

## ECONOMETRICS II

### Exercise session #7 (Ordered Probit and Logit)

**Problem 1.** Write down the log-likelihood function of ordered probit/logit models.

**Problem 2.** In an ordered probit model with four outcomes, you obtain the following results:

$$\hat{y}_i^* = 10 + 60x_1 - 70x_2$$

where  $\sigma = 100$ , with thresholds  $\mu_0 = -\infty$ ,  $\mu_1 = 0$ ,  $\mu_2 = 100$ ,  $\mu_3 = 200$ ,  $\mu_4 = +\infty$ .

- Evaluate  $\frac{\partial \mathbb{P}(100 < y_i^* \leq 200)}{\partial x_1}$  at  $x_1 = x_2 = 1$  and at  $x_1 = 2$  and  $x_2 = 1$ . What is the effect of corresponding unit increase in  $x_1$  (treating  $x_1$  as discrete) on  $\mathbb{P}(100 < y_i^* \leq 200)$ ?
- Evaluate  $\frac{\partial \mathbb{P}(y_i^* < 0)}{\partial x_2}$  at  $x_1 = x_2 = 1$  and at  $x_1 = 1$  and  $x_2 = 2$ . What is the effect of corresponding unit increase in  $x_2$  (treating  $x_2$  as discrete) on  $\mathbb{P}(y_i^* > 100)$ ?

**Problem 3.** You are interested in the effect of monthly income on monthly food expenditure in the following model:

$$\ln(\text{foodexp}_i) = \alpha + \beta \ln(\text{income}_i) + \epsilon_i$$

where  $\epsilon_i$  is distributed  $N(0, \sigma^2)$ .

However, instead of observing the actual value of food expenditure, you only observe whether the given individual's food expenditure falls into one of the following brackets:  $\langle 0, 5000]$ ,  $\langle 5000, 15000]$ , and more than 15000.

- Given the distributional assumption, how would you estimate the model? Write down the likelihood function and determine which parameters can be estimated.
- What is the effect of a 1% increase in income on the probability of having food expenditure above 15000 CZK?

**Problem 4.** You are asked to estimate the effect of preventive health expenditure on the number of days an individual is ill in a given year. However, instead of observing the actual number of days of illness, you only observe whether the number of days falls into one of the following categories:  $[0, 7]$  days,  $\langle 7, 14]$  days, and more than 14 days.

Assume that there are no or only few individuals who are not ill at all in a given year.

- Propose a model of the number of days of illness per year as a function of preventive health expenditure. State any assumptions you make and write down the log-likelihood function. Which parameters of the models are identified?
- Derive the effect of preventive health expenditure on the probability of being ill for more than two weeks. How would you summarize this effect for the whole sample?