

DERIVING  $MP_L(r)$  WITH HUMAN CAPITAL PER WORKER AT ANY SKILL LEVEL:

• P.N.F.N:  $y = A k^\alpha h^\beta$  (1)

$MP_L = r = ?$  (How does it depend on  $h$ ?  $y$ ?  $A$ ?)

• MARGINAL PRODUCT OF  $k$ :  $\frac{\partial y}{\partial k} = \alpha A k^{\alpha-1} h^\beta = r$  (2)

• FROM (1) get  $k(y)$  AND PLUG IN (2)

FROM (1)  $k^\alpha = \frac{y}{A h^\beta}$ ,  $k = \frac{y^{\frac{1}{\alpha}}}{A^{\frac{1}{\alpha}} h^{\beta/\alpha}}$  PWG-IN (2)

$$r = \alpha A \left( \frac{y^{\frac{1}{\alpha}}}{A^{\frac{1}{\alpha}} h^{\beta/\alpha}} \right)^{\alpha-1} h^\beta$$

$$r = \alpha A^{\frac{\alpha}{\alpha} - \frac{\alpha-1}{\alpha}} y^{\frac{\alpha-1}{\alpha}} h^{-\frac{\beta(\alpha-1)}{\alpha}} h^{\beta \frac{\alpha}{\alpha}}$$

$$r = \alpha A^{\frac{\alpha}{\alpha} - \frac{\alpha-1}{\alpha}} y^{\frac{\alpha-1}{\alpha}} h^{\frac{-\beta\alpha + \beta + \beta\alpha}{\alpha}}$$

$$r = \alpha A^{\frac{1}{\alpha}} y^{\frac{\alpha-1}{\alpha}} h^{\frac{\beta}{\alpha}}$$

$$r \equiv \frac{\alpha A^{\frac{1}{\alpha}} h^{\frac{\beta}{\alpha}}}{y^{\frac{1-\alpha}{\alpha}}}$$

- when  $y \uparrow$ ,  $r \downarrow$  (MARGINAL PRODUCT OF CAPITAL)
- when  $h \uparrow$ ,  $r \uparrow$
- when  $A \uparrow$ ,  $r \uparrow$
- when  $\alpha \uparrow$ ,  $r \uparrow$