Lecture 6. Explaining Economic Growth

Solow-Swan Model
• Solow Model:
  
  • Role of savings and population growth

  • The role of technology
The evolution of GDP per capita, 1960-2010

Log GDP

USA
UK
S. Korea
Singapore
Botswana
Guatemala
India
Nigeria

Solow-Swan Model of Economic Growth (1956)

- What drives an increase in GDP per capita in a long run?


- Dynamic general equilibrium model

- The model is only as good as its assumptions

**Economic environment (a set of assumptions)**

- A single composite good

- **Two factors** of production: capital and labor

- **Two agents:** firms and households

- A **closed** economy
Solow-Swan Model: Supply Side

Production function (technology)

- Maximum output for given inputs

Aggregate output

\[ Y = F(K, L) \]

Factor Inputs

- If the quantity of both inputs doubles, the output of potatoes also doubles

=> Constant returns to scale (CRS)

\[ 2Y = F(2K, 2L) \]
Properties of production function

- Output is a **positive** function of inputs $Y = F(K_{(+)}, L_{(+)})$

What would happened to GDP if **only one** input increases?

- **Diminishing returns** to factor inputs

For a **fixed** $L$, an increase in $K$ would lead to smaller and smaller increase in $Y$

For a **fixed** $K$, an increase in $L$ would lead to smaller and smaller increase in $Y$
Solow-Swan Model: GDP Per Capita

• Transforming model to **per capital** terms

• Divide **both sides** of production function by the size of labor force

\[ Y = F(K, L) \]

!! The level of capital per worker determines the level of output per worker

\[ \frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = F\left(\frac{K}{L}\right) \quad \text{Due to CRS} \]

\[ y = f(k) \]

**GDP per capita**  **Capital/labor ratio**

\[ y = f(k) = \sqrt{k} \]

• TE
Diminishing returns  \[ y = f(k) = \sqrt{k} \]

**Implication:** Countries with small capital stock \( k \) are more productive => grow faster
Experience of Germany and Japan after the WW II

<table>
<thead>
<tr>
<th>Country</th>
<th>Average annual growth rate of GDP per capita</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>6.6 %</td>
</tr>
<tr>
<td>Japan</td>
<td>6.8 %</td>
</tr>
<tr>
<td>France</td>
<td>9.6 %</td>
</tr>
<tr>
<td>USA</td>
<td>1.2 %</td>
</tr>
</tbody>
</table>

*Source: Blanchard et al (2010)*
Solow-Swan Model: Demand Side

Total output = Total expenditure = Total income

\[ Y = C + I \]

- Consumption
- Investment

A fixed fraction of HH income is saved

Exogenous savings rate \((s)\)

\[ I = sY \]
\[ c = Y - sY = (1 - s)Y \]

I & C per capital

\[ i = sy = sf(k) \]
\[ c = (1 - s)f(k) \]

Savings rate \((s)\) determines the allocation of income between \(C\) & \(I\)
Solow-Swan Model: Graphical Representation

Output per capita: \( y = \sqrt{k} \)

Investment: \( i = 0.3\sqrt{k} \)
Solow-Swan Model: Capital Accumulation

- Size of the labor force is fixed (no population growth)
- GDP per capital will increase only due to increase in capital stock

\[
\frac{Y_t}{L} = F\left( \frac{K_t}{L} \right)
\]

- Households’ savings are used as investment into capital accumulation \( K \)

\[ I = sY \]

- Investment is proportional to output: higher \( Y \) => higher \( sY \) => higher \( I \)

- Capital depreciates at an exogenous rate \( \delta \)

- Every year a fraction of capital \( \delta \) breaks down and becomes useless

\[
K_{t+1} = I_t + (1 - \delta)K_t
\]
• Capital accumulation

\[ K_{t+1} = I_t + (1 - \delta)K_t \]
\[ K_{t+1} = sY_t + (1 - \delta)K_t \]

• Change in capital from year t to year t+1

\[ K_{t+1} - K_t = sY_t - \delta K_t \]
\[ \Delta K = sY - \delta K \]
\[ \Delta k = sf(k) - \delta k \]

• If \( sY > \delta K \) capital stock increases
• If \( sY < \delta K \) capital stock decreases
Solow Model: Steady-State

- **Steady-state**: investment and depreciation just balance

\[ I = \delta K \rightarrow \Delta K = 0 \rightarrow \Delta k = 0 \]
**Steady-state**: the long-run equilibrium of the economy

The amount of savings per worker is just sufficient to cover the depreciation of the capital stock per worker.

- Economy will remain in the steady state (unless additional channels of growth are introduced)
  \[ \Delta k = sf(k^*) - \delta k^* = 0 \]
  \[ y^* = \sqrt{k^*} \rightarrow \Delta y = 0 \]

- Economy which is not in the steady state will go there => convergence to the constant level of output per worker over time.

- Different economies have **different steady state** value of capital.
Solow Model: Steady-State Level of Capital per Worker

- Convergence to steady state

\[ y = \sqrt{k} \]
\[ \delta k = 0.05k \]
\[ i = 0.3\sqrt{k} \]
Implications

- Savings rate (s) has **no effect** on the long-run **growth rate of GDP** per capita.
  - Increase in savings rate will lead to higher growth of output per capita for some time, but not forever.

- Saving rate is bounded by interval [0, 1]

- Savings rate determines the **level of GDP** per capita in a long run.

**Comparative statics**: Increase in savings rate
**Solow Model: Increase in Savings Rate**

- Savings rate increases from 30% to 40%

- Economy moves to a new steady state

\[ y = \sqrt{k} \]

\[ \delta k = 0.05k \]

\[ i = 0.4\sqrt{k} \]

\[ i = 0.3\sqrt{k} \]

- Economy moves to a **new steady state** => Higher capital and output per capita
A nation that devotes a large fraction of its income to savings will have a higher steady-state capital stock and a high level of income.
Summary

- GDP per capita is a function of per capita capital (capital/labor ratio) only.
- In the long run, capital/labor ratio reaches its steady state for the exogenous savings.
- In the steady state, per capita variables are constant => No growth in the long-run.
- Growth is possible only during the transition to steady state, but it is not sustainable.
Solow-Swan Model: Population Growth

- Labor force is growing at a constant rate $n$

\[ Y = F(K, L) \]

\[
\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = F\left(\frac{K}{L}\right)
\]

\[ K_{t+1} = I_t + (1 - \delta)K_t \]

\[ \Delta k = sf(k) - \delta k \]

\[ \Delta k = sf(k) - (\delta + n)k \]

- **Per capita capital** stock is affect by *investment, depreciation, and population growth*
Solow-Swan Model: Population Growth (Cont.)

- An increase in \( n \) reduces \( k^* \) and \( y^* \) =>
  
  => Economies with high rates of population growth will have **lower** GDP per capita
• Reduction of fertility should rise income per person in the long run

TE Chinese totalitarian policy of one child per couple