CEE Economic Growth & Development

Lecture 7. Economic Growth:

The Role of Technological Progress

Fall Semester, 2014

Class Outline

- The Solow model: Deriving steady state
- The Solow model and technological progress
- Growth accounting

The Vicious Circle of Poverty

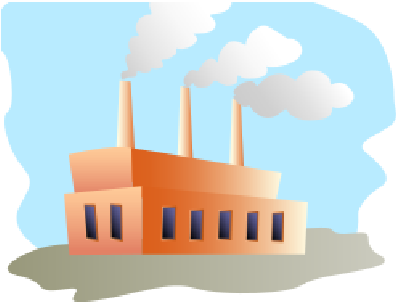
Low output and income

GDP

Low savings

POVERTY

Scarce investment capital



Scarce jobs, Poor capital

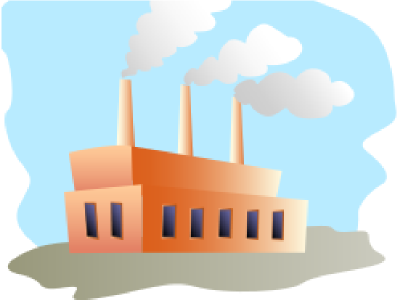
Breaking the Cycle

Better institutions

Higher output

GDP

ECONOMIC GROWTH



Investment capital

Better jobs, Technology

The World Bank

Solow-Swan Model of Economic Growth(1956)

▪ Overview

Production function $Y = F(K_{(+)}, L_{(+)})$

• **Diminishing returns** to factor inputs

GDP per capita

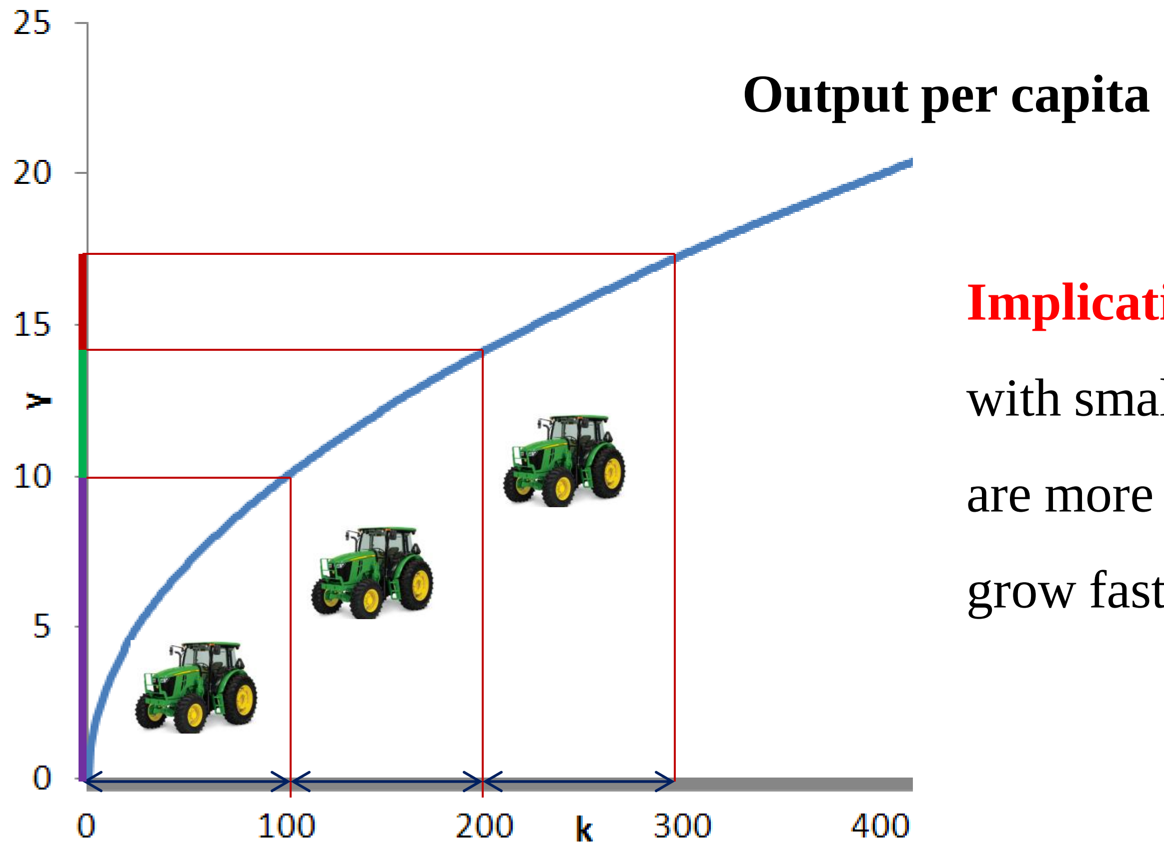
$$\frac{Y_t}{L} = F\left(\frac{K_t}{L}, 1\right) = F\left(\frac{K_t}{L}\right)$$

$$y_t = f(k_t)$$

$$y_t = \sqrt{k_t}$$

Diminishing Returns to Factor Inputs

$$y = f(k) = \sqrt{k}$$



Implication: Countries with small capital stock are more productive => grow faster

Economic Growth and Capital Accumulation

Increase in capital stock (K)

Investments of firms

New additions to the capital stock

Replacement of the worn-out capital (**depreciation**)

Net investment = Total savings – Replacement of the depreciated capital

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$K_{t+1} = sY_t + (1-\delta)K_t$$

$$\Delta K = sY - \delta K$$

Savings of households provide investment funds to firms

s - **exogenous** savings rate

Economic Growth and Capital Accumulation (Cont.)

Increase in capital stock (K)

Investments of firms

New additions to the capital stock

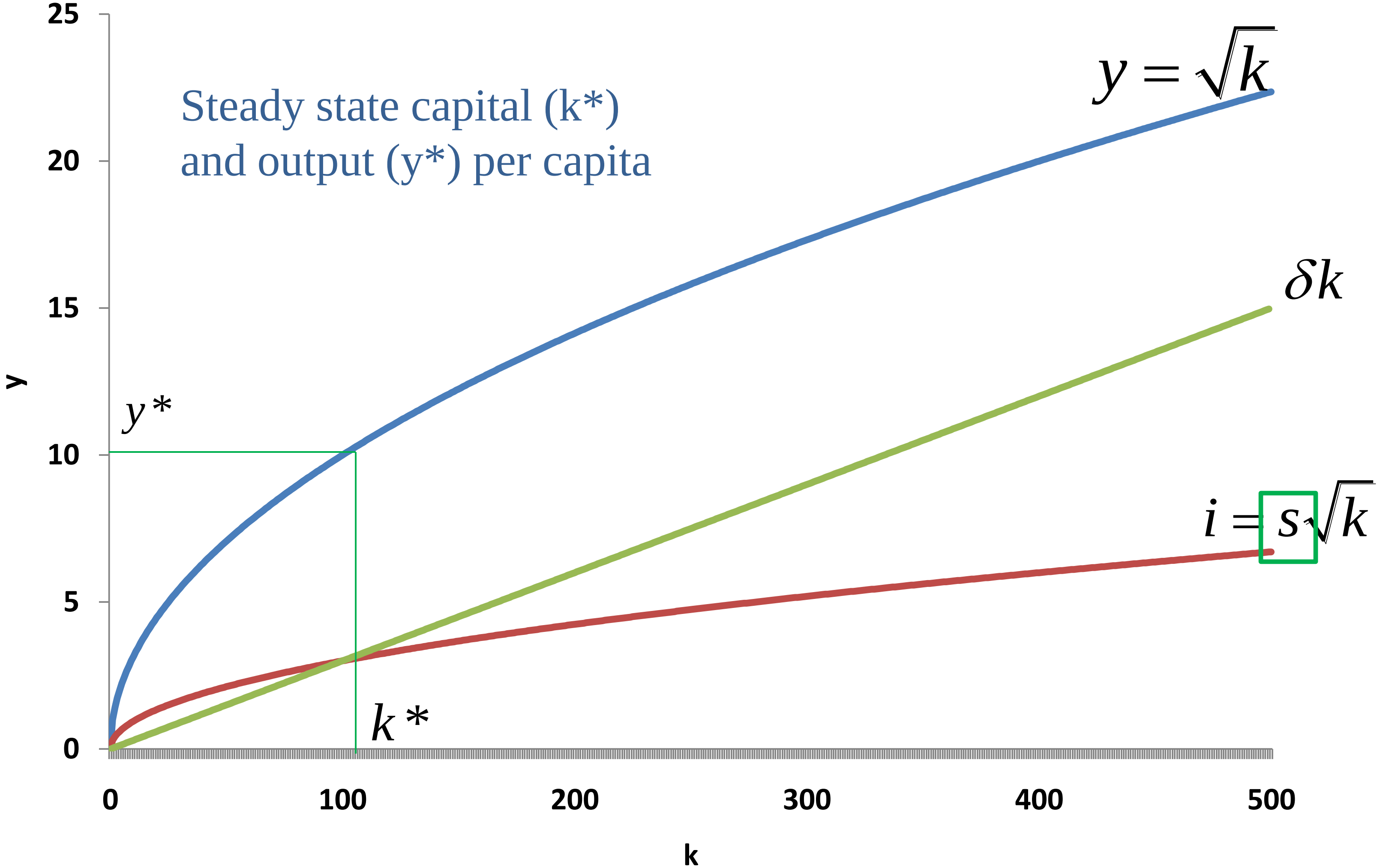
Replacement of the worn-out capital **(depreciation)**

$$\Delta K = sY - \delta K$$

- If $sY > \delta K$ capital stock is growing
- If $sY < \delta K$ capital stock is shrinking
- Break-even investment $sY = \delta K$

Steady State Level of Capital

- The economy would grow as long as $sf(k) > \delta k$



$$I = \delta K \rightarrow \Delta K = 0 \rightarrow \Delta k = 0$$

The Solow-Swan Model: Steady State

- **Steady state**: the **long-run** equilibrium of the economy
  - •Savings are just sufficient to cover the depreciation of the capital stock

- ❖ In the long run, capital per worker reaches its steady state for an **exogenous s**
- ❖ Increase in **s** leads to higher capital per worker and higher output per capita
- ❖ Output grows only during the transition to a new steady state (**not sustainable**)
- ❖ Economy will **remain** in the steady state (no further growth)
- ❖ Economy which is not in the steady state will go there => **Convergence**

Government policy response?

N!B! Savings rate is a fraction of wage, thus is bounded by the interval **[0, 1]**

The Solow-Swan Model: Numerical Example

Production function $$Y = F(K, L) = K^{0.5} L^{0.5}$$

Production function in **per capita terms** $$\frac{Y}{L} = \frac{K^{0.5} L^{0.5}}{L} = \frac{K^{0.5}}{L^{0.5}}$$

$$k = \frac{K}{L}; \quad y = \frac{Y}{L}$$

GDP per capita: $y = \sqrt{k}$

Savings rate: $s = 30\%$

Depreciation rate: $\delta = 10\%$

Initial stock of capital per worker: $k_0 = 4$

The Solow-Swan Model: Numerical Example (Cont.)

Year	k	y	i	c	δk	Δk
1	4					
2						
…						

Consumption: C = (1-s)Y

Consumption **per capita C/Y = c**

Steady state capital/labor ration:

$$s\sqrt{k} = \delta k \rightarrow k^* = \frac{s^2}{\delta^2}$$

Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
.						
.						
.						
10	5.602	2.367	1.657	0.710	0.560	0.150
.						
.						
.						
25	7.321	2.706	1.894	0.812	0.732	0.080
.						
.						
.						
100	8.962	2.994	2.096	0.898	0.896	0.002
.						
.						
.						
∞	9.000	3.000	2.100	0.900	0.900	0.000

The Solow-Swan Model: Convergence to Steady State

N!B! Regardless of k_0 , if two economies have the same **s, δ, N**, they will reach the **same** steady state

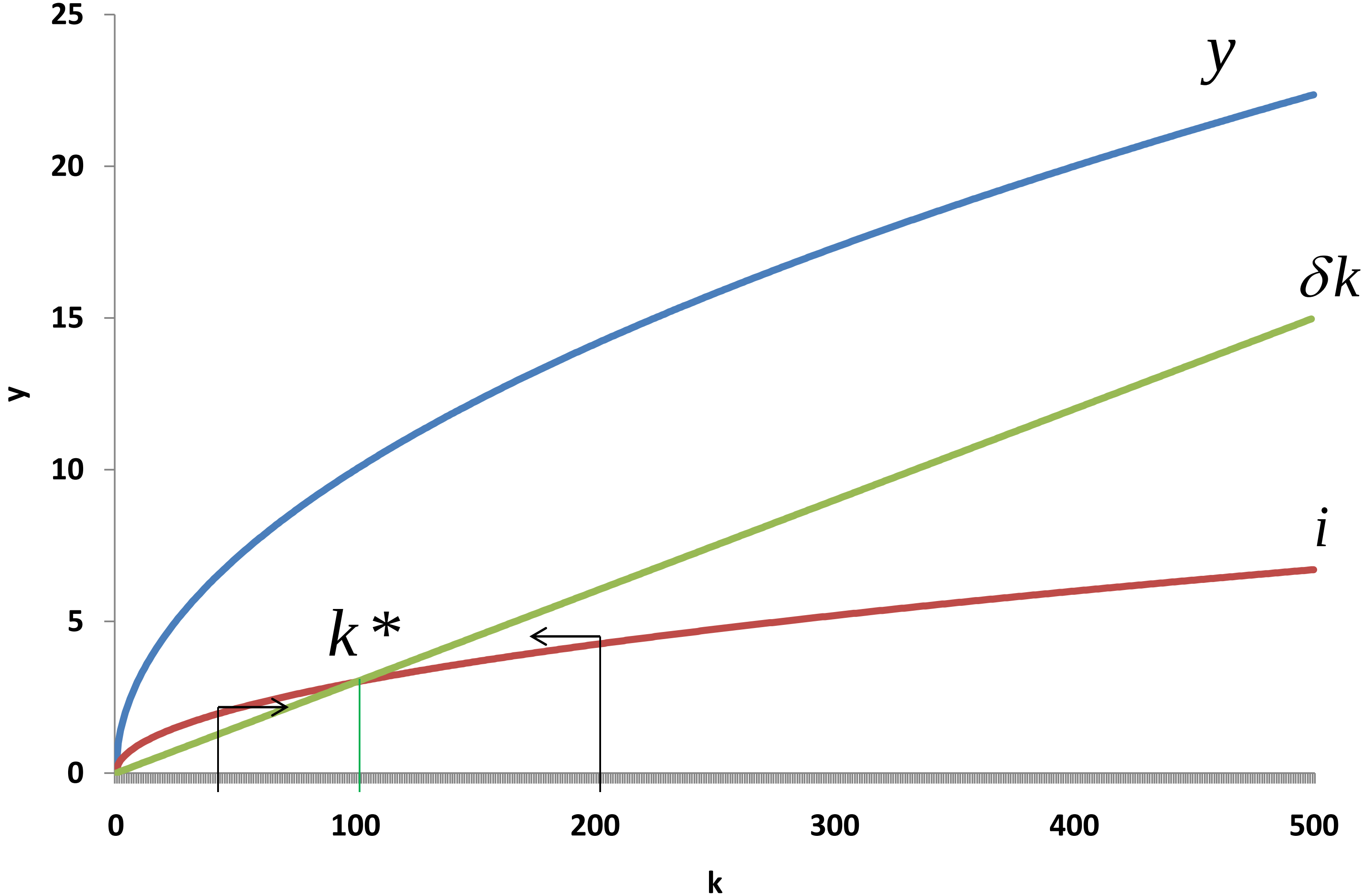
- If countries have the same steady state, poorest countries grow faster
- Not much convergence worldwide

  Different countries have different **institutions and policies**

- **Conditional convergence:** comparison of countries with similar savings rates

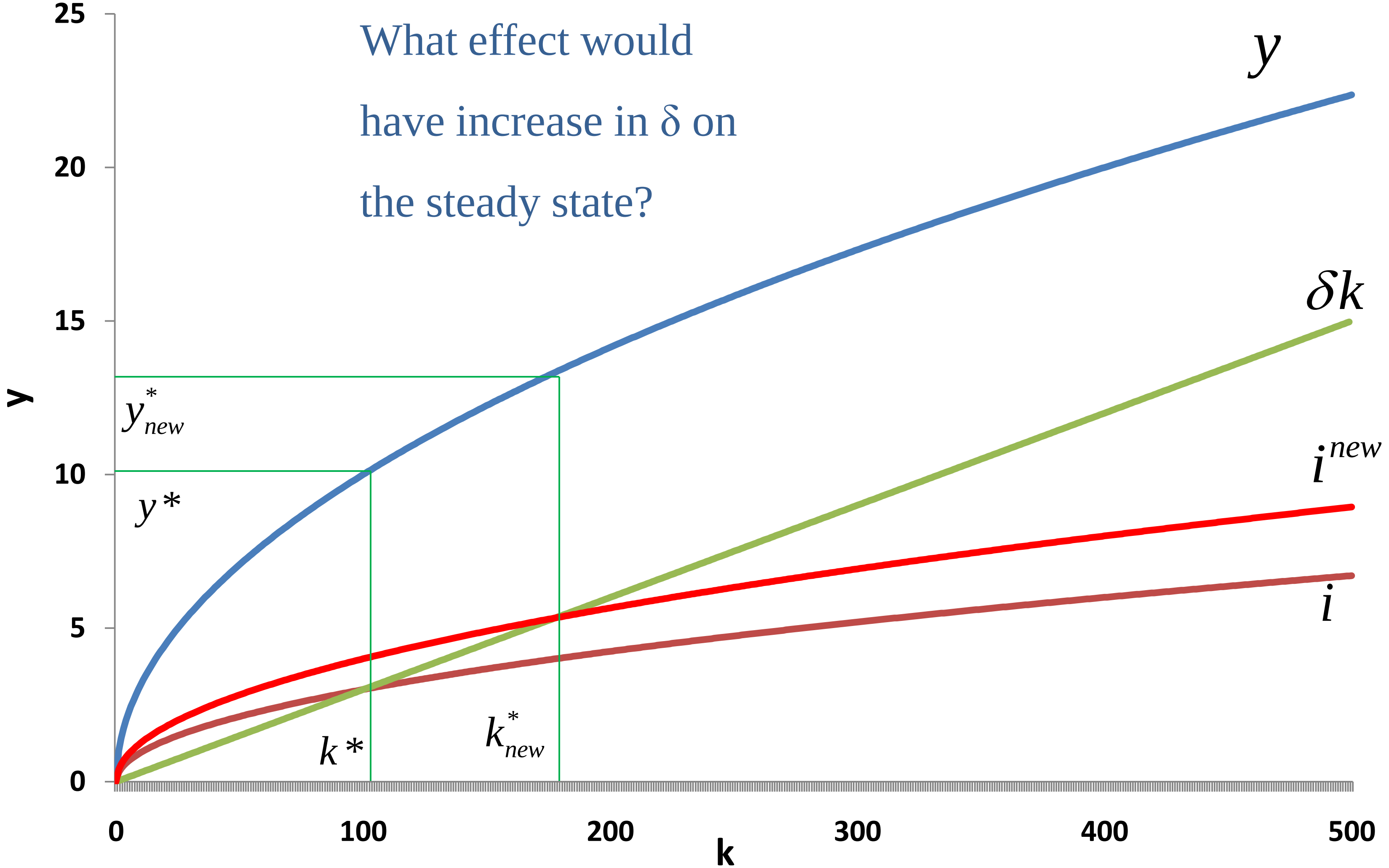
Solow Model: Convergence to Steady State

- **Convergence** to steady state

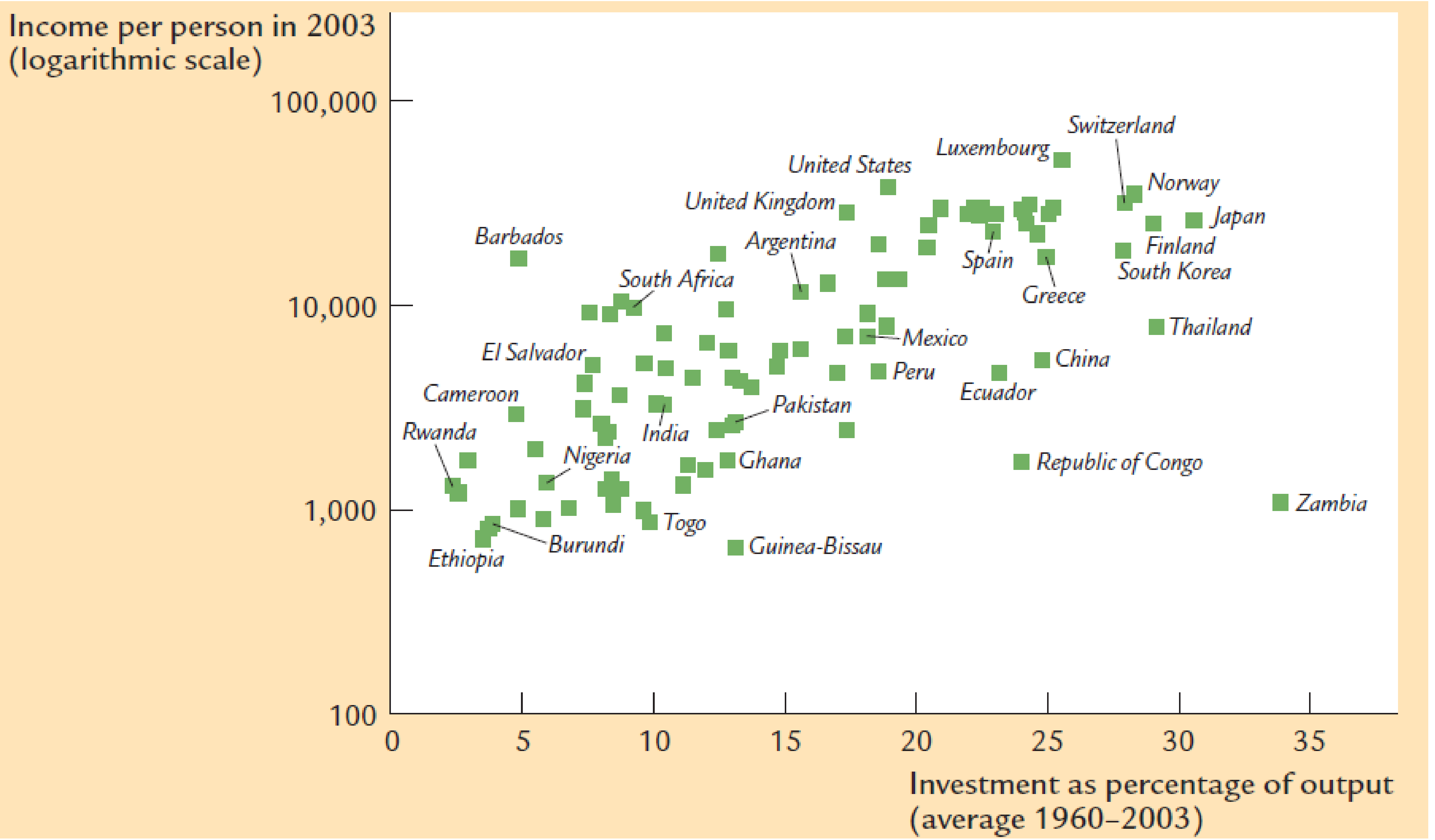


Solow Model: Increase in Savings Rate

- Savings rate increases from 30 % to 40 %



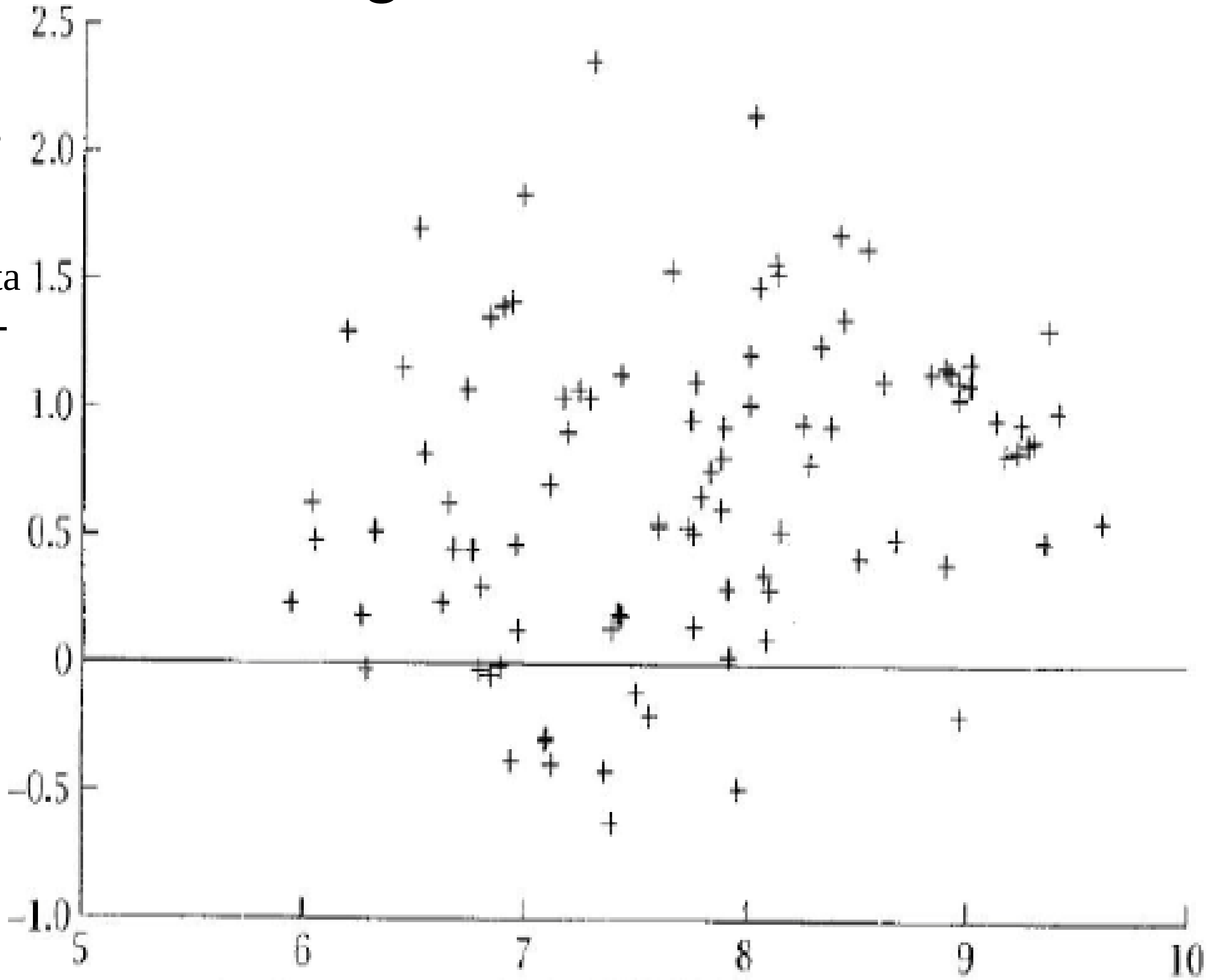
- Economy moves to a **new steady state** => Higher capital and output per capita



Source: Mankiw (2009)

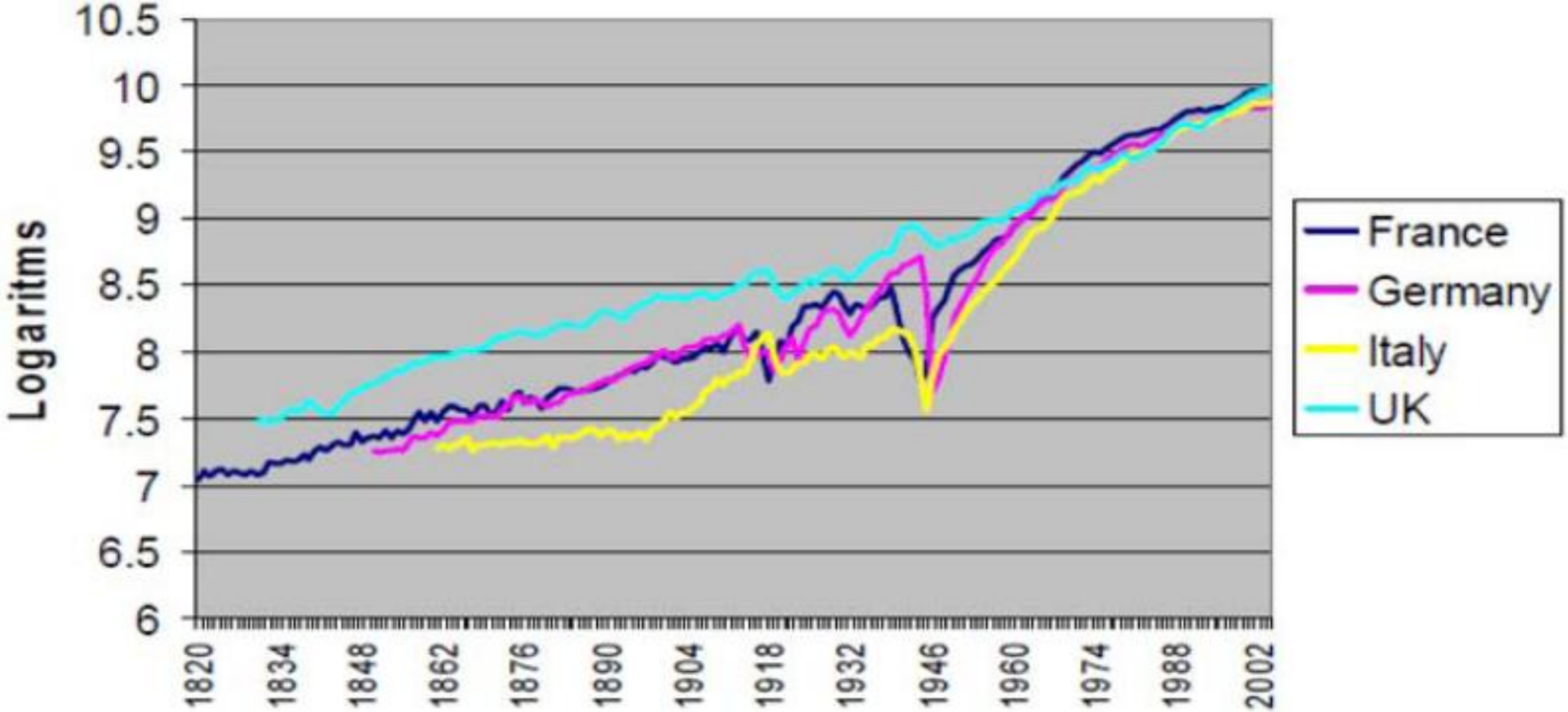
World Wide Convergence

Changes in Log **income** per capita in 1960 - 1990



Log income per capita in 1960 (100=1996)

Catch up amongst Europe's big 4



Output per head (US $ 1990)

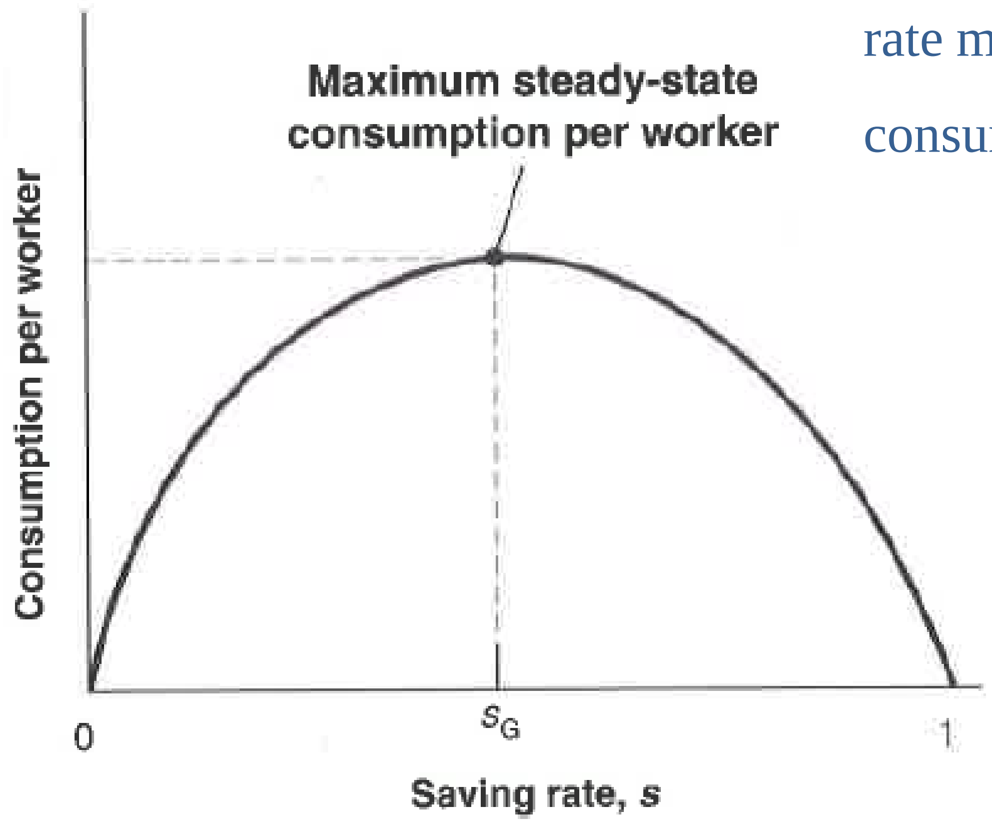
Source : Maddison and GGDC

The Golden Rule Level of Capital

- Increasing savings rate means less present consumption

What is the optimal savings rate?

N!B! Optimal savings rate maximizes consumption per capita



$$c^* = \sqrt{k^*} - \delta k^* \rightarrow \max$$

The Solow-Swan Model: Population Growth

- Labor force is growing at a constant rate **n =10%**

$$Y_t = F(K_t, L_t)$$

$$\Delta k = sy - \boxed{(\delta + n)}k$$

- **Per capita capital** stock is affect by *investment*, *depreciation*, and *population growth*

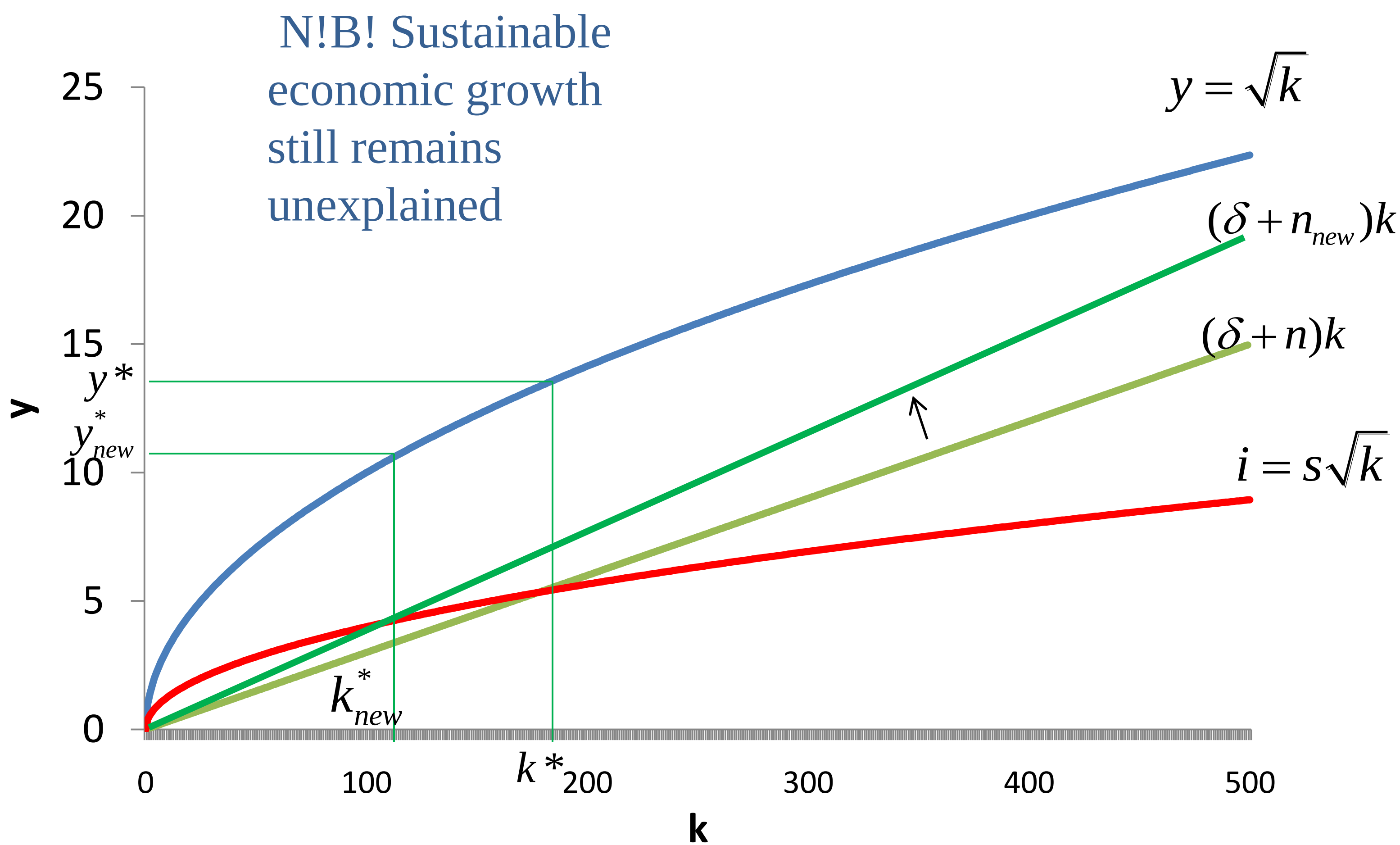
- **Steady state**:

$$s\sqrt{k} = (\delta + n)k \rightarrow k^* = \frac{s^2}{(\delta + n)^2}$$

- Population growth **increases** Y (level effect)
- Population growth **reduces** k* and y*

Solow-Swan Model: Population Growth (Cont.)

Economies with high rates of population growth will have **lower** GDP per capita



Government policy response?


Income per person in 2003 (logarithmic scale)
100,000
10,000
1,000
100
0
1
2
3
4
5
Population growth (percent per year; average 1960–2003)
Luxembourg
United States
Denmark
Norway
Canada
Australia
Hong Kong
Israel
South Korea
United Kingdom
Portugal
Uruguay
Brazil
Costa Rica
China
Jamaica
Guatemala
Jordan
India
Lesotho
Pakistan
Cote d'Ivoire
Gambia
Burundi
Guinea-Bissau
Ethiopia
Niger

The Role of Technological Progress

- Technological change, increase in factor productivity
  - ✓ Larger output with given quantities of capital and labor

$$Y = F(K, L, A)$$

- **State of technology (A)**

How does technological progress translates into larger output?

Labor-augmenting technological progress

$$Y = F(K, \boxed{A \cdot L})$$ **Effective labor**

- **A** as **labor efficiency**
- TP reduced number of workers needed to produce the same output
- TP increases output using the same number of workers

The Solow-Swan Model with Technological Progress

$$Y = F(\underset{(+)}{K}, \underset{(+)}{L}, \underset{(+)}{A})$$

- Technology is improving every year at the **exogenous rate (*g*)**

$$\frac{A_{t+1} - A_t}{A_t} = g$$

Production function: **GDP per effective labor**

$$Y = F(K, A \cdot L)$$

$$\frac{Y_t}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}\right)$$

The Solow-Swan Model with Technological Progress (Cont.)

- From **GDP per effective labor** to the **GDP per capita**?

$$Y = F(K, A \cdot L)$$

$$\frac{Y_t}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}\right)$$

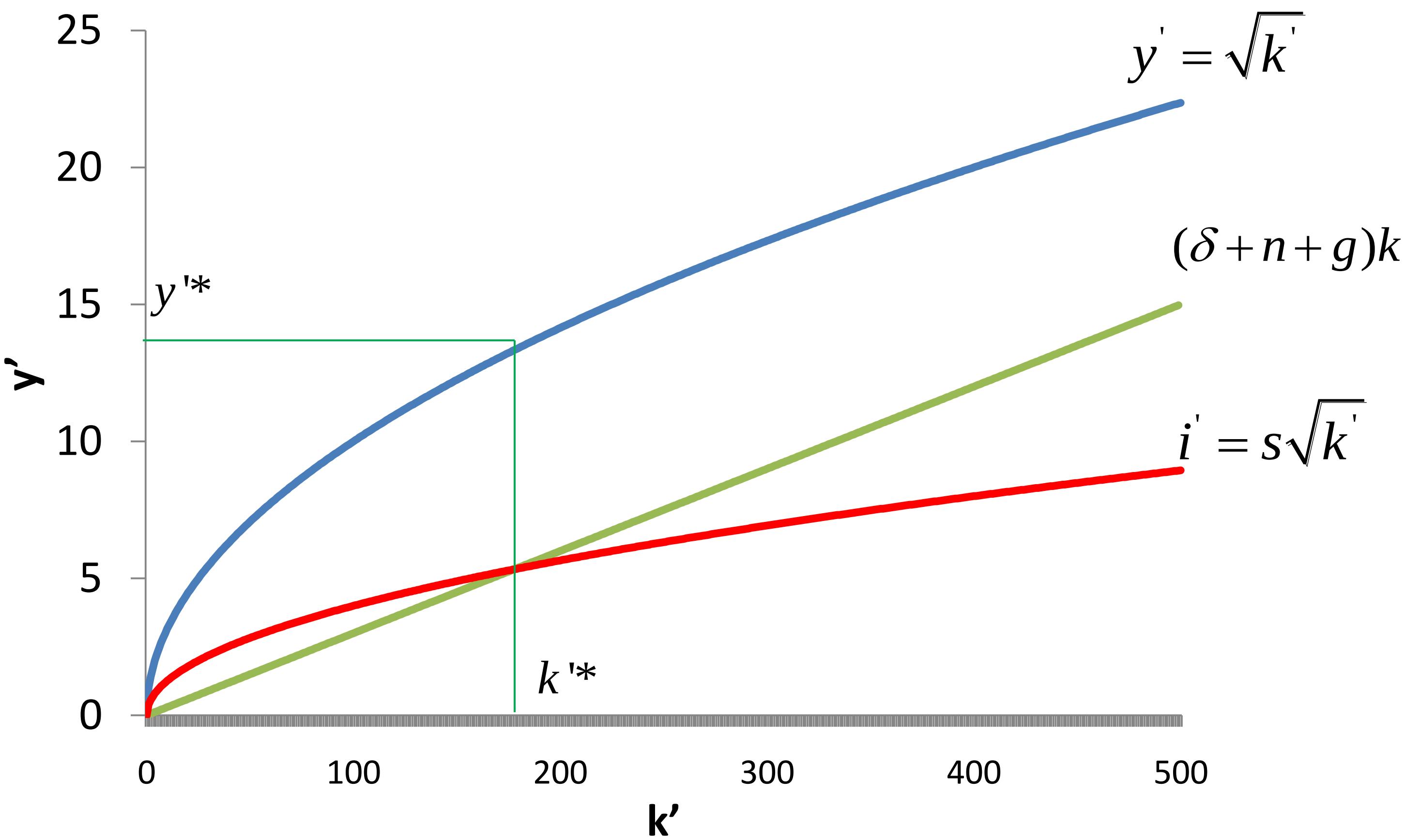
GDP per effective labor → $$y_t^{'} = f(k_t^{'})$$

↑ **Capital per effective labor**

- We are interested in **GDP per capita** $y = \frac{Y_t}{L_t} = A_t \cdot F\left(\frac{K_t}{A_t L_t}\right) = A_t \cdot f(k_t^{'})$

The Solow-Swan Model with Technological Progress (Cont.)

Steady state: Constant levels of capital and output **per effective worker**



The Solow-Swan Model: Technological Progress (Cont.)

- Capital and output per effective worker are constant in steady state
- **What about per capita variables?**

$$y^{*} = A_t \cdot \boxed{f(k_{*}^{'})}$$

GDP per capita grows at the rate **of technological progress (sustainable growth)**

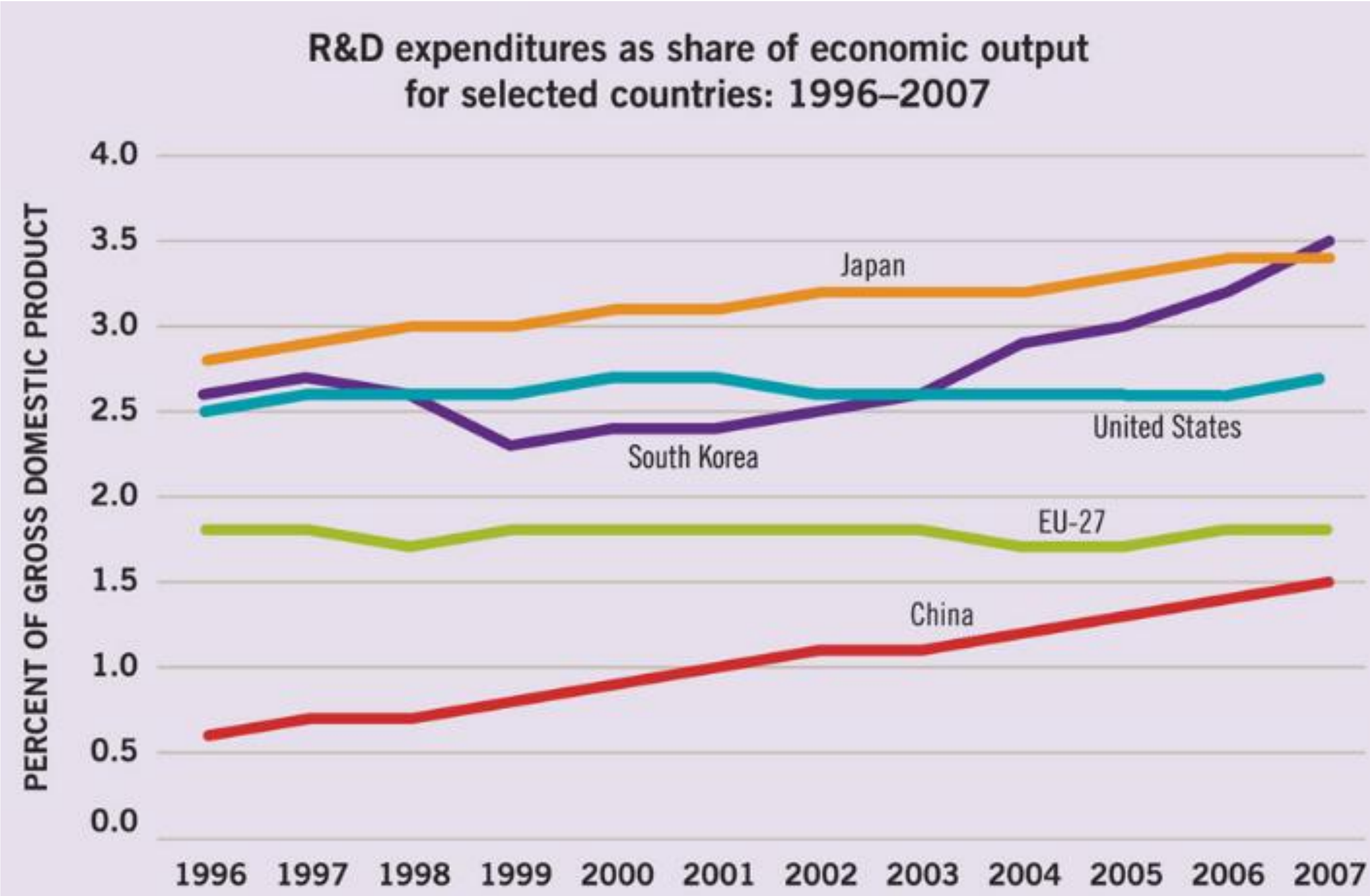
Balanced growth path: growth of variables at the same rate

•Per **capita variables** (capital, output and consumption) grow at a constant rate **g**

• **Per effective labor** variables are **not growing** in the steady state

N!B! Solow model explains 60 % of cross-country variation of the GDP per capita

by differences in savings rate and population growth


R&D expenditures as share of economic output
for selected countries: 1996–2007
PERCENT OF GROSS DOMESTIC PRODUCT
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Japan
United States
South Korea
EU-27
China

Growth Accounting

- Real GDP per capita growth rate for **Czech Republic** in 2011 was **1.7 %**
- Real GDP per capita growth rate for the **USA** in 2012 was **2.2 %**

How much of this growth is due to the factors' accumulation and/or technology?

Growth accounting: breakdown of observed growth of GDP into changes in inputs and technology

$$Y = F(A, K, L)$$

$$\Delta Y = \Delta A + \Delta K + \Delta L$$

Contribution of technology as a residual

$$\Delta A = \Delta Y - \Delta K - \Delta L$$

Growth Accounting (Cont.)

- **Capital** (**K**) increases by 1 unit

  What is the effect on output Y?

$$Y = F(A, K, L)$$

$$F(A, K+1, L) - F(A, K, L)$$

Marginal product of capita (MP_K)

TE Capital stock increased by 10 units and MP_K =0.1. What is the impact on GDP?

$$\Delta Y = 0.1 \cdot 10 = 1 \text{ unit}$$

Growth Accounting (Cont.)

- **Labor** (**L**) increases by 1 unit

What is the effect on output Y?

$$Y = F(A, K, L)$$

$$F(A, K, L+1) - F(A, K, L)$$

Marginal product of labor (MP_L)

TE Labor force increases by 10 units and MP_K =0.3.

$$\Delta Y = 0.3 \cdot 10 = 3 \text{ units}$$

Solow Residual

- Accounting for the increase in all components

$$Y = F(A, K, L)$$

$$\Delta Y = \boxed{MP_A \cdot \Delta A} + MP_K \cdot \Delta K + MP_L \cdot \Delta L$$

How to account for the technological change?

Calculate it as a **residual**

$$MP_A \cdot \Delta A = \Delta Y - MP_K \cdot \Delta K - MP_L \cdot \Delta L$$

Solow Residual: the left-over growth of output when growth attributed to the changes in labor and capital is subtracted

Solow Residual (Cont.)

- Where do we get marginal products of capital and labor?

$$\Delta Y = MP_A \cdot \Delta A + MP_K \cdot \Delta K + MP_L \cdot \Delta L$$

Mathematical manipulations

- Transforming changes to growth rates

GDP growth rate

$$\frac{\Delta Y}{Y} = MP_A \cdot \frac{\Delta A}{Y} + MP_K \cdot \frac{\Delta K}{Y} + MP_L \cdot \frac{\Delta L}{Y}$$

Unobservable technological change (g)

$$\frac{\Delta Y}{Y} = g + \frac{F_k K}{Y} \frac{\Delta K}{K} + \frac{F_L L}{Y} \frac{\Delta L}{L}$$

Solow Residual (Cont.)

$$\frac{\Delta Y}{Y} = g + MP_K \cdot \frac{\Delta K}{Y} + MP_L \cdot \frac{\Delta L}{Y}$$

$$\frac{\Delta Y}{Y} = g + \frac{MP_K \cdot K}{Y} \cdot \frac{\Delta K}{K} + \frac{MP_L \cdot L}{Y} \cdot \frac{\Delta L}{L}$$

Share of capital in output

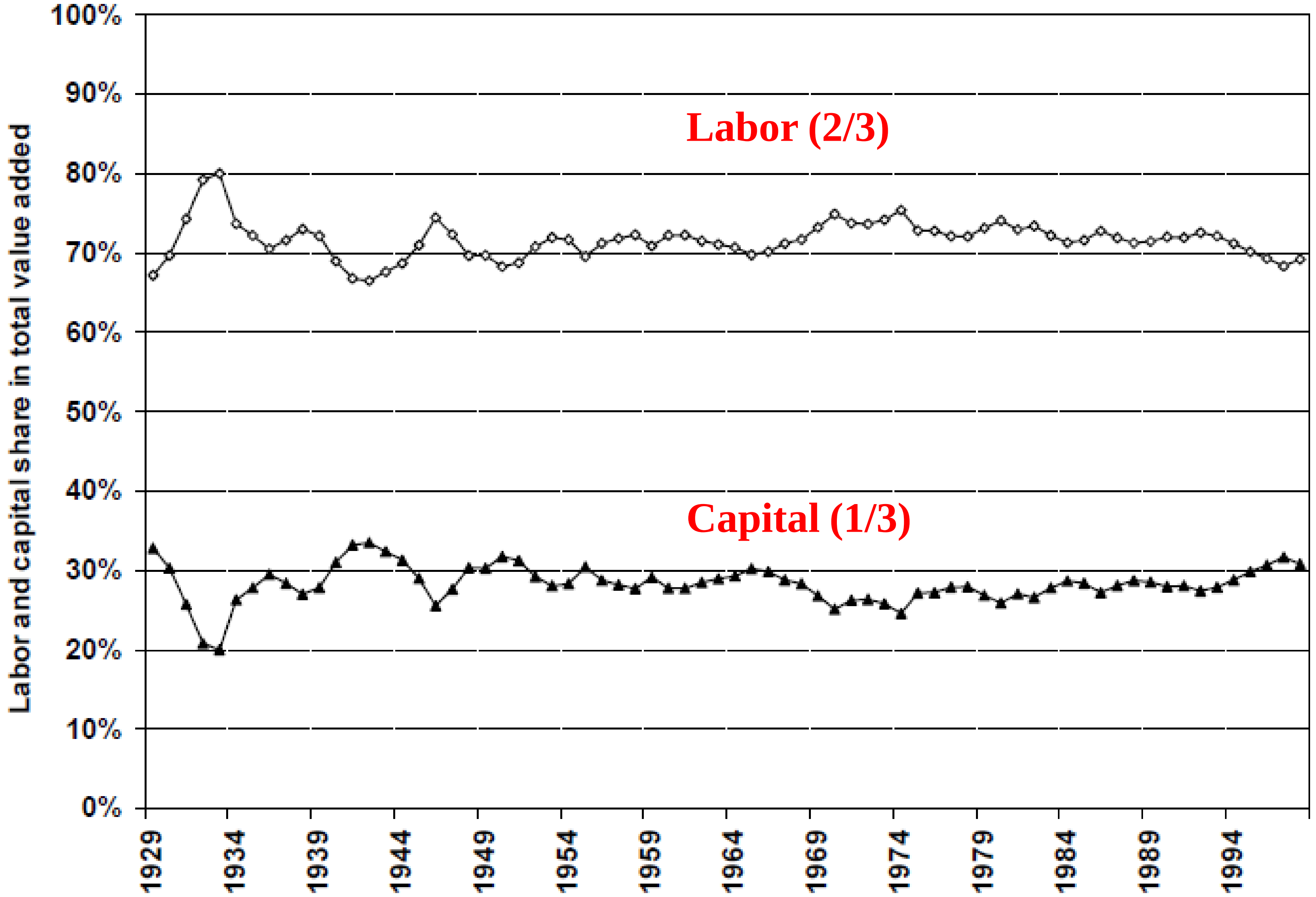
Share of labor in output

N!B! Key assumption: Factors of production are **paid marginal product**

- **Wages** and **rental rate** of capital reflect productivity of factors

$$\frac{\Delta Y}{Y} = g + \alpha \cdot \frac{\Delta K}{K} + \beta \cdot \frac{\Delta L}{L}$$

Historical Factor Shares



Source: Acemoglu, 2009

Accounting for Economic Growth in the United States

Years	Output Growth $\Delta Y/Y$	=	Capital $\alpha \Delta K/K$	+	Labor $(1-\alpha)\Delta L/L$	+	Total Factor Productivity $\Delta A/A$
			SOURCE OF GROWTH				
			(average percentage increase per year)				
1948–2007	3.6		1.2		1.2		1.2
1948–1972	4.0		1.2		0.9		1.9
1972–1995	3.4		1.3		1.5		0.6
1995–2007	3.5		1.3		1.0		1.3

Source: Mankiw, 2009

Growth Accounting for a Sample of Countries

Country	(1) Growth Rate of GDP	(2) Contribution from Capital	(3) Contribution from Labor	(4) TFP Growth Rate
		Panel A: OECD Countries, 1947–73		
Canada ($\alpha = 0.44$)	0.0517	0.0254 (49%)	0.0088 (17%)	0.0175 (34%)
France[a] ($\alpha = 0.40$)	0.0542	0.0225 (42%)	0.0021 (4%)	0.0296 (54%)
Germany[b] ($\alpha = 0.39$)	0.0661	0.0269 (41%)	0.0018 (3%)	0.0374 (56%)
Italy[b] ($\alpha = 0.39$)	0.0527	0.0180 (34%)	0.0011 (2%)	0.0337 (64%)
Japan[b] ($\alpha = 0.39$)	0.0951	0.0328 (35%)	0.0221 (23%)	0.0402 (42%)
Netherlands[c] ($\alpha = 0.45$)	0.0536	0.0247 (46%)	0.0042 (8%)	0.0248 (46%)
U.K.[d] ($\alpha = 0.38$)	0.0373	0.0176 (47%)	0.0003 (1%)	0.0193 (52%)
U.S. ($\alpha = 0.40$)	0.0402	0.0171 (43%)	0.0095 (24%)	0.0135 (34%)
		Panel B: OECD Countries, 1960–95		
Canada ($\alpha = 0.42$)	0.0369	0.0186 (51%)	0.0123 (33%)	0.0057 (16%)
France ($\alpha = 0.41$)	0.0358	0.0180 (53%)	0.0033 (10%)	0.0130 (38%)
Germany ($\alpha = 0.39$)	0.0312	0.0177 (56%)	0.0014 (4%)	0.0132 (42%)
Italy ($\alpha = 0.34$)	0.0357	0.0182 (51%)	0.0035 (9%)	0.0153 (42%)
Japan ($\alpha = 0.43$)	0.0566	0.0178 (31%)	0.0125 (22%)	0.0265 (47%)
U.K. ($\alpha = 0.37$)	0.0221	0.0124 (56%)	0.0017 (8%)	0.0080 (36%)
U.S. ($\alpha = 0.39$)	0.0318	0.0117 (37%)	0.0127 (40%)	0.0076 (24%)