_O they would be the optimal solution because still the market will be covered. The rigorous result for the optimal quality choice in the second stage is stated in proposition 2 below.

Proposition 2. Let $\frac{3}{8}\overline{\theta} < \underline{\theta} < \frac{2\overline{\theta}}{3}$ and the bounds on quality functions satisfy the relationship $\overline{F} = \overline{G} > \underline{G} = \underline{F} \geq \frac{7}{9}\overline{F}$. Then, the optimal quality choices of the four entrants in the second stage will be $f(A_1) = \overline{F}; g(B_1) = \overline{G}; f(A_2) = \underline{F}$ and $g(B_2) = \underline{G}$ provided that the quality of the outside option χ_O satisfies the condition $0 < \chi_O \leq \frac{(4\overline{\theta} - \underline{\theta})\underline{F}^2 - 2(2\overline{\theta} - 3\underline{\theta})\overline{F}^2}{5\underline{\theta}}$.

Proof: see Appendix B

Again as in the proposition 1, the requirement for identical range of the goods' qualities $(\overline{F} = \overline{G}; \underline{F} = \underline{G})$ is just a generalization of the less restrictive particular conditions for natural duopoly. Dependent on the form of the assumed functional relationship between the quality of the bundle and the qualities of the goods in it, both the duopoly market structure and the optimal quality choice could hold also with differing ranges (i.e. $\overline{F} \neq \overline{G}$ and $\underline{G} \neq \underline{F}$) of the goods' qualities.

The restriction from above on the quality of the outside option ensures that the market would still be covered when the goods (A2 and B2) that form the bottom-quality bundle are assigned the lower-bound quality values $\underline{F}$ and $\underline{G}$.[14]

through only changing its good's quality. Therefore, with minimum and multiplicative functional relationships stronger exogenous restrictions on qualities are required for a third B-type firm to be prevented from entry.

[14] A trivial market situation for which the condition for covered market will always hold is when the outside option is of quality zero i.e. $\chi_O = 0$.

3.3. Solution of the model –subgame perfect equilibrium

Proposition 3 below defines formally the subgame perfect equilibrium that results from the solution of the model's three-stage game for any of the three functional forms assumed to represent the relationship between the good qualities in a bundle.

Proposition 3. Let $\frac{3}{8}\overline{\theta} < \underline{\theta} < \frac{2\overline{\theta}}{3}$ and the bounds on quality functions satisfy the relationship $\overline{F} = \overline{G} > \underline{G} = \underline{F}$ such that $0 \leq \chi_O \leq \frac{\left(4\overline{\theta} - \underline{\theta}\right)\underline{F}^2 - 2\left(2\overline{\theta} - 3\underline{\theta}\right)\overline{F}^2}{5\underline{\theta}}$. Then, for an infinitesimally small positive entry fee $\varepsilon \to 0$ and any number *m, n>2* of potential single-product entrants of each type of goods, A and B, there exists a Perfect Subgame Nash Equilibrium in which for any of the three considered functional relationships between the qualities of the goods in a bundle:

1. only the two top-quality goods of both types A and B enter the market
2. the optimal quality choices of the four entrants in the second stage is $f\left(A_1\right) = \overline{F}; g\left(B_1\right) = \overline{G}; f\left(A_2\right) = \underline{F}$ and $g\left(B_2\right) = \underline{G}$
3. entrants charge their products prices $p_1^A = p_1^B = p_1 = \frac{\left(3\overline{\theta} - 2\underline{\theta}\right)\left(\overline{F} - \underline{F} + \overline{G} - \underline{G}\right)}{5}$, $p_2^A = p_2^B = p_2 = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)\left(\overline{F} - \underline{F} + \overline{G} - \underline{G}\right)}{5}$,

   at which only two bundles, A1B1 and A2B2, have positive market shares, and yield the following positive profits:

$$\Pi_1^A = \Pi_1^B = \frac{\left(3\overline{\theta} - 2\underline{\theta}\right)^2\left(\overline{F} - \underline{F} + \overline{G} - \underline{G}\right)}{25} - \varepsilon,$$
$$\Pi_2^A = \Pi_2^B = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)^2\left(\overline{F} - \underline{F} + \overline{G} - \underline{G}\right)}{25} - \varepsilon.$$

Proof: Corollary from the proofs of proposition 1 and 2 (See Appendices A and B)

The main outcome of driving out the middle-quality bundles from the market is described solely by the subgame solution of the model in the pricing stage (bullet-point 3 above). The economic explanation why this outcome that otherwise cannot be optimal in a single-good market turns out to be equilibrium in the market for bundles is that it results from two effects that characterize the bundle markets:

- On the one hand, each firm could sell its good as a part of more than a single bundle, respectively tagged by different qualities. However, it cannot set separate prices conditional on the bundle quality tag under which its good is sold.

- On the other hand, the closer are the qualities between bundles the tougher is the competition between firms which drives their prices down.

Respectively, the top-quality firms face a trade-off, to try to resist the competitive pressure from below by charging sufficiently low price at which all the bundles containing their goods would have positive demand or instead to give it in by charging a high price at which only the best bundles could be sold. Moreover, they gain from differentiating their goods as much as possible from the other lower quality goods of the same type in the market. Therefore, in a duopoly market the middle quality[15] bundles suffer tougher competition than the top and bottom-quality bundles which makes it optimal for firms to set high prices which drive the middle-quality bundles out from the market in equilibrium.

One could argue that when consumers agree on the ranking of the qualities of the goods of each type this still does not imply that they must also agree on the uniform ranking of the qualities of their bundles. In a duopoly market, however, it is straightforward to show that the equilibrium outcome could still hold even if there are two groups of consumers differing in their valuations of the bundles. This result is stated in proposition 4 below:

[15] Note that due to the general restriction on the goods' qualities $\left(\overline{F} = \overline{G}; \underline{F} = \underline{G}\right)$, in the equilibrium defined by proposition 3 the bundles A1B2 and A2B1 have identical quality for each of the three functional relationships (resp. $\overline{F} + \underline{G} = \underline{F} + \overline{G}$, $\overline{F}\underline{G} = \underline{F}\overline{G}$ and $Min\left(\overline{F}\underline{G}\right) = Min\left(\underline{F}\overline{G}\right)$). Moreover, if the relationship is assumed to be of minimum functional form, the two bundles A1B2 and A2B1 have the same quality as the bottom bundle A2B2 ($Min\left(\overline{F}\underline{G}\right) = Min\left(\underline{F}\overline{G}\right) = Min\left(\underline{F}\underline{G}\right)$) and therefore it is not very precise to call them "middle-quality" in this particular case.

Proposition 4. Let $\frac{3}{8}\overline{\theta} < \underline{\theta} < \frac{2\overline{\theta}}{3}$ and the bounds on quality functions satisfy the relationship $\overline{F} = \overline{G} > \underline{G} = \underline{F}$ such that $0 \le \chi_O \le \frac{(4\overline{\theta} - \underline{\theta})\underline{F}^2 - 2(2\overline{\theta} - 3\underline{\theta})\overline{F}^2}{5\underline{\theta}}$. However, a share λ of consumers value the good of A-type by function $f(A_i), i = 1,..,m$ and the good of B-type by function $g(B_j), j = 1,..,n$ while the rest $(1-\lambda)$ value the good of A-type by function $g(A_i), i = 1,..,m$ and the good of B-type by function $f(B_j), j = 1,..,n$. Then, for an infinitesimally small positive entry fee $\varepsilon \to 0$ and any number *m, n>2* of potential single-product entrants of each type of goods, A and B, there exists a Perfect Subgame Nash Equilibrium in which for any of the three considered functional relationships between the qualities of the goods in a bundle:

1. only the two top-quality goods of both types A and B enter the market
2. the optimal quality choices of the four entrants in the second stage is $f(A_1) = \overline{F}; g(B_1) = \overline{G}; f(A_2) = \underline{F}$ and $g(B_2) = \underline{G}$
3. entrants charge their products prices $p_1^A = p_1^B = p_1 = \frac{(3\overline{\theta} - 2\underline{\theta})(\overline{F} - \underline{F} + \overline{G} - \underline{G})}{5}$, $p_2^A = p_2^B = p_2 = \frac{(2\overline{\theta} - 3\underline{\theta})(\overline{F} - \underline{F} + \overline{G} - \underline{G})}{5}$,

   at which only two bundles, A1B1 and A2B2, have positive market shares, and yield the following positive profits:

   $$\Pi_1^A = \Pi_1^B = \frac{(3\overline{\theta} - 2\underline{\theta})^2(\overline{F} - \underline{F} + \overline{G} - \underline{G})}{25} - \varepsilon,$$
   $$\Pi_2^A = \Pi_2^B = \frac{(2\overline{\theta} - 3\underline{\theta})^2(\overline{F} - \underline{F} + \overline{G} - \underline{G})}{25} - \varepsilon.$$

Proof: Note that in a duopoly market there could be only two distinct orders of the bundle qualities. In both orders A1B1 is the best bundle while A2B2 is the worst independent on the quality choices in the second stage. Respectively, the two groups of consumers could disagree only on the order of the middle quality bundles. However, the conditions of proposition 3 ensure that middle-quality good have no market share which makes their ordering irrelevant for the equilibrium solution. Therefore, the payoff functions and corresponding optimal choices of the firms remain the same in proposition 4. Q.E.D.

Generally speaking, the outcome of having the middle qualities out of the market at equilibrium is a result not of the homogenous ranking of the bundle qualities by consumers. It is rather a consequence of the large quality difference between the best bundle and the rest of the bundles that raises an incentive for the firms to charge both top quality goods prices which are perceived to be prohibitively high by the low-taste consumers.

4. Conclusion

This paper considers a model of price competition in a vertically differentiated market for bundles of complementary products manufactured by different firms. The situation analyzed differs from the case considered in the classical literature on vertical differentiation where the goods are standardly assumed to be single products each offered by a different firm and consumed separately one from another.

What makes the solution of the new model special is that distinct from the standard case, here the foreclosure of the sales of a bundle does not imply zero market shares of all the other bundles that have lower quality. The main result presented in the paper implies that in a duopoly market (i.e. with two goods of a type) if a middle-quality bundle is more differentiated from the best than from the worst bundle, the suppliers of the goods would be better-off of setting higher prices which would foreclose the middle-quality bundle in favor of the bottom-quality one. In fact, this is a manifestation of the maximal differentiation principle which characterizes the markets with vertical differentiation in general. By driving out the middle-quality bundle(s), the top- and bottom-quality bundles maximize the quality differential between each other which provides them with more market power and allows them to gain higher profits.

The market exclusion of the middle qualities in a natural duopoly is shown to be an equilibrium outcome for three different ways (additive, multiplicative and minimizing) in which the qualities of the goods could interact each with another when defining the quality of the bundle in which they take part. The solution of the model between the respective three setups, however, differs only in the quality-choice stage. The subgame equilibria in the other two stages, the entry stage and the pricing stage, are independent on the functional relationship between the qualities of the goods and the quality of the bundle they form together.

The assumed functional form of the interaction between good qualities matters only for defining the bundle quality differentiation that would result from the solution of the quality-choice stage. Respectively, in order this differentiation to be large enough for middle qualities to be driven out from the market, different restrictions need to be imposed on the range in which good qualities are allowed to vary. The range should be most restricted in the case of multiplicative interaction between qualities and least restrictive in the case of additive relationship between the two. The intuition is that when the qualitative characteristics of the two goods are incompatible and add each to another, the variance of the one could always compensate for the variance of the other and the restriction could be focused only on the quality range of a single type of good. However, in the case when the goods' qualities limit each other's effect on the bundle's quality (multiplicative relationship) or the bundle's quality is given by the smaller of the two good's qualities (minimum relationship), the restriction should be imposed on the quality range of the both types of goods. Nevertheless, as it is shown in the paper, a general condition on quality ranges could be defined at which all quality relations would lead to market exclusion of the middle-quality bundles in a natural duopoly equilibrium.

The economic intuition behind the market exclusion of the middle-quality goods provides a new explanation for the competitive behavior observed in some of the real-world examples of markets for complementary goods. It presents the observed self-selection of qualities in some markets - i.e. the top (resp. bottom) quality good of a type is purchased only together with the top (resp. bottom) quality good of another type - as a straightforward result from the price competition between the firms. Moreover, for this result to occur, it is not required consumers to agree on the quality ordering of the bundles. That is, the very bundles must not be vertically differentiated. What is sufficient to assume is that only the goods are vertically differentiated which itself ensures that the best (resp. the worst) bundle is the one consisting of the best (resp. the worst) goods of each type available in the market.

Finally, by relaxing of some of its restrictive conditions the model presented in the paper could be extended in several further directions. First, the condition for natural duopoly could be replaced by entry barrier like patent fee or government regulation which would restrict the number of entrants exogenously thus making redundant the requirement for covered market. This would significantly simplify the computations, decrease most of the restrictive conditions on the width of the market and would thus allow for the analysis of the price competition in different market structures (e.g. with third entrant). Since the middle-quality market exclusion occurs as a result of the firms' interaction in the quality-choice and pricing stages, however, making the entry decision exogenous would not affect the equilibrium outcome. Second, it would be useful to check whether the exclusion of the middle-quality bundles would still occur in equilibrium given that firms compete not in prices but in quantities in the last stage. Third, the equilibrium result derived in the present paper could be used as a benchmark for exploring the social welfare effect of the tying behavior observed in most of the high-tech markets for systems nowadays.

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Appendix A: Derivation of the condition for natural duopoly – proposition 1 and 2

To derive the condition for exactly two firms of a type to enter in the first stage, the following five-step procedure could be followed.

<u>Step 1</u> (Analogous to Shaked and Sutton, 1982) Assume *m* firms of type A and *n* firms of type B enter the market. Only initially, also suppose a condition on the good quality ranges, $f(A_i) \in [\underline{F}, \overline{F}]$ and $g(B_j) \in [\underline{G}, \overline{G}]$, such that the bundle qualities would be ranked in lexicographical order. Derive the condition for at most A1 and A2 to have positive market shares in equilibrium.

It is useful to start with derivation of the condition for the particular case when consumers rank the bundle qualities in lexicographic order[16]:

$$\chi(A1B1) > \chi(A1B2) > ... > \chi(A1Bn) \geq \chi(A2B1) > ... > \chi(AmBn) \quad \text{(i)}$$

Ordering (i) will be at stake as long as $\chi(AiBn) \geq \chi(Ai+1B1), i = 1,...,m$. The latter condition, however, imposes different restrictions on the range of good qualities dependent on how they interact each with another when defining the qualities of the bundles:

- Additive function

If the good qualities add to each other, the quality of the bundle would be given as a sum of the qualities of the two goods of type A and B in it. The respective condition for lexicographic ordering would look as follows:

$$f(A_i) - f(A_{i+1}) \geq g(B_1) - g(B_n), i = 1,...,m \quad \text{(ii)}$$

i.e. the largest indivisible unit of measuring the difference between the qualities of the goods of type B is at most equal to the smallest indivisible unit of measuring the difference between the qualities of the goods of type A.

[16] This is a weak definition of lexicographic ordering. The strong one requires strict inequality $\chi(A1Bn) > \chi(A2B1)$ to hold. The succinct statement of the general solution for all the three setups, however, is facilitated by the assumption for identical ranges of the two types of qualities $(\underline{F} = \underline{G} \text{ and } \overline{F} = \overline{G})$ which is feasible only when condition (i) is allowed to hold with identity between $\chi(A1B2)$ and $\chi(A2B1)$ in the case of duopoly market $(n = 2)$.

- Multiplicative function

If the good qualities multiply each other's magnitude, the quality of the bundle could be represented as a multiplicative function of the qualities of the two goods of type A and B in it. The corresponding condition for lexicographic ordering would look as follows:

$$\frac{f(A_i)}{f(A_{i+1})} \geq \frac{g(B_1)}{g(B_n)}$$

which after subtracting 1 from both sides is equivalent to the following inequality:

$$\frac{f(A_i) - f(A_{i+1})}{f(A_{i+1})} > \frac{g(B_1) - g(B_n)}{g(B_n)}$$

The above condition is compatible with (ii) as long as the following inequality holds:

$$g(B_n) \geq f(A_{i+1}) \geq \frac{f(A_{i+1})[g(B_1) - g(B_n)]}{f(A_i) - f(A_{i+1})} \qquad \text{(iii)}$$

- Minimum function

If in a bundle the magnitude of the higher good quality is limited down to the magnitude of the other good quality, the quality of the bundle could be represented as a minimum function of the qualities of the two goods of type A and B in it. The corresponding condition for lexicographic ordering would look as follows:

$$Min(A_i, B_n) > Min(A_{i+1}, B_1)$$

which holds only if one of the following three inequalities is true:

$$f(A_i) \geq g(B_1) > g(B_n) \geq f(A_{i+1})$$

$$g(B_1) \geq f(A_i) > g(B_n) \geq f(A_{i+1})$$

$$g(B_1) > g(B_n) \geq f(A_i) > f(A_{i+1})$$

Any of the above involves (iii) and could hold together with (ii). The general solution for all the three setups presented in the paper, however, corresponds to the following union of the first two conditions:

$$f(A_i) = g(B_1) > g(B_n) = f(A_{i+1}) \qquad \text{(iv)}$$

Thus, (ii) and (iv) appear as necessary conditions for the lexicographic ordering in (i) to be valid for any of the three assumed functional relationships between the qualities of the goods and the quality of their bundle. As one could easily guess from the statement of the equilibrium solution of the model in the paper, the assumption for lexicographical ordering is not crucial for the final result. It is introduced here only temporarily to facilitate the derivation of the condition for natural duopoly. Later on in step 3, the assumption will be released to show that the condition for natural duopoly would still apply to the markets with non-lexicographical ordering of the bundle qualities. So, the conditions (ii) and (iv) are assumed to hold only for the purpose of the analysis in the first two steps of the current procedure.

Given the lexicographical bundle quality ordering in (i) at a covered market $(\theta_{mn/O} \leq \underline{\theta})$, the profit-maximization problems of the firms offering products of type A would look as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^A = p_1^A\left[\left(\bar{\theta} - \frac{(p_1^A + p_n^B) - (p_2^A + p_1^B)}{d_{1n/21}}\right)\right]$$

$$\underset{p_2^A}{Max}\,\Pi_2^A = p_2^A\left[\left(\frac{(p_1^A + p_n^B) - (p_2^A + p_1^B)}{d_{1n/21}} - \frac{(p_2^A + p_n^B) - (p_3^A + p_1^B)}{d_{2n/31}}\right)\right]$$

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$$\underset{p_m^A}{Max}\,\Pi_m^A = p_m^A\left[\left(\frac{(p_{m-1}^A + p_n^B) - (p_m^A + p_1^B)}{d_{m-1n/m1}} - \underline{\theta}\right)\right]$$

After slight modification, the corresponding first-order conditions for optimality could be expressed by the following system of equations[17]:

$$\bar{\theta} = 2\theta_{1n/21} + \frac{(p_2^A + p_1^B) - p_n^B}{d_{1n/21}} \qquad \text{(v)}$$

$$\theta_{1n/21} = 2\theta_{2n/31} + \frac{(p_3^A + p_1^B) - p_n^B}{d_{2n/31}} + \frac{p_2^A}{d_{1n/21}} \qquad \text{(vi)}$$

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$$\theta_{m-2n/m-11} = 2\theta_{m-1n/m1} + \frac{(p_m^A + p_1^B) - p_n^B}{d_{m-1n/m1}} + \frac{p_{m-1}^A}{d_{m-2n/m-11}} \qquad \text{(vii)}$$

$$\underline{\theta} = 2\theta_{m-1n/m1} - \frac{(p_{m-1}^A + p_n^B) - p_1^B}{d_{m-1n/m1}}$$

[17] Apparently, the expressions for profits are quadratic functions of the firms' prices with respect to which they are maximized. It is trivial to show that their second derivative is negative. That is, the second-order condition for optimality always holds.

Combining (v), (vi) and (vii) leads to the relationship between the marginal taste parameters presented below:

$$\bar{\theta} > 2\theta_{1n/21} > 4\theta_{2n/31}$$

By induction it could be extended to hold for all $\theta_{in/i+11}$, $i = 1,2,...,m$, as follows:

$$\bar{\theta} > 2\theta_{1n/21} > 4\theta_{2n/31} > ... > 2^{m-1}\theta_{m-1n/m1} \qquad \text{(viii)}$$

Hence, it is sufficient to set the lower bound of the market to be high enough for at most A1 and A2 to have a positive demand in equilibrium:

$$\underline{\theta} \geq \frac{\bar{\theta}}{4} \text{ implies that } \underline{\theta} > \theta_{2n/31} > ... > 2^{m-3}\theta_{m-1n/m1} \qquad \text{(ix)}$$

Step 2 Given that only two firms of type A enter, derive a condition for A2 to have positive demand at most when bundled with B1 and B2.

In compliance with the assumptions for lexicographical ordering of the bundle qualities (i) and covered market, the profit-maximization problems of the firms offering products of type B look as follows:

$$\underset{p_1^B}{Max}\,\Pi_1^B = p_1^B\left[\left(\overline{\theta} - \frac{\left(p_1^A + p_1^B\right) - \left(p_1^A + p_2^B\right)}{d_{11/12}}\right) + \left(\frac{\left(p_1^A + p_n^B\right) - \left(p_2^A + p_1^B\right)}{d_{1n/21}} - \frac{\left(p_2^A + p_1^B\right) - \left(p_2^A + p_2^B\right)}{d_{21/22}}\right)\right]$$

$$\underset{p_2^B}{Max}\,\Pi_2^B = p_2^B\left[\left(\frac{\left(p_1^A + p_1^B\right) - \left(p_1^A + p_2^B\right)}{d_{11/12}} - \frac{\left(p_1^A + p_2^B\right) - \left(p_1^A + p_3^B\right)}{d_{12/13}}\right) + \left(\frac{\left(p_2^A + p_1^B\right) - \left(p_2^A + p_2^B\right)}{d_{21/22}} - \frac{\left(p_2^A + p_2^B\right) - \left(p_2^A + p_3^B\right)}{d_{22/23}}\right)\right]$$

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$$\underset{p_n^B}{Max}\,\Pi_n^B = p_n^B\left[\left(\frac{\left(p_1^A + p_{n-1}^B\right) - \left(p_1^A + p_n^B\right)}{d_{1n-1/1n}} - \frac{\left(p_1^A + p_n^B\right) - \left(p_2^A + p_1^B\right)}{d_{1n/21}}\right) + \left(\frac{\left(p_2^A + p_{n-1}^B\right) - \left(p_2^A + p_n^B\right)}{d_{2n-1/2n}} - \underline{\theta}\right)\right]$$

After slight modification, the corresponding first-order conditions for optimality are given by the following system of equations:

$$\overline{\theta} + \theta_{1n/21} = 2\theta_{11/12} + 2\theta_{21/22} + p_2^B\left(\frac{1}{d_{11/12}} + \frac{1}{d_{21/22}}\right) + \frac{p_1^B}{d_{1n/21}} \qquad \text{(x)}$$

$$\theta_{11/12} + \theta_{21/22} = 2\theta_{12/13} + 2\theta_{22/23} + p_2^B\left(\frac{1}{d_{11/12}} + \frac{1}{d_{21/22}}\right) + p_3^B\left(\frac{1}{d_{12/13}} + \frac{1}{d_{22/23}}\right) \qquad \text{(xi)}$$

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$$\theta_{1n-2/1n-1} + \theta_{2n-2/2n-1} = 2\theta_{1n-1/1n} + 2\theta_{2n-1/2n} + p_{n-1}^B\left(\frac{1}{d_{1n-2/1n-1}} + \frac{1}{d_{2n-2/2n-1}}\right) + p_n^B\left(\frac{1}{d_{1n-1/1n}} + \frac{1}{d_{2n-1/2n}}\right) \text{(xii)}$$

$$\theta_{1n-1/1n} + \theta_{2n-1/2n} = \theta_{1n/21} + \underline{\theta} + p_n^B\left(\frac{1}{d_{1n-1/1n}} + \frac{1}{d_{2n-1/2n}} + \frac{1}{d_{1n/21}}\right)$$

By adding $\overline{\theta}$ to both sides of the equality in (v) the following condition could be shown to hold:

$$\frac{3\overline{\theta}}{2} > \overline{\theta} + \theta_{1n/21}$$

which in combination with (x)-(xii) implies the following relationship between the marginal taste parameters:

$$\overline{\theta} > \frac{4}{3}\theta_{21/22} > \frac{8}{3}\theta_{22/23} > \ldots > \frac{2^n}{3}\theta_{1n-1/1n} \qquad \text{(xiii)}$$

Hence, for at most B1 and B2 to have positive demand when bundled with A2, it is sufficient the following condition to hold:

$$\underline{\theta} > \frac{3}{8}\overline{\theta} \qquad \text{(xiv)}$$

Step 3 Relax the assumption for lexicographic ordering (ii)-(iv) of the bundle qualities and derive the condition for (xiv) to imply at most B1 and B2 to have positive demand when bundled with A2 in markets with non-lexicographic bundle quality ranking

The procedure of deriving condition (xiv) in step 1 and 2 could still apply to markets with non-lexicographic ranking of the bundle qualities. To show that, first the possible options of ranking orders will be analyzed.

Note that whatever is the bundle quality ranking always the bundle of the best ranked goods of both types, A1B1, will have the top quality.

Next, note that the lowest-quality bundle containing A1 should always be A1Bn while the worst-quality bundle containing A2 should be respectively A2Bn.

Also, the best-quality bundles containing respectively A2 and B2 should always be A2B1 and A3B1.

Hence, as long as A1Bn is better than A2B1 and A2Bn is better than A3B1, condition (v) and (vi) still hold, so that condition (viii) is partially valid as shown below:

$$\underline{\theta} \geq \frac{\overline{\theta}}{4} \text{ implies that } \underline{\theta} > \theta_{2n/31} \qquad \text{(xv)}$$

Condition (xv) is sufficient for (ix)-(xii) to be still valid. Therefore, the restriction of (xiv) would still ensure that at most B1 and B2 have positive demand when bundled with A2.

It remains to check whether condition (xiv) would be still sufficient when A1Bn is worse than A2B1 and A2Bn is worse than A3B1. Denote by k $(k = 2,...,n-1)$ the index of the B-type good in the lowest-quality bundle of A1 which is still better than A2B1 and by l $(l = 2,...,n-1)$[18] the index of the B-type good in the lowest quality bundle of A2 which is still better than A3B1. Then, two general rankings of the bundles could be distinguished, when A3B1 is better than A1B*k+1* $(\chi_{11} > ... > \chi_{1k} > \chi_{21} > ... > \chi_{2l} > \chi_{31} > ... > \chi_{mn})$ and when it is worse $(\chi_{11} > ... > \chi_{1k} > \chi_{21} > ... > \chi_{2l} > \chi_{1k+1} > ... > \chi_{mn})$.

[18] In fact, *l* could be assigned also a value of 1 but this case is not interesting for the purpose of the current paper because in equilibrium A2B2 would not have a sufficiently high rank to drive out A2B1. Moreover, it is precluded by the analysis in step 4.

Let A3B1 be better than A1B*k+1*. Then, if the supplier of good A1 sells it only in combination with B1 to B*k*, its profit-maximization problem will look as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^A = p_1^A\left[\left(\bar{\theta} - \frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}}\right)\right] \qquad \text{(xvi)}$$

Its solution implies that $\bar{\theta} > 2\theta_{1k/21}$ i.e. $D_1^A = \bar{\theta} - \theta_{1k/21} > \theta_{1k/21}$.

Analogously, if good A2 sells it only in combination with B1 to B*l*, its profit-maximization problem will look as follows:

$$\underset{p_2^A}{Max}\,\Pi_2^A = p_2^A\left[\left(\frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}} - \frac{\left(p_2^A + p_l^B\right) - \left(p_3^A + p_1^B\right)}{d_{2l/31}}\right)\right] \qquad \text{(xvii)}$$

Its solution implies that $\theta_{1k/21} > 2\theta_{2l/31}$ i.e. $D_2^A = \theta_{1k/21} - \theta_{2l/31} > \theta_{2l/31}$.

Respectively, if bundles A1B*k+1* to A1B*n* and A2B*l+1* to A2B*n* have positive market shares, the profit-maximizing problems of the two firms supplying A1 and A2 will be as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^A = p_1^A\left[\left(\bar{\theta} - \frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}}\right) + D_{1k+1} + \ldots + D_{1n}\right]$$

$$\underset{p_2^A}{Max}\,\Pi_2^A = p_2^A\left[\left(\frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}} - \frac{\left(p_2^A + p_l^B\right) - \left(p_3^A + p_1^B\right)}{d_{2l/31}}\right) + D_{2l+1} + \ldots + D_{2n}\right]$$

Since A1B*k* is better than any A1B*k+h* $\left(k = 2,\ldots,n-1; h = 1,\ldots,n-k\right)$, its equilibrium market share should be less sensitive to price $\left(\left|\frac{\partial D_{1k+h}}{\partial p_1^A}\right| > \left|\frac{\partial D_{1k}}{\partial p_1^A}\right|\right)$ and respectively larger $\left(D_{1k+h} < D_{1k+h}\right)$. Hence, the optimal price here must be lower, i.e. $p_1^A\left(D_1^A > \sum_{j=1}^{k} D_{1j}\right) < p_1^A\left(D_1^A = \sum_{j=1}^{k} D_{1j}\right)$. At lower p_1^A, however, the market share of the bundles from A1B1 to A1B*k* will be larger $D_1^A > \bar{\theta} - \theta_{1k/21} > \theta_{1k/21}$.

Analogously, it could be shown that the equilibrium demand for the bundles from A2B1 to A2B*l* will also be larger when A2B*l+1* to A2B*n* have positive market shares.

Hence, as long as A1B*k+1* is worse than A3B1 and $k, l \geq 3$, the restriction of (xv) $\underline{\theta} \geq \frac{\bar{\theta}}{4}$ is sufficient for (ix)-(x) to be still valid. Thus, the restriction of (xiv) would still ensure that at most B1 and B2 have positive demand when bundled with A2.

For k and/or $l < 3$, $\underline{\theta} \geq \frac{\bar{\theta}}{4}$ still implies that $\underline{\theta} > \theta_{2l/31}$ and the restriction of (xv) is sufficient for similar identities as (ix)-(x) to hold as in the example ($k = l = 2$) shown below:

$$\bar{\theta} + \theta_{1k/21} = 2\theta_{11/12} + 2\theta_{21/22} + \frac{p_2^B}{d_{11/12}} + \frac{p_2^A - p_3^A}{d_{21/31}} + \frac{p_1^B}{d_{1k/21}}$$

$$\theta_{11/12} + \theta_{21/22} = 2\theta_{12/21} + 2\theta_{22/31} + \frac{p_2^A + p_1^B - p_1^A}{d_{12/21}} + \frac{p_3^A + p_1^B - p_2^A}{d_{22/31}} + p_2^B\left(\frac{1}{d_{11/12}} + \frac{1}{d_{21/22}}\right)$$

For the restriction of (xiv) to ensure that at most B1 and B2 have positive demand when bundled with A2, the following addition conditions on prices must hold in equilibrium:

$$p_2^A + p_1^B \geq p_1^A \qquad \text{(xviii)}$$

$$p_1^B \geq p_2^A$$

which will be proven (see step 4) to be satisfied with inequality for the particular equilibrium studied in the paper.

Thus, when A1B*k+1* is worse than A3B1 the restriction of (xiv) ensures that at most B1 and B2 have positive demand when bundled with A2.

Alternatively, let A3B1 be worse than A1B*k+1*. Then, if the supplier of good A1 sells it only in combination with B1 to B*k*, its profit-maximization problem will still be given by (xvi).

Respectively, if the supplier of good A2 sells it only in combination with B1 to B*l*, its profit-maximization problem will look similar but not exactly as in (xvii):

$$\underset{p_2^A}{Max}\,\Pi_2^A = p_2^A\left[\left(\frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}} - \frac{\left(p_2^A + p_l^B\right) - \left(p_1^A + p_{k+1}^B\right)}{d_{2l/1k+1}}\right)\right]$$

Its solution implies that $\theta_{1k/21} > 2\theta_{2l/1k+1}$ i.e. $D_2^A = \theta_{1k/21} - \theta_{2l/1k+1} > \theta_{2l/1k+1}$.

Respectively, if bundles A1B*k+1* to A1B*n* and A2B*l+1* to A2B*n* are present, the profit-maximizing problems of the two firms supplying A1 and A2 will be as follows:

$$\underset{p_1^A}{Max}\,\Pi_1^A = p_1^A\left[\left(\bar{\theta} - \frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}}\right) + D_{1k+1} + \ldots + D_{1n}\right]$$

$$\underset{p_2^A}{Max}\Pi_2^A = p_2^A\left[\left(\frac{\left(p_1^A + p_k^B\right) - \left(p_2^A + p_1^B\right)}{d_{1k/21}} - \frac{\left(p_2^A + p_l^B\right) - \left(p_3^A + p_1^B\right)}{d_{2l/1k+1}}\right) + D_{2l+1} + \ldots + D_{2n}\right]$$

From here on the proof that the restriction of (xiv) ensures that at most B1 and B2 have positive demand when bundled with A2 is analogous to the case when A3B1 is better than A1B*k+1*.

In summary, the results from the analysis in this step imply that the condition (xiv) for at most B1 and B2 to have positive demand when bundled with A2 in markets with lexicographic bundle quality ranking also applies to markets with non-lexicographic ranking.

<u>Step 4</u> Suppose that when condition (xiv) holds the suppliers of A1 and A2 believe that only the bundles A1B1 and A2B2 will have positive market share. Check the conditions at which this outcome could be the subgame equilibrium in the pricing stage.

The corresponding profit maximization problems of the four firms that produce the goods in the two bundles, A1B1 and A2B2, look as follows:

$$\underset{p_1^A}{Max}\Pi_1^A = p_1^A\left[\left(\overline{\theta} - \frac{\left(p_1^A + p_1^B\right) - \left(p_2^A + p_2^B\right)}{d_{11/22}}\right)\right] \tag{xix}$$

$$\underset{p_1^B}{Max}\Pi_1^B = p_1^B\left[\left(\overline{\theta} - \frac{\left(p_1^A + p_1^B\right) - \left(p_2^A + p_2^B\right)}{d_{11/22}}\right)\right] \tag{xx}$$

$$\underset{p_2^A}{Max}\Pi_2^A = p_2^A\left[\left(\frac{\left(p_1^A + p_1^B\right) - \left(p_2^A + p_2^B\right)}{d_{11/22}} - \underline{\theta}\right)\right] \tag*{(xxi))}$$

$$\underset{p_2^B}{Max}\Pi_2^B = p_2^B\left[\left(\frac{\left(p_1^A + p_1^B\right) - \left(p_2^A + p_2^B\right)}{d_{11/22}} - \underline{\theta}\right)\right] \tag{xxii}$$

The solution for the equilibrium prices yields the following expressions in term of the quality difference $d_{11/22}$:

$$p_1^A = p_1^B = p_1^* = \frac{\left(3\overline{\theta} - 2\underline{\theta}\right)}{5} d_{11/22} \tag{xxiii}$$

$$p_2^A = p_2^B = p_2^* = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)}{5} d_{11/22} \tag{xxiv}$$

Note that no optimal price of a good of given type is exceeded by a good with the same or lower quality index of the same type or another. Hence, the above equilibrium prices satisfy with inequality both requirements of the condition (xviii) in the previous step. Therefore, indeed having $\underline{\theta} > \frac{3}{8}\bar{\theta}$ ensures that A2 will have positive demand in bundle with at most two goods of type B - B1 and B2, for any bundle quality ordering.

Since the prices of the equally indexed goods of each type are identical and A1B2 is strictly better than A2B1, the latter bundle would be driven out from the market at these prices. It remains to check the condition for A1B2 and A1B3 to be driven out as well.

The condition for A1B2 to have no positive market share at the optimal prices derived above is as follows:

$$D_{12} = \theta_{11/12} - \theta_{12/22} = \frac{\left(p_1^A + p_1^B - p_1^A - p_2^B\right)}{d_{11/12}} - \frac{\left(p_1^A + p_2^B - p_2^A - p_2^B\right)}{d_{12/22}} = \left(p_1^* - p_2^*\right)\left(\frac{1}{d_{11/12}} - \frac{1}{d_{12/22}}\right) \leq 0$$

i.e.

$$d_{12/22} \leq d_{11/12} \text{ or } \chi_{12} \leq \frac{\chi_{11} + \chi_{22}}{2}$$

The further representation of the condition in terms of single good quality depends on what type of functional relationship is assumed between the good qualities in the bundle:

- Given summation function, the condition would look as follows:

$$f(A_1) - f(A_2) \leq g(B_1) - g(B_2) \qquad \text{(xxv)}$$

- Given multiplication function, the condition would look as follows:

$$\frac{f(A_2)}{f(A_1)} + \frac{g(B_1)}{g(B_2)} \geq 2 \qquad \text{(xxvi)}$$

- Given minimization function, the condition would look as follows[19]:

$$g(B_2) \leq \frac{f(A_1) + f(A_2)}{2} \text{ if } g(B_1) \geq f(A_1) > g(B_2) \geq f(A_2) \qquad \text{(xxvii)}$$

or

$$g(B_2) \leq \frac{g(B_1) + f(A_2)}{2} \text{ if } f(A_1) \geq g(B_1) > g(B_2) \geq f(A_2) \qquad \text{(xxviii)}$$

Note that conditions (xxv)-(xxviii) hold always when

$$f(A_1) = g(B_1) \text{ and } f(A_2) = g(B_2) \qquad \text{(xxix)}$$

[19] Note that in the case of minimization functional relationship between goods' qualities and the qualities of their bundle, for bundle A1B2 to be better than A2B1, the following ordering of goods' qualities must be valid: $f(A_1) > g(B_2) > f(A_2)$.

Next, the condition could be derived for B3 to stay out of the market in equilibrium:

$$D_{13} = \theta_{11/13} - \theta_{13/22} = \frac{\left(p_1^A + p_1^B - p_1^A - p_3^B\right)}{d_{11/13}} - \frac{\left(p_1^A + p_3^B - p_2^A - p_2^B\right)}{d_{13/22}} = \frac{d_{11/22}}{5}\left[\left(3\overline{\theta} - 2\underline{\theta}\right)d_{13/22} - \left(4\underline{\theta} - \overline{\theta}\right)d_{11/13}\right] \leq 0$$

i.e.

$$d_{13/22} \leq \frac{\left(4\underline{\theta} - \overline{\theta}\right)}{\left(3\overline{\theta} - 2\underline{\theta}\right)} d_{11/13} \text{ or } \chi_{13} \leq \frac{\chi_{11}\left(4\underline{\theta} - \overline{\theta}\right) + \chi_{22}\left(3\overline{\theta} - 2\underline{\theta}\right)}{2\left(\overline{\theta} + \underline{\theta}\right)}$$

Again, the representation of the condition in terms of single good quality depends on what type of functional relationship is assumed between the good qualities in the bundle:

- Given summation function, the condition would look as follows:

$$g(B_3) \leq \frac{\left(2\underline{\theta} - 3\overline{\theta}\right)f(A_1) + \left(4\underline{\theta} - \overline{\theta}\right)g(B_1) + \left(3\overline{\theta} - 2\underline{\theta}\right)\left[f(A_2) + g(B_2)\right]}{2\left(\overline{\theta} + \underline{\theta}\right)} \qquad \text{(xxx)}$$

which always holds because $\frac{\left(4\underline{\theta} - \overline{\theta}\right)g(B_1) + \left(3\overline{\theta} - 2\underline{\theta}\right)g(B_2)}{2\left(\overline{\theta} + \underline{\theta}\right)} > g(B_2) > g(B_3)$

- Given multiplication function, the condition would look as follows:

$$g(B_3) \leq \frac{\left(4\underline{\theta} - \overline{\theta}\right)f(A_1)g(B_1) + \left(3\overline{\theta} - 2\underline{\theta}\right)f(A_2)g(B_2)}{2f(A_1)\left(\overline{\theta} + \underline{\theta}\right)} \qquad \text{(xxxi)}$$

which always holds as long as $\frac{f(A_1)}{f(A_2)} \leq \frac{9}{7} < \frac{3\overline{\theta} - 2\underline{\theta}}{2\left(\overline{\theta} + \underline{\theta}\right) - \left(4\underline{\theta} - \overline{\theta}\right)\frac{g(B_1)}{g(B_2)}}$ for $\underline{\theta} \geq \frac{3}{8}\overline{\theta}$.

- Given minimization function, the condition would look as follows:

$$g(B_3) \leq \frac{\left(4\underline{\theta} - \overline{\theta}\right)f(A_1) + \left(3\overline{\theta} - 2\underline{\theta}\right)f(A_2)}{2\left(\overline{\theta} + \underline{\theta}\right)} \text{ if } g(B_1) \geq f(A_1) > g(B_2) \geq f(A_2) \qquad \text{(xxxii)}$$

which holds as long as $\frac{f(A_1)}{g(B_2)} \geq 2 - \frac{f(A_2)}{g(B_2)} > \frac{2\left(\overline{\theta} + \underline{\theta}\right) - \left(3\overline{\theta} - 2\underline{\theta}\right)\frac{f(A_2)}{g(B_2)}}{\left(4\underline{\theta} - \overline{\theta}\right)}$ for $\underline{\theta} < \frac{2}{3}\overline{\theta}$.

or

$$g(B_3) \leq \frac{\left(4\underline{\theta} - \overline{\theta}\right)g(B_1) + \left(3\overline{\theta} - 2\underline{\theta}\right)f(A_2)}{2\left(\overline{\theta} + \underline{\theta}\right)} \text{ if } f(A_1) \geq g(B_1) > g(B_2) \geq f(A_2) \qquad \text{(xxxiii)}$$

which holds as long as $\frac{g(B_1)}{g(B_2)} \geq 2 - \frac{f(A_2)}{g(B_2)} > \frac{2(\overline{\theta}+\underline{\theta}) - (3\overline{\theta}-2\underline{\theta})\frac{f(A_2)}{g(B_2)}}{(4\underline{\theta}-\overline{\theta})}$ for $\underline{\theta} < \frac{2}{3}\overline{\theta}$.

Note that conditions (xxx), (xxxii) and (xxxiii) hold always (for $\frac{3}{8}\overline{\theta} \leq \underline{\theta} < \frac{2}{3}\overline{\theta}$) given that the identities in (xxix) are valid. For also condition (xxxi) to hold, the qualities of the goods must be restricted further as follows:

$$f(A_1) = g(B_1) > f(A_2) = g(B_2) \geq \frac{7}{9} f(A_1) \qquad \text{(xxxiv)}$$

<u>Step 5</u> Derive a condition for A2B2 to have positive demand.

The condition for A2B2 to have positive demand is:

$$\theta_{11/22} - \underline{\theta} = \frac{2\overline{\theta}-3\underline{\theta}}{5} > 0 \text{ which holds for any } \underline{\theta} < \frac{2\overline{\theta}}{3} \qquad \text{(xxxv)}$$

Thus, conditions (xiv), (xxxiv) and (xxxv) ensure that there exists an equilibrium at which only A1B1 and A2B2 will have a positive demand. This completes the proof of proposition 1.

Appendix B: Proof of Proposition 2

Given that the conditions for natural duopoly hold, the optimal profits of the four firms in the market could be expressed in terms of the bundle qualities by substituting for the subgame-equilibrium solution in the pricing stage:

$$\Pi_1^A = \Pi_1^B = \frac{\left(3\overline{\theta} - 2\underline{\theta}\right)^2}{25} d_{11/22} = \frac{\left(3\overline{\theta} - 2\underline{\theta}\right)^2}{25} \left(\chi_{11} - \chi_{22}\right)$$

$$\Pi_2^A = \Pi_2^B = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)^2}{25} d_{11/22} = \frac{\left(2\overline{\theta} - 3\underline{\theta}\right)^2}{25} \left(\chi_{11} - \chi_{22}\right)$$

Note that all profits are strictly increasing in the quality χ_{11} of bundle A1B1 and strictly decreasing in the quality χ_{22} of bundle A2B2. Hence, the optimal quality choice is given by the corner solution where A1 and B1 choose the upper bounds of the intervals in which their type qualities vary i.e. $\overline{F}$ and $\overline{G}$. Respectively, A2 and B2 choose the lowest qualities at which the market would be still covered and no fifth entrant would have an incentive to enter the market.

The condition for covered market is as follows:

$$\theta_{22/O} = \frac{2 p_2^*}{\left(\chi_{22} - \chi_O\right)} = \frac{2\left(2\overline{\theta} - 3\underline{\theta}\right)\left(\chi_{11} - \chi_{22}\right)}{5\left(\chi_{22} - \chi_O\right)} \leq \underline{\theta}$$

That is, the lowest quality of A2B2 at which the market is covered is given by the following expression:

$$\chi_{22} = \frac{2\left(2\overline{\theta} - 3\underline{\theta}\right)\chi_{11} + 5\underline{\theta}\chi_O}{\left(4\overline{\theta} - \underline{\theta}\right)}$$

which looks as follows when the relationship between good qualities is assumed to be:

- additive function: $f\left(A_2\right) + g\left(B_2\right) = \dfrac{2\left(2\overline{\theta} - 3\underline{\theta}\right)\left(\overline{F} + \overline{G}\right) + 5\underline{\theta}\chi_O}{\left(4\overline{\theta} - \underline{\theta}\right)}$
- multiplicative function: $f\left(A_2\right) g\left(B_2\right) = \dfrac{2\left(2\overline{\theta} - 3\underline{\theta}\right)\overline{F}\,\overline{G} + 5\underline{\theta}\chi_O}{\left(4\overline{\theta} - \underline{\theta}\right)}$
- minimization function: $f\left(A_2\right) = \dfrac{2\left(2\overline{\theta} - 3\underline{\theta}\right)\overline{F} + 5\underline{\theta}\chi_O}{\left(4\overline{\theta} - \underline{\theta}\right)}$ for

  $\overline{G} = g\left(B_1\right) \geq f\left(A_1\right) = \overline{F} > g\left(B_2\right) \geq f\left(A_2\right)$ or,

  $f\left(A_2\right) = \dfrac{2\left(2\overline{\theta} - 3\underline{\theta}\right)\overline{G} + 5\underline{\theta}\chi_O}{\left(4\overline{\theta} - \underline{\theta}\right)}$ for $\overline{F} = f\left(A_1\right) \geq g\left(B_1\right) = \overline{G} > g\left(B_2\right) \geq f\left(A_2\right)$

Note that whether the producers of the lower-quality goods, A2 and B2, would find it optimal to choose the lower quality bounds, $\underline{F}$ and $\underline{G}$, or an internal solution, depends on how large is the quality of the outside option χ_O. When it is low enough, the worst feasible qualities could be assigned to A2 and B2 and still the market will be covered. The respective restriction on χ_O for each of the three functional forms is given below:

- additive function: $0 \le \chi_O \le \dfrac{(4\overline{\theta}-\underline{\theta})(\underline{F}+\underline{G})-2(2\overline{\theta}-3\underline{\theta})(\overline{F}+\overline{G})}{5\underline{\theta}}$
- multiplicative function: $0 \le \chi_O \le \dfrac{(4\overline{\theta}-\underline{\theta})\underline{F}\,\underline{G}-2(2\overline{\theta}-3\underline{\theta})\overline{F}\,\overline{G}}{5\underline{\theta}}$
- minimization function: $0 \le \chi_O \le \dfrac{(4\overline{\theta}-\underline{\theta})\underline{F}-2(2\overline{\theta}-3\underline{\theta})\overline{F}}{5\underline{\theta}}$ for $\overline{G} \ge \overline{F} > \underline{G} \ge \underline{F}$

or

$$0 \le \chi_O \le \frac{(4\overline{\theta}-\underline{\theta})\underline{F}-2(2\overline{\theta}-3\underline{\theta})\overline{G}}{5\underline{\theta}} \text{ for } \overline{F} \ge \overline{G} > \underline{G} \ge \underline{F}\,.$$

Hence, the quality choice of $f(A_1)=\overline{F}; g(B_1)=\overline{G}; f(A_2)=\underline{F}$ and $g(B_2)=\underline{G}$ will be subgame equilibrium in a market with range of the taste parameters given by condition (xiv) and (xxxv) as long as the supports of the quality functions, $f(A_i)$ and $g(B_j)$, and χ_O comply with the following restrictions:

$$\overline{F}=\overline{G}>\underline{G}=\underline{F}\ge\frac{7}{9}\overline{F} \qquad \text{(xxxvi)}$$

which ensures the validity of condition (xxxiv) and

$$0 \le \chi_O \le \frac{(4\overline{\theta}-\underline{\theta})\underline{F}^2-2(2\overline{\theta}-3\underline{\theta})\overline{F}^2}{5\underline{\theta}} \qquad \text{(xxxvii)}$$

which ensures the validity of the restrictions on χ_O for all the three considered functional forms above.

This completes the proof of Proposition 2.

Appendix X: Exogenously set duopoly – key results

For simplicity, let's assume that $\underline{\theta} = 0$.

Profit-maximization problems:

$$\underset{p_1^A}{Max}\,\Pi_1^A = p_1^A\left(\overline{\theta} - \theta_{11/22}\right) = p_1^A\left(\overline{\theta} - \frac{p_1^A + p_1^B - p_2^A - p_2^B}{d_{11/22}}\right)$$

$$\underset{p_1^A}{Max}\,\Pi_1^B = p_1^B\left(\overline{\theta} - \theta_{11/22}\right) = p_1^B\left(\overline{\theta} - \frac{p_1^A + p_1^B - p_2^A - p_2^B}{d_{11/22}}\right)$$

$$\underset{p_2^A}{Max}\,\Pi_2^A = p_2^A\left(\theta_{11/22} - \theta_{22/O}\right) = p_2^A\left(\frac{p_1^A + p_1^B - p_2^A - p_2^B}{d_{11/22}} - \frac{p_2^A + p_2^B}{d_{22/O}}\right)$$

$$\underset{p_2^B}{Max}\,\Pi_2^B = p_2^B\left(\theta_{11/22} - \theta_{22/O}\right) = p_2^B\left(\frac{p_1^A + p_1^B - p_2^A - p_2^B}{d_{11/22}} - \frac{p_2^A + p_2^B}{d_{22/O}}\right)$$

The corresponding system of first-order optimality conditions is as follows:

$$2p_1^A + p_1^B - p_2^A - p_2^B = \overline{\theta} d_{11/22}$$

$$p_1^A + 2p_1^B - p_2^A - p_2^B = \overline{\theta} d_{11/22}$$

$$d_{22/O}\,p_1^A + d_{22/O}\,p_1^B - 2\left(d_{11/22} + d_{22/O}\right)p_2^A - \left(d_{11/22} + d_{22/O}\right)p_2^B = 0$$

$$d_{22/O}\,p_1^A + d_{22/O}\,p_1^B - \left(d_{11/22} + d_{22/O}\right)p_2^A - 2\left(d_{11/22} + d_{22/O}\right)p_2^B = 0$$

Respectively, the solution for prices in terms of quality differentials is given by the expressions below:

$$p_1^A = p_1^B = \frac{3\overline{\theta} d_{11/22}\left(d_{11/22} + d_{22/O}\right)}{9d_{11/22} + 5d_{22/O}}$$

$$p_2^A = p_2^B = \frac{2\overline{\theta} d_{11/22} d_{22/O}}{9d_{11/22} + 5d_{22/O}}$$

Respectively, the marginal taste parameters could be expressed as follows:

$$\theta_{22/O} = \frac{4\bar{\theta} d_{11/22}}{9d_{11/22} + 5d_{22/O}}$$

$$\theta_{11/22} = \frac{2\bar{\theta}\left(3d_{11/22} + d_{22/O}\right)}{9d_{11/22} + 5d_{22/O}}$$

Note, that $\theta_{22/O}$ is positive which implies that market would not be covered in equilibrium.

Hence, the optimal profits look as follows:

$$\Pi_1^A = \Pi_1^B = \frac{9\bar{\theta}^2 d_{11/22}\left(d_{11/22} + d_{22/O}\right)^2}{\left(9d_{11/22} + 5d_{22/O}\right)^2}$$

$$\Pi_2^A = \Pi_2^B = \frac{4\bar{\theta}^2 d_{11/22} d_{22/O}\left(d_{11/22} + d_{22/O}\right)^2}{\left(9d_{11/22} + 5d_{22/O}\right)^2}$$

To prove that these profits are really the equilibrium ones, it needs to be validated that at the derived prices the middle quality bundles A1B2 and A2B1 will have zero demand i.e. $\theta_{11/12} \leq \theta_{12/22}$. After substitution for the price expressions in terms of quality differences, the latter inequality takes the following form:

$$\theta_{11/12} = \frac{\bar{\theta} d_{11/22}\left(3d_{11/22} + d_{22/O}\right)}{d_{11/12}\left(9d_{11/22} + 5d_{22/O}\right)} \leq \frac{\bar{\theta} d_{11/22}\left(3d_{11/22} + d_{22/O}\right)}{d_{12/22}\left(9d_{11/22} + 5d_{22/O}\right)} = \theta_{12/22}$$

Or after simplification:

$$d_{12/22} \leq d_{11/12}$$

That is, the quality differential of the second-best bundle from the best bundle needs to be at most equal to the quality differential from the worst bundle.

The representation the above expressions in terms of the goods qualities depends on the assumption for their functional relationship when defining the bundle's quality, as shown below.

First, if it is assumed that the quality of the one type of good adds to the quality of the other type of good in the bundle, the optimal solution for the prices in terms of goods' qualities takes the following form:

$$p_1^A = p_1^B = \frac{3\bar{\theta}\left(\chi_1^A - \chi_2^A + \chi_1^B - \chi_2^B\right)\left(\chi_1^A + \chi_1^B - \chi_O\right)}{9\left(\chi_1^A + \chi_1^B\right) - 4\left(\chi_2^A + \chi_2^B\right) - 5\chi_O}$$

$$p_2^A = p_2^B = \frac{2\bar{\theta}\left(\chi_1^A - \chi_2^A + \chi_1^B - \chi_2^B\right)\left(\chi_2^A + \chi_2^B - \chi_O\right)}{9\left(\chi_1^A + \chi_1^B\right) - 4\left(\chi_2^A + \chi_2^B\right) - 5\chi_O}$$

Second, if it is assumed that the quality of the one type of good enhances the quality of the other type of good in the bundle, the optimal solution for the prices in terms of goods' qualities takes the following form:

$$p_1^A = p_1^B = \frac{3\bar{\theta}\left(\chi_1^A \chi_1^B - \chi_2^A \chi_2^B\right)\left(\chi_1^A \chi_1^B - \chi_O\right)}{9\chi_1^A \chi_1^B - 4\chi_2^A \chi_2^B - 5\chi_O}$$

$$p_2^A = p_2^B = \frac{2\bar{\theta}\left(\chi_1^A \chi_1^B - \chi_2^A \chi_2^B\right)\left(\chi_2^A \chi_2^B - \chi_O\right)}{9\chi_1^A \chi_1^B - 4\chi_2^A \chi_2^B - 5\chi_O}$$

Third, if it is assumed that the lower quality of the two types is also the bundle quality, the optimal solution for the prices in terms of goods' qualities takes the following form:

$$p_1^A = p_1^B = \frac{3\bar{\theta}\left(Min\left(\chi_1^A, \chi_1^B\right) - Min\left(\chi_2^A, \chi_2^B\right)\right)\left(Min\left(\chi_1^A, \chi_1^B\right) - \chi_O\right)}{9Min\left(\chi_1^A, \chi_1^B\right) - 4Min\left(\chi_2^A, \chi_2^B\right) - 5\chi_O}$$

$$p_2^A = p_2^B = \frac{2\bar{\theta}\left(Min\left(\chi_1^A, \chi_1^B\right) - Min\left(\chi_2^A, \chi_2^B\right)\right)\left(Min\left(\chi_2^A, \chi_2^B\right) - \chi_O\right)}{9Min\left(\chi_1^A, \chi_1^B\right) - 4Min\left(\chi_2^A, \chi_2^B\right) - 5\chi_O}$$

The respective expressions of the optimal profits in terms goods' qualities are as follows:

- additive function: