CEE Growth & Development

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Lecture 5

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The production function I

This is our way to summarize economic activity in the simplest possible terms:

Inputs This is what is used by the economy:

- Capital, K, which captures means of production like buildings, machines, infrastructure. Anything that you need to invest and which stockpiles with time.
- Labor, L, which captures the working population. This is also stands in for population in growth models, and it grows at a natural rate.

Output Production, Y, which stands in for all the goods and services produced by the economy. This is what we use to describe GDP, and we use the change in Y to model economic growth. Growth in Y/L then stands for growth in GDP per capita.

The production function II

$$Y = F(K, L)$$

Key question: How does the output Y change when we change the amount of outputs?

Typical assumption: The production function has *constant returns to scale*. That is, if we scale both inputs by a factor z, the output will also change by that factor:

$$F(zK, zL) = zF(K, L)$$

Output per worker (GDP/capita)

We are often interested in the output per worker, a stand in for the concept of GDP/capita. The constant returns to scale assumption helps us make a general definition:

$$
\begin{aligned}
\frac{1}{L}Y &= \frac{1}{L}F(K, L) \\
&= F(K/L, L/L) = F(K/L, 1)
\end{aligned}
$$

So the output/worker only depends on the how much capital is available to each worker. L does not feature in the equation except in K/L, so this result is independent of the size of the population. You will have higher GDP/capita if you have more capital per worker, regardless of the country size
(US v. India, Czech Republic v. Bolivia)

Output per worker, continued

For that reason, we often use a per-capita representation of the production function (it is valid for countries of all sizes):

$$\frac{Y}{L} = F(\frac{K}{L}, 1)$$
$$y = f(k)$$

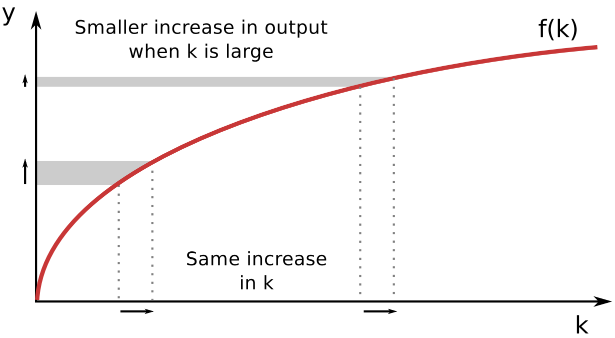
Where

y: output per worker (or GDP/capita)

k: capital per worker (industrialization)

Diminishing marginal product

Another very common assumption: As k increases, its effect on y decreases.



Significance: A 10% increase in capital/worker in Bolivia would have a higher impact on output/worker than the same increase in Czech Republic.

The Cobb-Douglas production function

It is hard to keep talking about $F(K, L)$ without giving it a proper shape. The most common function we use is the Cobb-Douglas production function:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$$

A: total factor productivity (TFP), a multiplier needed to model advances in technology

α: a parameter that captures how K and L combine to produce Y

The per/worker variety is equally simple

$$\begin{aligned} y = Y/L &= \frac{AK^{\alpha}L^{1-\alpha}}{L^{\alpha}L^{1-\alpha}} \\ &= A(K/L)^{\alpha}(L/L)^{1-\alpha} \\ &= Ak^{\alpha} \end{aligned}$$

The Cobb-Douglas production function, continued

What is the marignal product of capital?

$$Y = AK^{\alpha}L^{1-\alpha}$$
$$\partial Y/\partial K = \alpha AK^{\alpha-1}L^{1-\alpha}$$

What share of the total output goes to pay for capital?

$$\frac{MP_K K}{Y} = \frac{\alpha AK^{\alpha-1}L^{1-\alpha}K}{AK^{\alpha}L^{1-\alpha}} = \alpha$$

What share of the total output goes to pay for labor?

$$\frac{MP_L L}{Y} = \frac{(1-\alpha)AK^{\alpha}L^{-\alpha}L}{AK^{\alpha}L^{1-\alpha}} = 1-\alpha$$

Fun fact: The empirical observation that shares of output going to pay for capital and labor are constant led Cobb and Douglas to come up with this function.