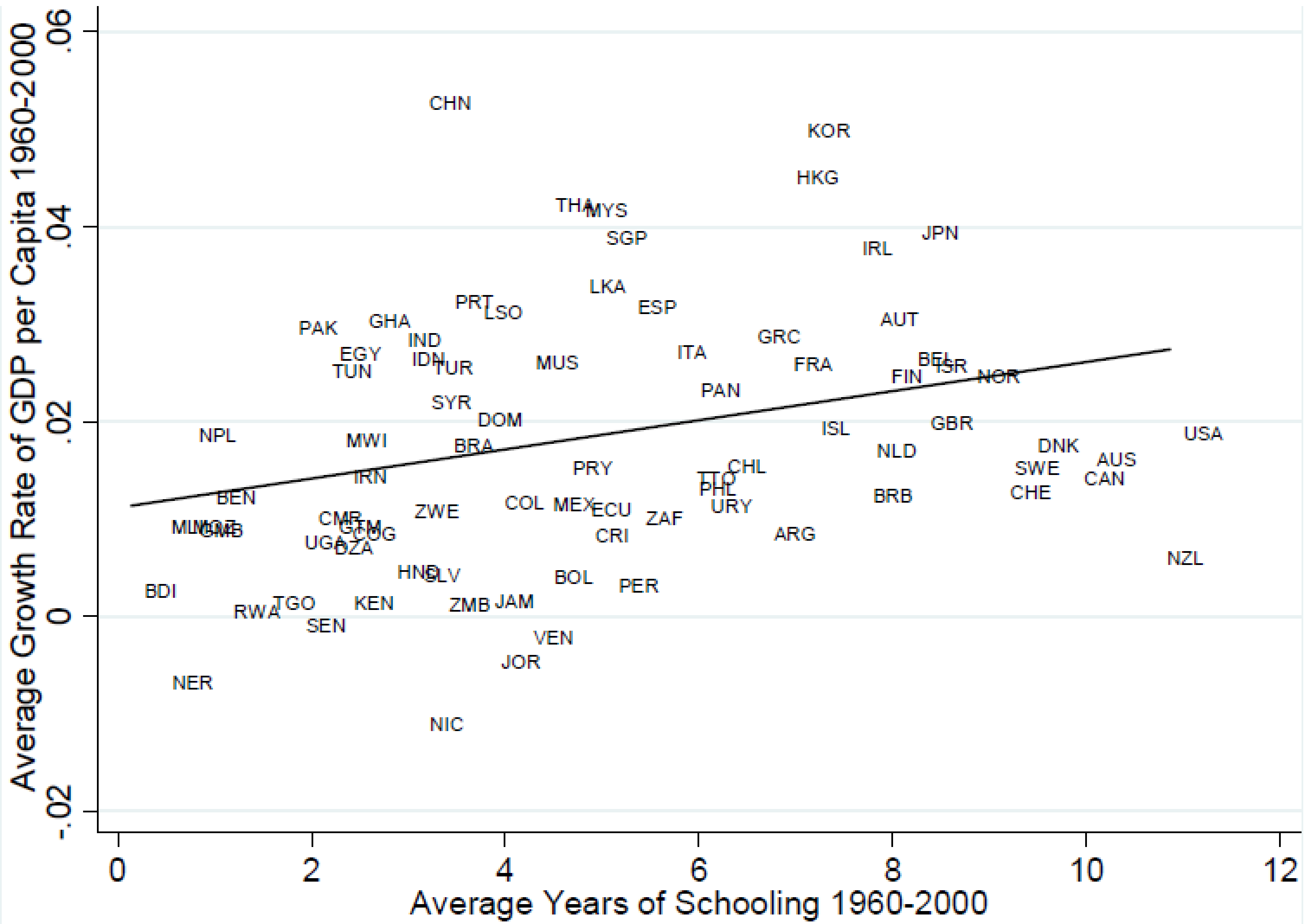
Average Growth Rate of GDP per Capita 1960-2000
Average Years of Schooling 1960-2000
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Average Growth Rate of GDP per Capita 1960-2000
Average Investment Rate 1960-2000
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Lecture 6. Explaining Economic Growth

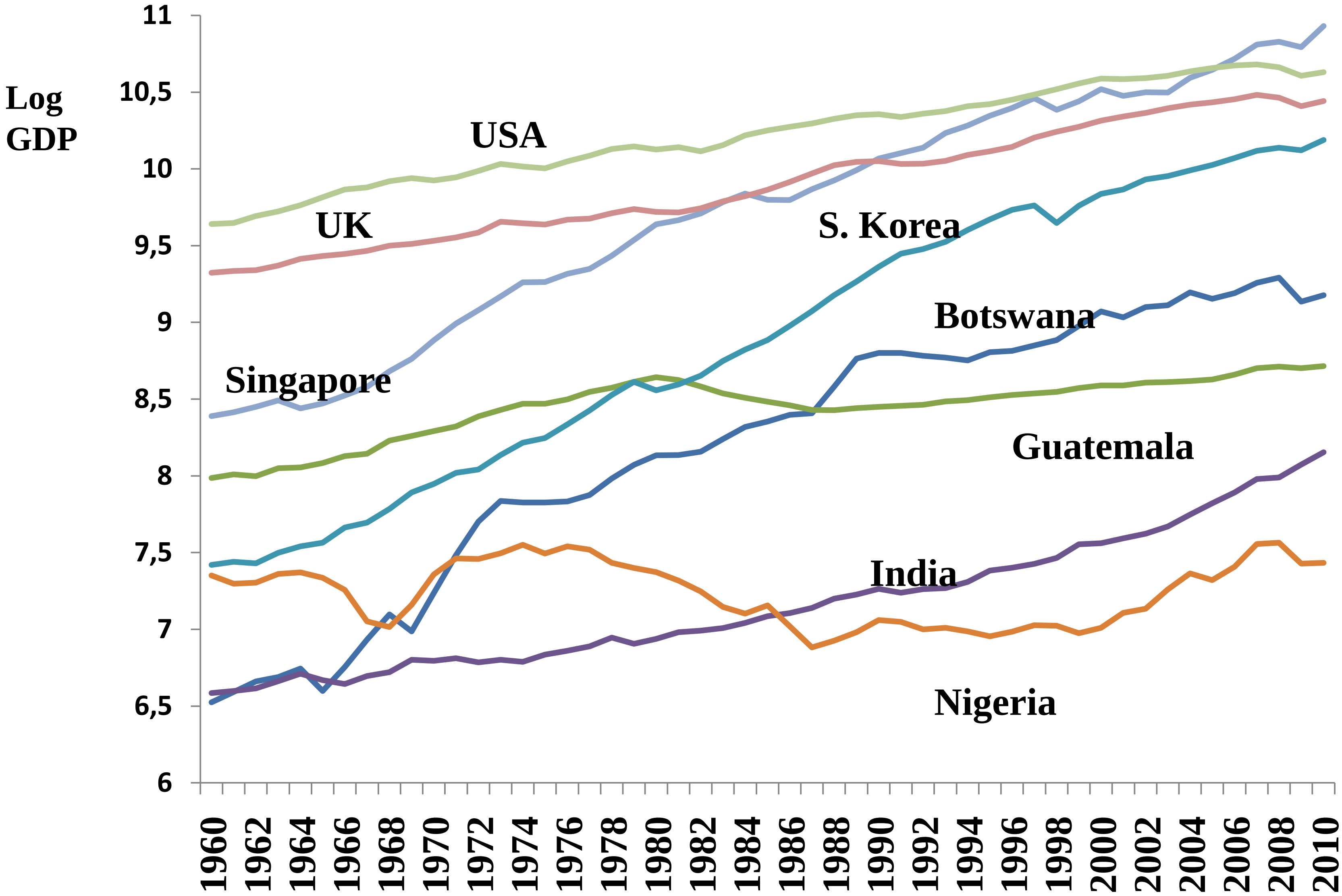
Solow-Swan Model

Fall Semester, 2014

Class Outline

- Solow Model: Role of savings and population growth

The evolution of GDP per capita, 1960-2010



Solow-Swan Model of Economic Growth(1956)

- What drives an increase in GDP per capita **in a long run**?

Robert Solow (1956).“A Contribution to the Theory of Economic Growth,” QJE

• Dynamic general equilibrium model

• The model is only as good as its assumptions

Economic environment (a set of assumptions)

• A single composite good

• **Two factors** of production: capital and labor

• **Two agents**: firms and households

• A **closed** economy

Solow-Swan Model: Supply Side

Production function (technology)

- Maximum output for given inputs

Aggregate output

$$\boxed{Y = F(K, L)}$$

Capital **Labor**

Factor Inputs

- If the quantity of **both** inputs doubles, the output of potatoes also doubles

=> Constant returns to scale (CRS)

$$2Y = F(2K, 2L)$$

Solow-Swan Model: Supply Side (Cont.)

Properties of production function

- Output is a **positive** function of inputs $Y = F(K_{(+)}, L_{(+)})$

What would happened to GDP if **only one** input increases?

- **Diminishing returns** to factor inputs

For a **fixed L**, an increase in K would lead to smaller and smaller increase in Y

For a **fixed K**, an increase in L would lead to smaller and smaller increase in Y

Solow-Swan Model: GDP Per Capita

- Transforming model to **per capital** terms

• Divide **both sides** of production function by the size of labor force

$$Y = F(K, L)$$

N!B! The level of **capital per worker** determines the level of **output per worker**

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = F\left(\frac{K}{L}\right)$$

Due to CRS

$$y = f(k)$$

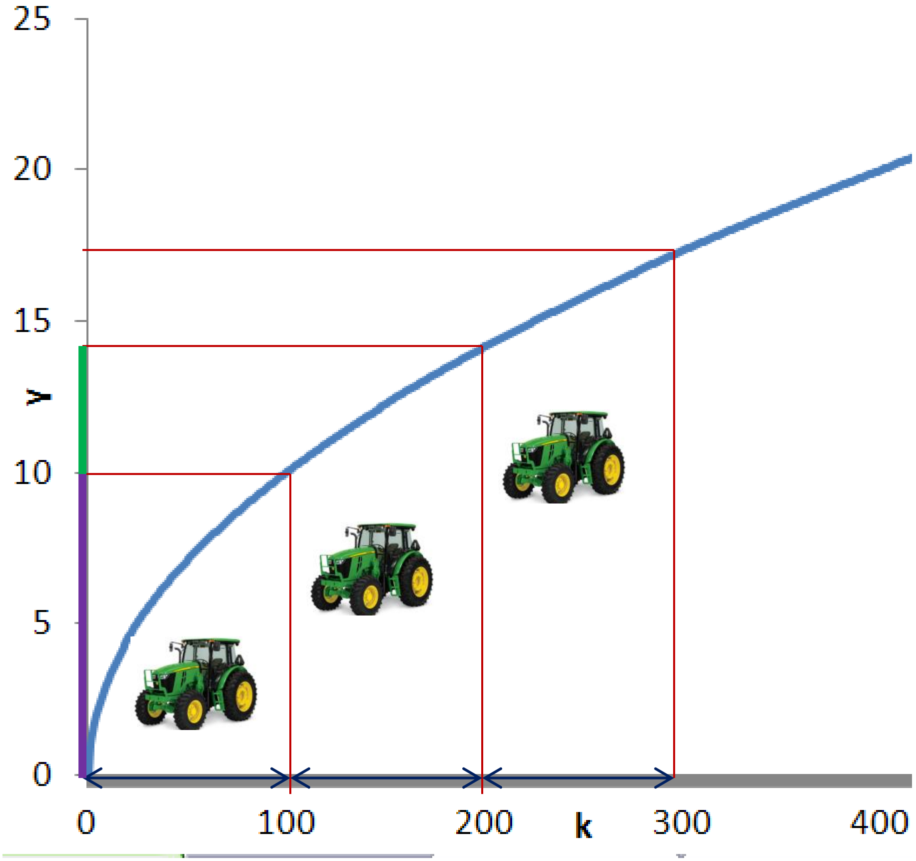
GDP per capita **Capital/labor ratio**

$$y = f(k) = \sqrt{k}$$

- **TE**

Solow-Swan Model: Diminishing Returns

Diminishing returns $y = f(k) = \sqrt{k}$



Implication: Countries with small capital stock (k) are more productive => grow faster

Solow-Swan Model: Diminishing Returns (Cont.)

TE Experience of Germany and Japan after the WW II

Country	Average annual growth rate of GDP per capita	
	1950-1960	**1980-1990**
Germany	6.6 %	1.9 %
Japan	6.8 %	3.4 %
France	9.6 %	2.8%
USA	1.2 %	2.3 %

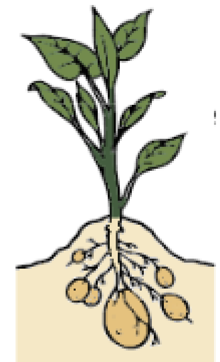
Source: Blanchard et al (2010)

Solow-Swan Model: Demand Side

Total output = Total expenditure = Total income

$$Y = C + I$$

Consumption **Investment**



- A fixed fraction of HH income is saved
- Exogenous **savings rate (s)**

$$I = sY$$

$$C = Y - sY = (1-s)Y$$

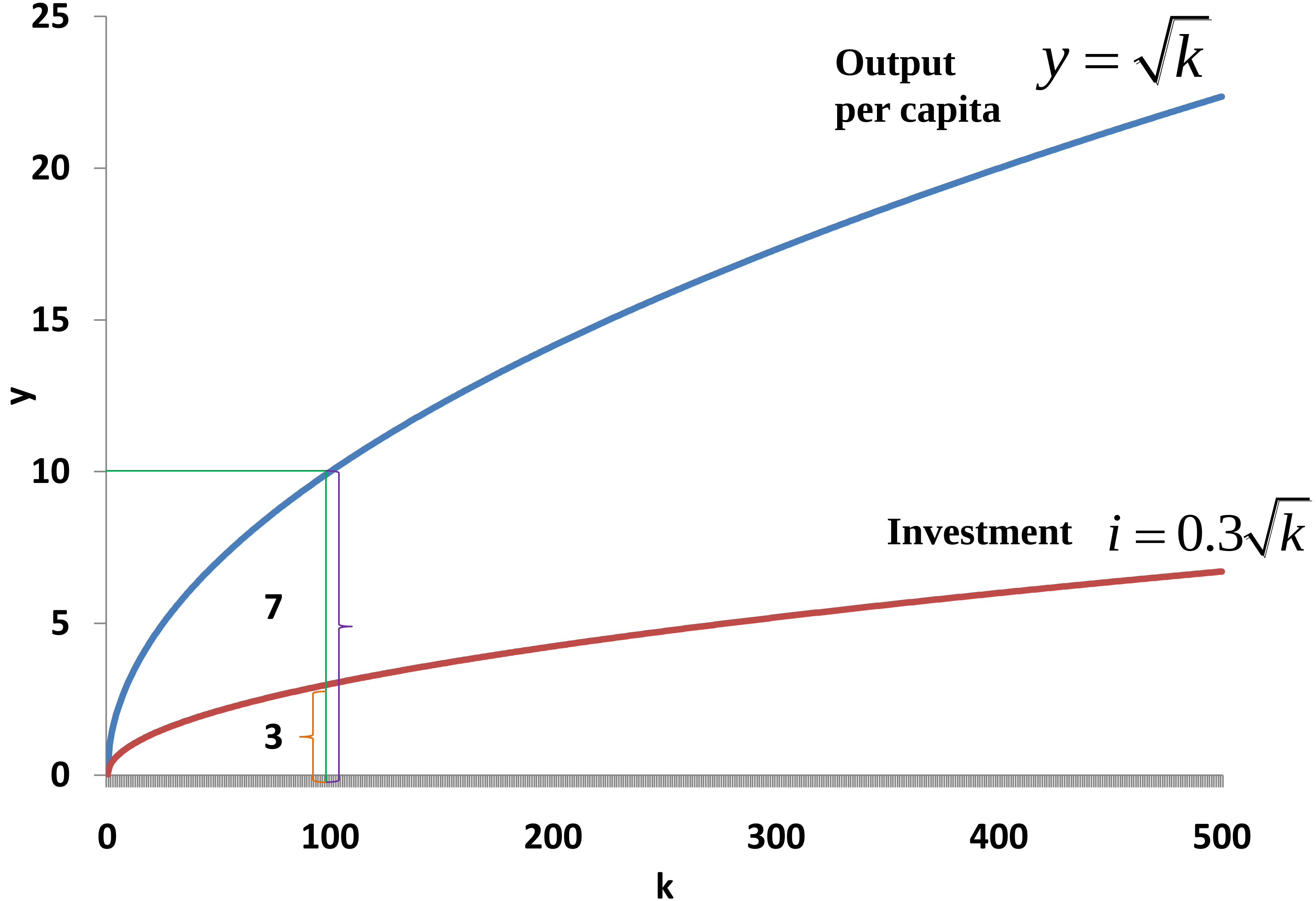
I & C per capita

$$i = sy = sf(k)$$

$$c = (1-s)f(k)$$

Savings rate (s) determines the allocation of income between C & I

Solow-Swan Model: Graphical Representation



Solow-Swan Model: Capital Accumulation

- Size of the labor force is fixed (**no population growth**)
- GDP per capital will increase only due to increase in capital stock

$$\frac{Y_t}{L} = F\left(\frac{K_t}{L}\right)$$

- Households' savings are used as investment into **capital accumulation** K

$$I = sY$$

- Investment is proportional to output: higher Y => higher sY=> higher I

- Capital **depreciates** at an exogenous rate **δ**
- Every year a fraction of capital δ breaks down and becomes useless

$$K_{t+1} = I_t + (1-\delta)K_t$$

Solow-Swan Model: Capital Accumulation (Cont.)

- Capital **accumulation**

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$K_{t+1} = sY_t + (1-\delta)K_t$$

- Change in capital from year t to year t+1

$$K_{t+1} - K_t = sY_t - \delta K_t$$

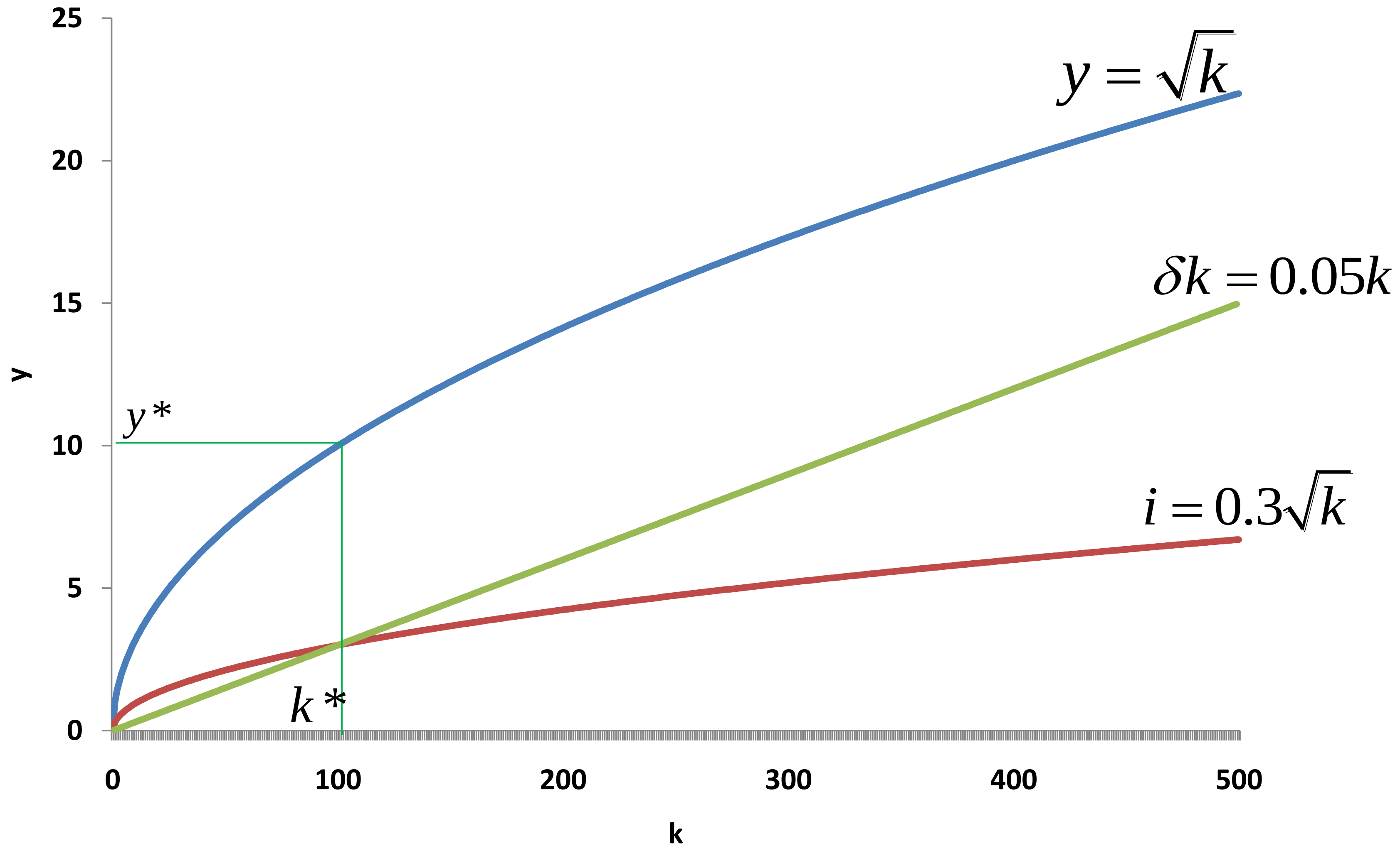
$$\Delta K = sY - \delta K$$

$$\Delta k = sf(k) - \delta k$$

- If $sY > \delta K$ capital stock increases
- If $sY < \delta K$ capital stock decreases

Solow Model: Steady-State

- **Steady-state**: investment and depreciation just **balance**



$$I = \delta K \rightarrow \Delta K = 0 \rightarrow \Delta k = 0$$

Solow Model: Steady-State (Cont.)

- **Steady-state**: the long-run equilibrium of the economy

The amount of savings per worker is just sufficient to cover the depreciation of the capital stock per worker

• Economy will remain in the steady state (unless additional channels of growth are introduced)

$$\Delta k = sf(k^*) - \delta k^* = 0$$

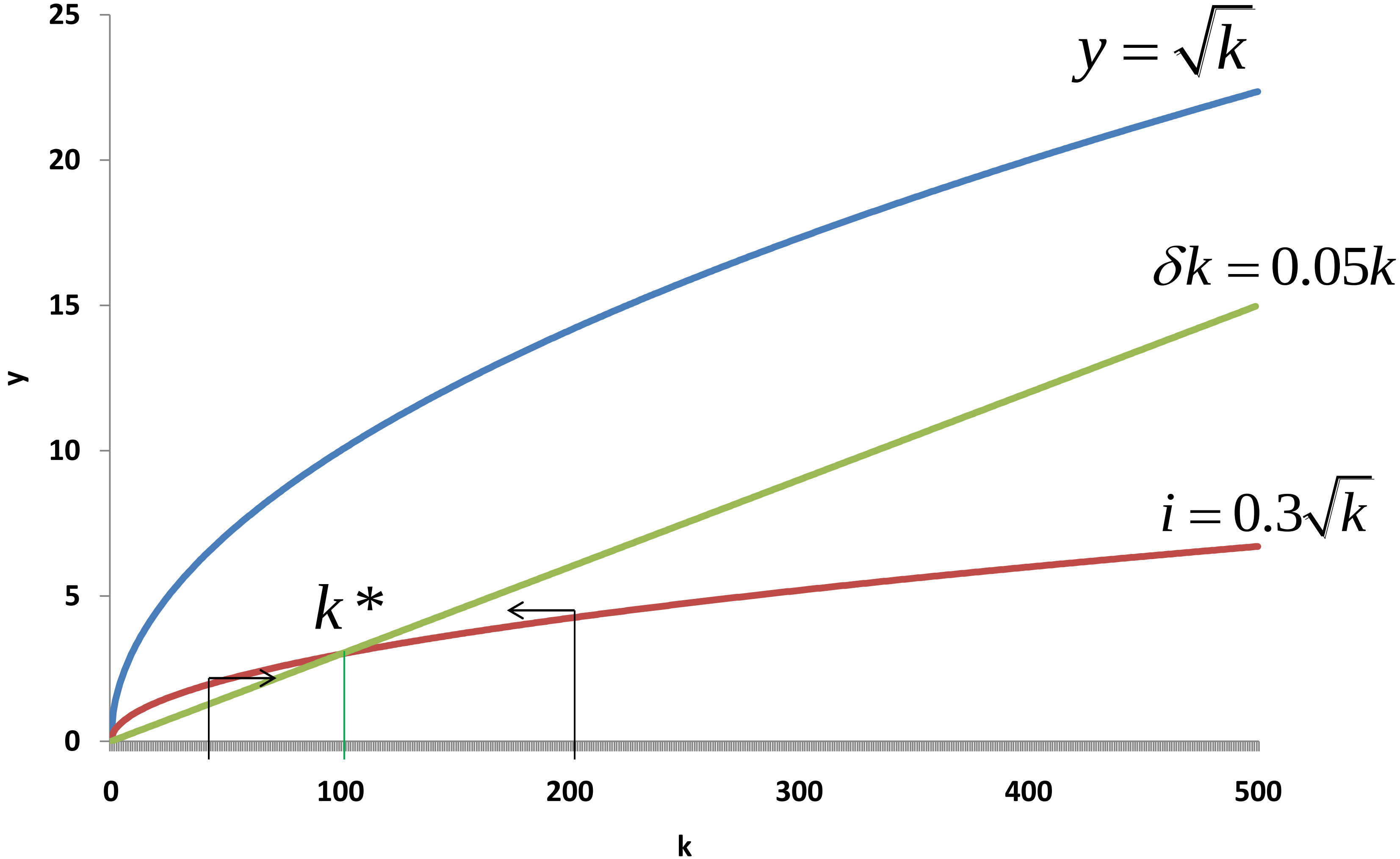
$$y^* = \sqrt{k^*} \rightarrow \Delta y = 0$$

• Economy which is not in the steady state will go there => convergence to the constant level of output per worker over time

• Different economies have **different steady state** value of capital

Solow Model: Steady-State Level of Capital per Worker

- **Convergence** to steady state



Solow Model: Steady-State (Cont.)

Implications

- Savings rate (s) has **no effect** on the long-run **growth rate of GDP** per capita

➢ Increase in savings rate will lead to higher growth of output per capita for some time, but not forever.

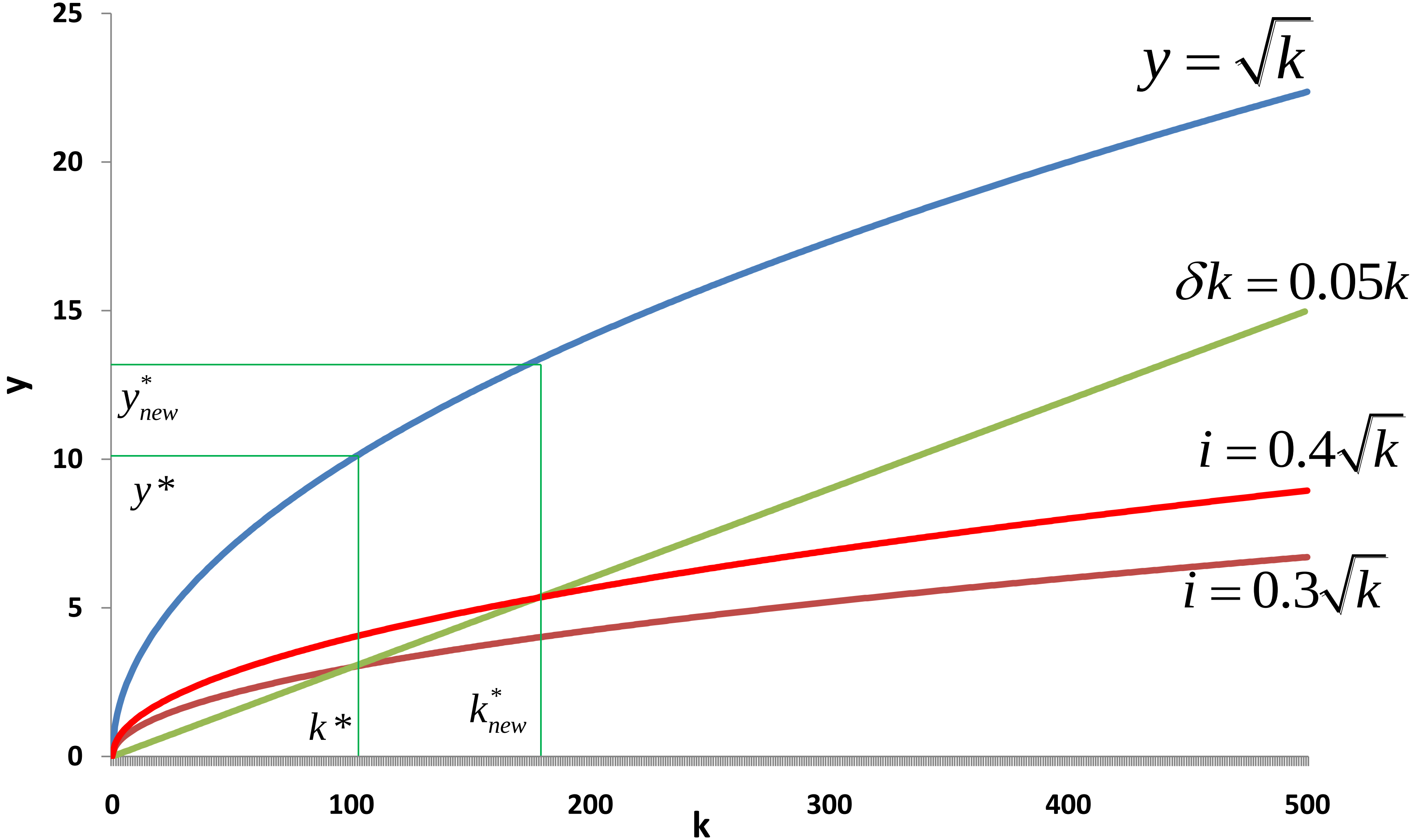
➢ Saving rate is bounded by interval [0, 1]

- Savings rate determines the **level of GDP** per capita in a long run

Comparative statics: Increase in savings rate

Solow Model: Increase in Savings Rate

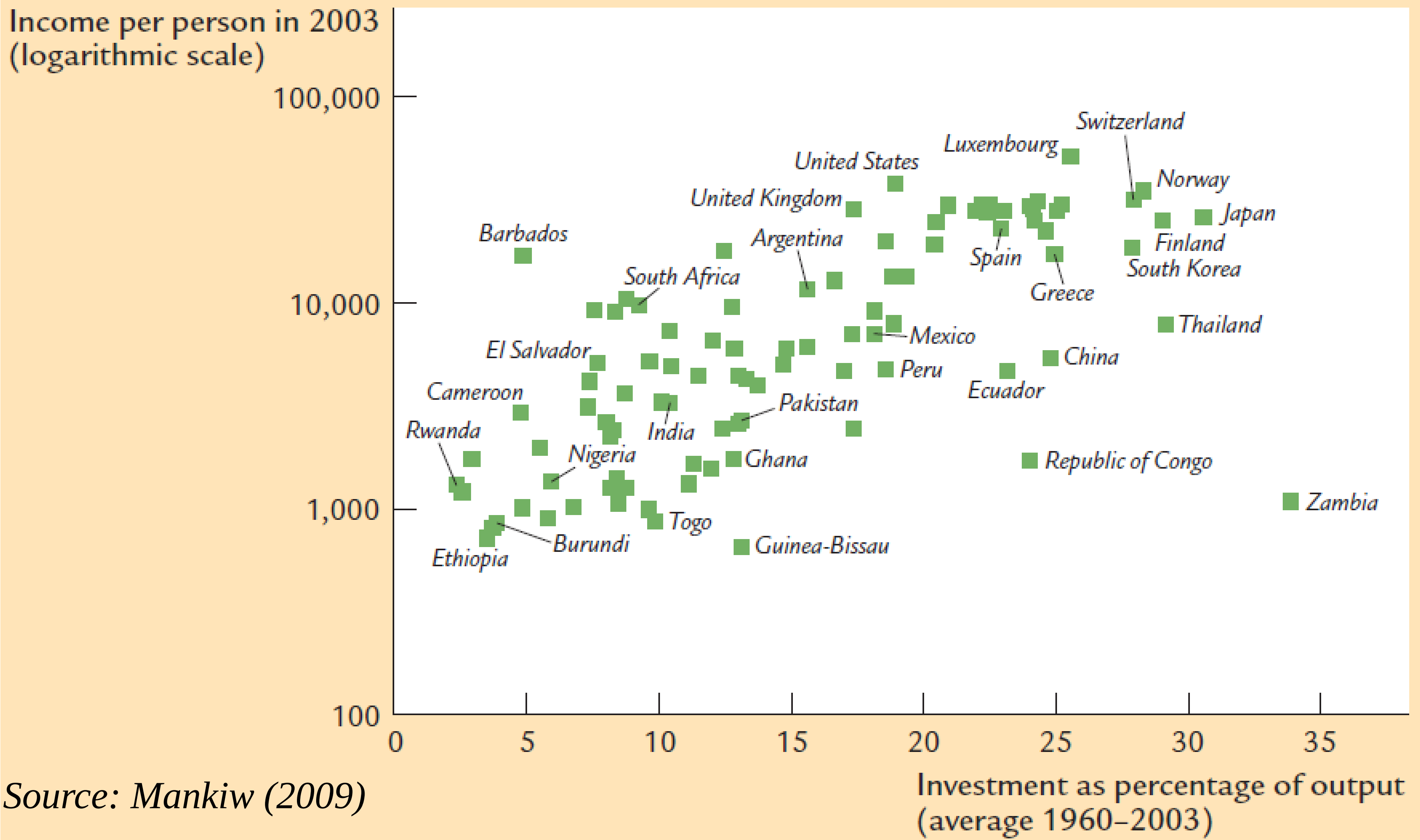
- Savings rate increases from 30 % to 40 %



- Economy moves to a **new steady state** => Higher capital and output per capita

Solow Model: The Role of Savings

- A nation that devotes a large fraction of its income to savings will have a higher steady-state capital stock and a high level of income



Source: Mankiw (2009)

Solow Model: The Role of Savings (Cont.)

Summary

- GDP per capita is a function of per capita capital (capital /labor ratio) **only**
- In the long run, capital/labor ratio reaches its steady state for the exogenous **s**
- In the steady state, per capita variables are constant => No growth in the long-run
- Growth is possible only during the transition to steady state, but it is not sustainable

Solow-Swan Model: Population Growth

- Labor force is growing at a constant rate **n**

$$Y = F(K, L)$$

$$\frac{Y_t}{L_t} = F\left(\frac{K}{L}, 1\right) = F\left(\frac{K_t}{L_t}\right)$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$\Delta k = sf(k) - \delta k$$

$$\Delta k = sf(k) - (\delta + n)k$$

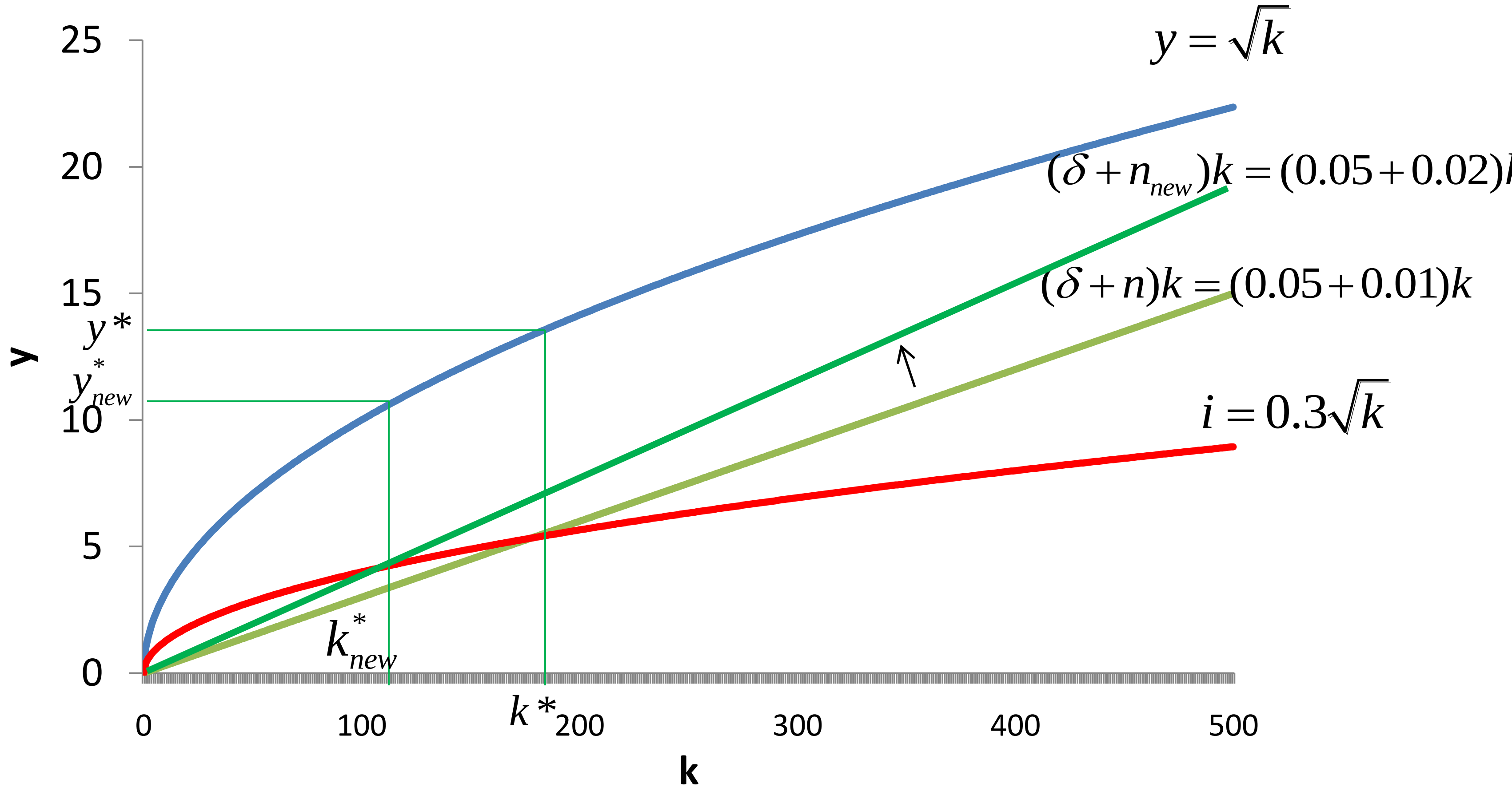
- **Per capita capital** stock is affect by *investment*, *depreciation*, and *population growth*

- The amount of investment necessary to keep per capita capital stock constant

Solow-Swan Model: Population Growth (Cont.)

• An increase in n reduces k* and y* =>

=> Economies with high rates of population growth will have **lower** GDP per capita



Solow-Swan Model: Population Growth (Cont.)

• Reduction of fertility should rise income per person in the long run

TE Chinese totalitarian policy of one child per couple