Economic growth

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Previous lecture

Classical model:

- output determined by factors of production
- distribution of income determined by marginal products
- flexible prices and loanable fund equilibirium
- classical dichotomy - real economy separate from monetary factors

Essentially, all the action is on the supply side.

- more justifiable in the long run than in the short-run

Today

- over long-run, economy usually grows
- think about long-run growth in terms of above model
  - Solow (1956)[2]
- look at some empirics

[2] A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, Vol. 70, No. 1

Side note

Economists usually focus on real GDP per capita as measure of growth. Why?

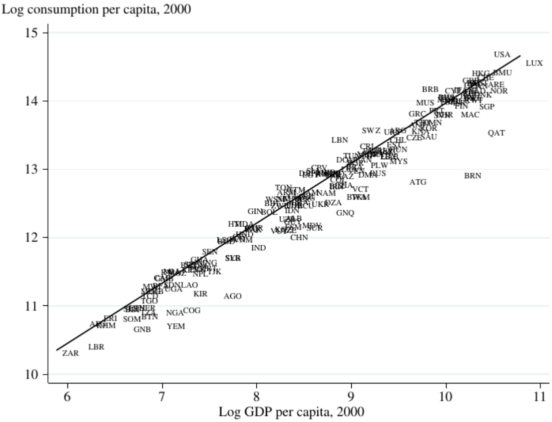
GDP correlated with:

- private consumption
- other measures of quality of life, e.g. life expectancy
- even happiness

Of course, it has limitations too:

- accounts only for market activities
- often ignores externalities (pollution,...)

GDP vs. consumption



Source: Acemoglu (2009)[3]

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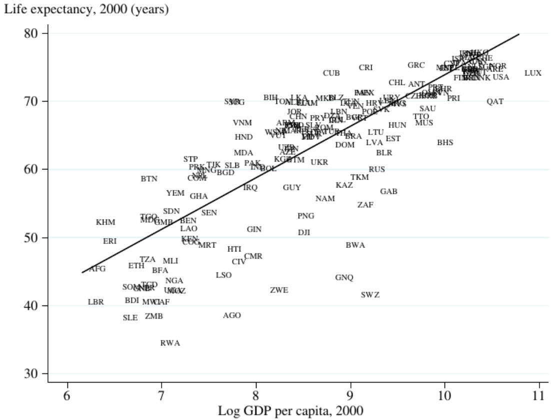
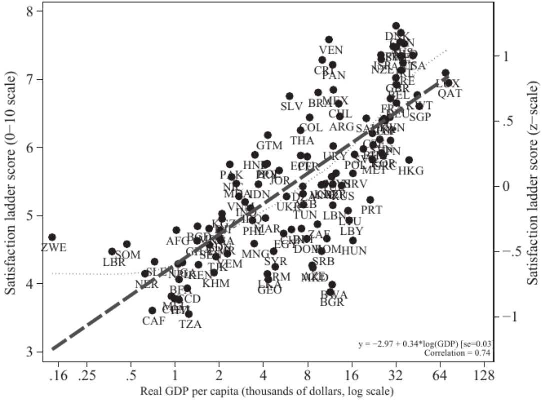
[3] *Introduction to Modern Economic Growth*, Princeton University Press

GDP vs. life expectancy



Source: Acemoglu (2009)

GDP vs. happiness



Source: Sacks, Stevenson & Wolfers (2012)[4]

[4]The New Stylized Facts About Income and Subjective Well-Being. *Emotion*, Vol. 12 No. 6

Motivation

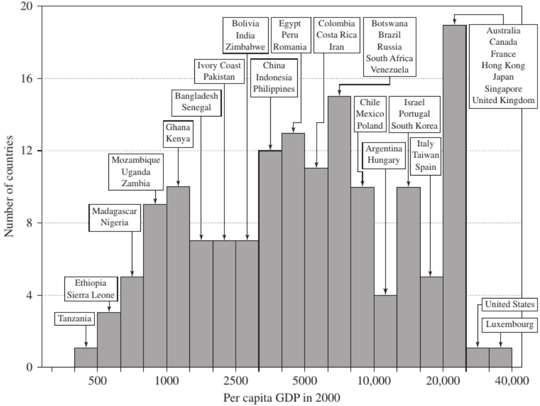
Why care about growth?

- even small differences in growth rate compound over time

$$Y_t = Y_0 \times (1+g)^t$$

  - if $g = 0.02$, over a century $\frac{Y_{100}}{Y_0} = 7.24$
  - if $g = 0.025$, over a century $\frac{Y_{100}}{Y_0} = 11.81$
- at the same time, there are large cross-sectional differences across countries
  - in current output
  - in past growth rates

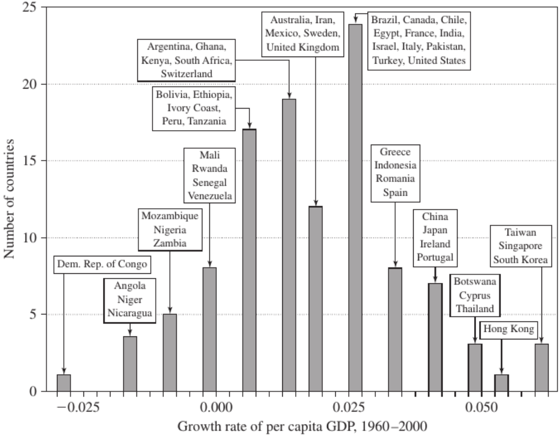
Current output



Source: Barro & Sala-i-Martin (2004)[5]

[5] *Economic Growth, 2nd ed.* MIT Press

Past growth



Source: Barro & Sala-i-Martin (2004)

Robert Lucas:

> *I do not see how one can look at figures like these without seeing them representing possibilities. Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: once one starts to think about them, it is hard to think about anything else.*[6]

[6] On the Mechanics of Economic Development. *Journal of Monetary Economics*. 22 July, 1988, pp. 5.

Production function

We'll start with most obvious explanation. Output is a function of input factors:

$$Y_t = F_t(K_t, L_t)$$

- t - time index
- prod. function itself can change over time (technological progress)
- but for now assume no progress, i.e. $F_t() = F()$

Can growth be explained through accumulation of capital and population increase?

Production function

Assume constant returns to scale:

$$F(cK, cL) = cF(K, L)$$

Choose $c = 1/L$, and denote per-capita values by lowercase

- $y = Y/L$
- $k = K/L$

Then we have

$$y = \frac{Y}{L} = \frac{1}{L}F(K, L) = F\left(\frac{K}{L}, \frac{L}{L}\right) = F(k, 1) = f(k)$$

Thus, with CRS output per capita is function of capital per worker ratio only.

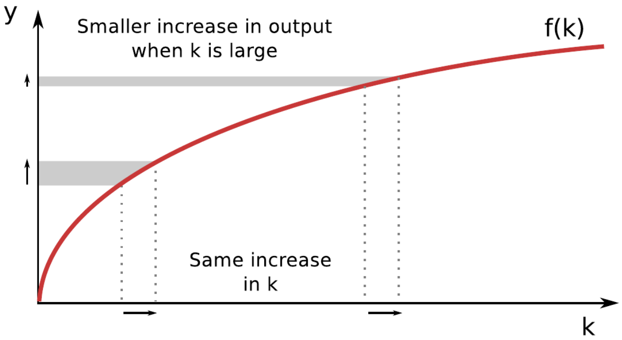
Production function

Assume that $f(k)$ has these properties:

- $f(0) = 0$
  - capital is essential input
- increasing
  - more machines per worker makes them more efficient...
- concave
  - ...but at decreasing pace
- $\lim_{k \to 0} f'(k) = \infty$
- $\lim_{k \to \infty} f'(k) = 0$
  - a.k.a. Inada conditions - these will make math easier

Production function

Typical shape:



Production function

Recall Cobb-Douglas:

$$F(K, L) = AK^{\alpha}L^{1-\alpha}$$

Then

$$f(k) = F(k, 1) = A \times k^{\alpha} \times 1^{1-\alpha} = A \times k^{\alpha}$$

E.g. for $\alpha = 0.5$ it looks like square root of k.

Factor prices

Recall last lecture - competitive prices are given by marginal products.

- wage $= \frac{\partial F(K,L)}{\partial L}$
- rental rate of capital $= \frac{\partial F(K,L)}{\partial K}$

With a bit of math, one can show

$$\frac{\partial F(K,L)}{\partial K} = f'(k)$$
$$\frac{\partial F(K,L)}{\partial L} = f(k) - kf'(k)$$

i.e. marginal products also depend just on capital-labor ratio.

How factors evolve

- assume that labor grows at fixed rate

$$L_t = (1 + g_L)L_{t-1}$$

- capital accumulates from investment, but also depreciates

$$K_t = (1 - \delta)K_{t-1} + I_{t-1}$$

  - K is stock variable, I is flow variable
  - without new investment, capital would decrease (machines breaking down...)
  - alternatively, think that portion of investment goes toward maintenance of existing capital

Investment

All output must be either consumed or used for investment:

$$Y_t = C_t + I_t$$

- we implicitely assume final output can be converted one-to-one into new capital

Assume that households save constant share of output/income:

$$C_t = (1 - s)Y_t, \ I_t = sY_t$$

- s: savings rate

Model dynamics

Now we have fully specified the model:

$$\begin{aligned} Y_t &= F(K_t, L_t) \\ I_t &= sY_t \\ L_{t+1} &= (1+g_L)L_t \\ K_{t+1} &= (1-\delta)K_t + I_t \end{aligned}$$

- for given initial conditions K_0, L_0, we can simulate the system above and obtain sequence $\{K_t, L_t, Y_t, I_t\}_{t=1}^{\infty}$
- how will the system behave over time?

Model dynamics

For simplicity, assume constant labor, i.e. $g_L = 0$ and $L_t = \bar{L}$.

Rewrite everything else in per capita terms:

- output
$$y_t = f(k_t)$$
- investment
$$i_t = \frac{I_t}{L_t} = \frac{sY_t}{\bar{L}} = sy_t$$
- capital accumulation
$$k_{t+1} = \frac{K_{t+1}}{\bar{L}} = \frac{(1-\delta)K_t + I_t}{\bar{L}} = (1-\delta)k_t + i_t$$

Steady state

We obtain a single difference equation for k:

$$k_{t+1} = (1 - \delta)k_t + sf(k_t)$$

A *steady state* is value k^* such that

$$k^* = (1 - \delta)k^* + sf(k^*)$$

- if capital-labor ratio is k^*, it will be same in next period...
- ...and forever after

Steady state

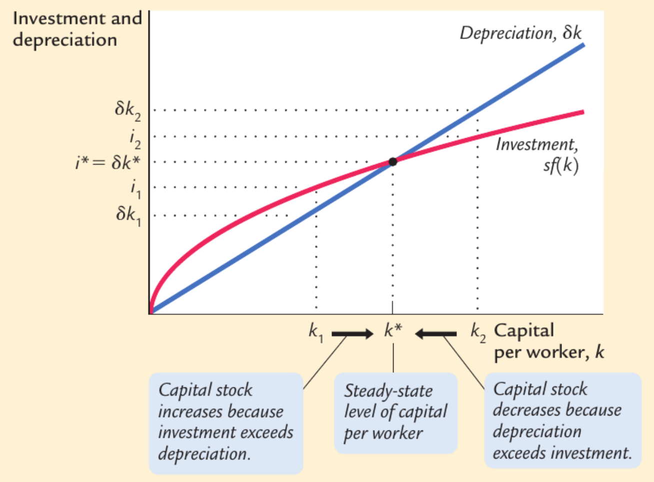
Claim: there are two steady states:

- $k^* = 0$ (the trivial one)
- $k^* > 0$ that solves $\delta k^* = sf(k^*)$
  - investment exactly covers depreciation

Moreover:

- for any $k_0 > 0$, the trajectory $k_1, k_2, \dots$ converges to the positive steady state

Steady state



Source: Mankiw, figure 8-4

Example

Cobb-Douglas technology: $f(k) = Ak^{\alpha}$

Steady state:

$$\delta k^* = sA(k^*)^{\alpha} \Rightarrow k^* = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}}$$

Steady state capital-labor ratio:

- increases with productivity A
- increases with saving rate s
- decreases with depreciation δ

The same holds for steady state output per capita.

Saving rate

Solow model predicts that output depends on saving rate.

- more saving -> more investment -> more capital accumulation -> more output

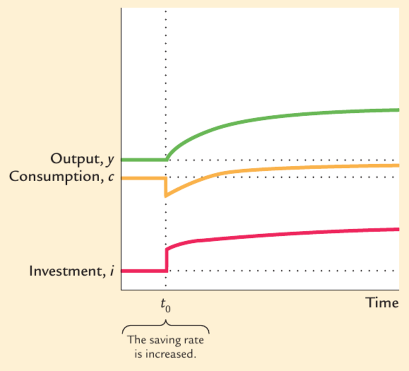
However, there is no growth in the steady state

- eventually, marginal product of new capital falls below depreciation rate
- thus Solow model can potentially explain cross-sectional differences in output
- but it cannot explain sustained growth

Transition dynamics

Let's say we're in steady state, and then saving rate goes up.

- thus consumption decreases in favor of investment
- over time capital accumulates, leading to higher output and typically higher consumption



Source: Mankiw, figure 8-10

Golden rule

So is more savings always good? No.

- higher s leads to higher output in steady state
- but we should care about consumption

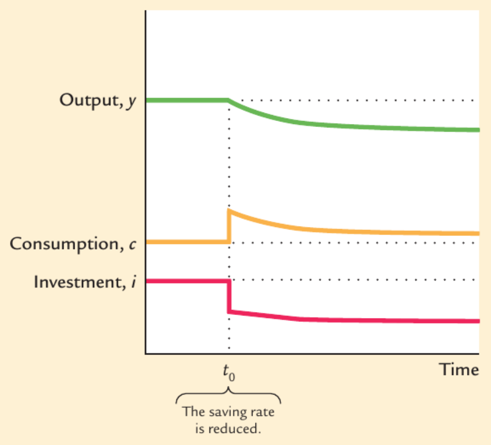
$$c^* = \frac{C^*}{\bar{L}} = \frac{(1-s)Y^*}{\bar{L}} = (1-s)y^* = (1-s)f(k^*)$$

$$= f(k^*) - \delta k^*$$

This is maximized if $f'(k^*) = \delta$ ("golden rule")

- so marginal product of capital equals depreciation rate
- if saving rate is too high, so that $f'(k^*) < \delta$, economy is dynamically inefficient
- decreasing saving rate will increase consumption both in steady state and along transition

Dynamic inefficiency



Source: Mankiw, figure 8-9

Population growth

What changes if population grows ($g_L > 0$)?

- essentially, growth drags K/L ratio down - same amount of machines must be spread over more workers
- thus the effective depreciation rate is $\hat{\delta} = \delta + g_L$

$$k_{t+1} = (1 - \hat{\delta})k_t + i_t$$

- otherwise everything stays the same

In steady state:

- population and total output Y grow at rate g_L
- per capita output is constant
- higher population growth leads to lower k^* and thus lower output per capita

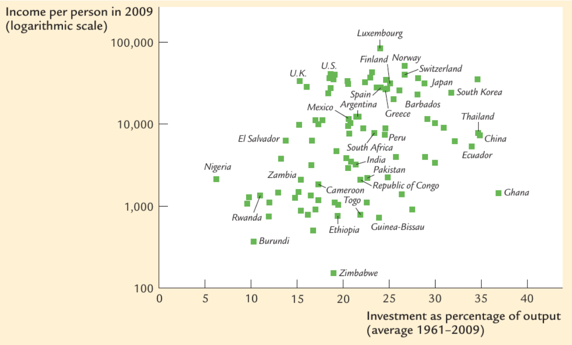
Empirics

Solow model predicts:

- steady state output per capita is positive function of saving rate
  - thus if current cross-country differences can be explained as different steady states, saving rate should be positively correlated with output
- growth takes place only along transition path to steady state
  - however, during transition, economy further away from steady state grows faster
  - thus within a group of homogeneous countries (same steady state) the model predicts convergence - initially poorer countries grow faster

Empirics

Saving rate vs. output:



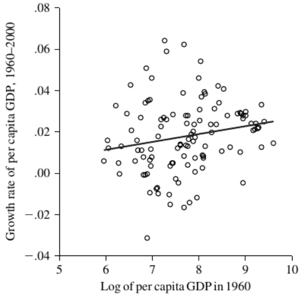
Correlation: 0.25 ($R^2 = 0.063$)

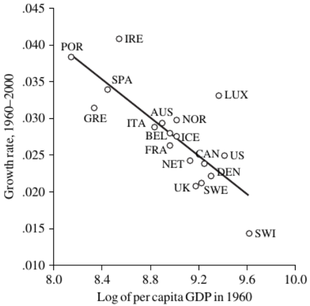
- correlation has correct sign, but kind of weak

Source: Mankiw, figure 8-6

Empirics

Convergence:





- no convergence in wide sample of countries
- better results among just developed countries

Source: Barro & Sala-i-Martin (2004)

Summary

- Solow model predicts convergence of output per capita to steady state
- steady state output depends mainly on saving rate
- with usual assumptions on production function, no sustained growth

Possible extensions

- add exogenous technological growth
- derive consumption function from optimizing behavior - Ramsey model
- endogenous growth theory - explain causes of technological growth
  - e.g. production of new "ideas" through research sector