CERGE-EI
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Econometrics II
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Exercise Session 1

Review Excercises

1. You have two independent samples of individuals in the labor force in two different countries (first sample has a size n_1, the second n_2). Each sample contains a binary indicator U_{ci} describing whether and individual i from country c is unemployed $U_{ci} = 1$ or works $U_{ci} = 0$. Based on the estimates form the two samples test whether the unemployment rates in the two countries are equal. [Assumethat $U_{ci} = 1$ with probability p_c]

2. Assume a regression model $Y = X\beta + \epsilon$, where X is a $N \times k$ matrix of explanatory variables, β is the $k \times 1$ vector of coefficients, and ϵ is the $N \times 1$ vector of error terms.
   - Write down the matrix expression for $\hat{\beta}^{OLS}$ and $VAR(\hat{\beta}^{OLS})$.
   - What are the basic assumptions for $\hat{\beta}^{OLS}$ to be BLUE?
   - What is the additional assumptions for $\hat{\beta}^{OLS}$ to be distributed Normally?
   - What are the minimum necessary conditions that assure that $\hat{\beta}^{OLS}$ is consistent?
   - What is the additional assumption that assures that $\hat{\beta}^{OLS}$ is efficient?

3. Assume a regression model $y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \epsilon_i$ estimated by OLS. All the assumptions hold for $\hat{\beta} \sim AN(\beta, \sigma^2(X'X)^{-1})$, where
   $X = [1 \;\; x \;\; z]$.
   - What is the estimate of the effect of x_i on y_i?
   - Test that $\beta_1 = 0$
   - Test that $\beta_1 = \beta_2$
   - construct the confidence interval for $\frac{\beta_2}{\beta_1}$

4. Assume a model $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$ estimated by OLS. What is the estimate of the effect of x_i on y_i? What is the estimate of the avarage partial effect of x_i on $E(y_i|x_i)$?

5. You have a sample of N individuals from the Czech Republic. You observe their earnings y_i, their gender, and the region they work in.

   - Assume a regression model $y_i = a + \epsilon_i$, where a is a constant and ϵ_i is an error term with 0 mean. What is $\hat{a}^{OLS}$?
   - You believe that individuals who work in Prague earn more on avarage. You construct a dummy variable $D_i^P = 1$ if an individual works in Prague and 0 otheerwise. You then construct a variable $D_i^N P = 1$ if an individual does not work in Prague and 0 otheerwise. What are the OLS estimates in the following models?

     $y_i = a + bD_i^P + u_i$

     $y_i = a + bD_i^{NP} + u_i$

     $y_i = b_1 D_i^P + b_2 D_i^{NP} + u_i$

     $y_i = a + b_1 D_i^P + b_2 D_i^{NP} + u_i$
   - You also believe that men earn more than women, and the difference is greater in Prague. You estimate the following model, where $D_i^M = 1$ if i is a man and 0 otherwise.

     $y_i = b_0 + b_1 D_i^P + b_2 D_i^M + b_3 D_i^P D_i^M + u_i$

     Express the avarage earnings for women and men, in and outsied Prague respectively, in terms of the coefficients.
   - You believe that wages vary sistematically accross R regions. You construct R binary indicators of the region an individual works in: you have a set of R dummy variables $D_i^r = 1$ if an individual i works in region r and 0 otherwise. What are the OLS estiamtes in the following model?

     $y_i = \sum_{r=1}^{r=R} b_r D_i^r + u_i$

     Can you add a constant term? What is the mean of the estimated residuals?