ANSWER KEY HW 2

2.9 CAPITAL TAXATION

At $t=0$ investment gets taxed at rate τ

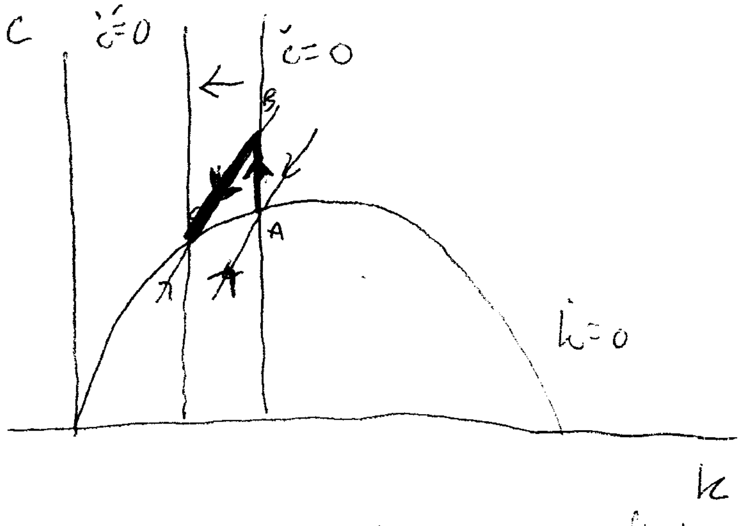
$$\Rightarrow r(t) = (1-\tau) f'(k(t)) \text{ after } t=0$$

Tax revenue is returned to households via <u>lump sum</u> transfers. Policy is <u>unanticipated</u>.

a) $\dot{c}=0$ locus becomes $\dfrac{\dot{c}}{c} = \dfrac{(1-\tau)f'(k) - \rho - \theta g}{\theta}$

Since $r(t)$ does not enter the $\dot{k}$ equation $\dot{k}=0$ is unchanged.

Since $(1-\tau)f'(k) < f'(k)$ $\dot{c}=0$ shifts left

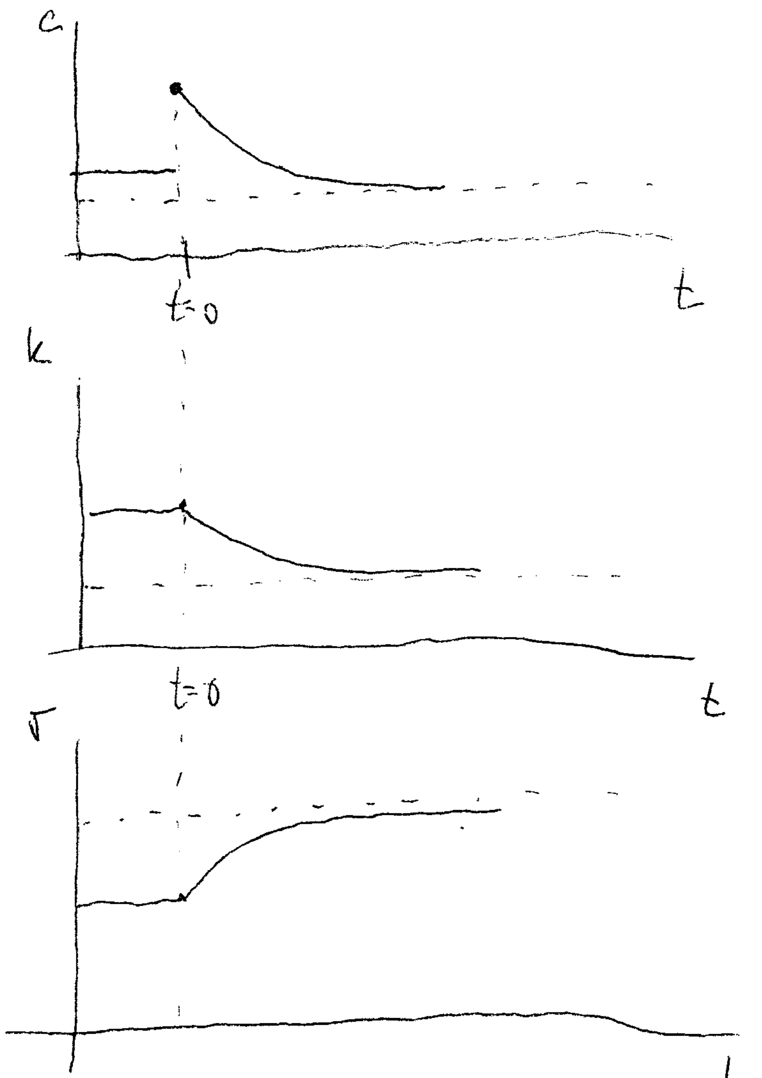


b) At time zero, since the policy is unanticipated consumption will <u>jump up</u> from point A in diagram above to point B. Then it will gradually fall along a new stable path

$\longrightarrow$

Note: The assumption that the policy is unanticipated is what allows for a discontinuous jump in consumption to be consistent with optimizing behavior of the households.

Time path of consumption, capital stock and interest rates



Another note: A number of people have been asking about the convexity of the transition path $\frac{\partial^2 k}{\partial t^2}$ and $\frac{\partial^2 c}{\partial t^2}$

Basically it depends on the slope of the stable path, i.e. $\frac{\dot{c}}{\dot{k}}$

but as economy gets back to BGP it has to ~~be~~ become convex.

2.9 cont.

c) As can be seen from the diagram, after adjustment has taken place $k_{NEW} < k_{OLD}$ and $c_{NEW} < c_{OLD}$

Note that the fact that $c_{NEW} < c_{OLD}$ ~~vaaresu~~ follows from the fact that initially the economy was at an undistorted BGP, hence to the left of where $\dot{k}=0$ attains a maximum.

d) Tax rates differ among countries.

i) Show that $\dfrac{\partial\left(\frac{y^*-c^*}{y^*}\right)}{\partial \tau} < 0$

Let $s = \dfrac{f(k^*)-c^*}{f(k^*)}$. On BGP $f(k^*)-c^* = (n+g)k^*$

constant savings

$$\Rightarrow s = \frac{(n+g)k^*}{f(k^*)}$$

$$\Rightarrow \frac{\partial s}{\partial \tau} = \frac{(n+g)}{f(k^*)}\frac{\partial k^*}{\partial \tau} - \frac{(n+g)k^*}{f(k^*)^2} f'(k^*)\frac{\partial k^*}{\partial \tau}$$

$$= \frac{n+g}{f(k^*)}\frac{\partial k^*}{\partial \tau}\left[1 - \frac{k^* f'(k^*)}{f(k^*)}\right]$$

$$= \underset{(+)}{\frac{s}{k^*}}\ \underset{(-)}{\frac{\partial k^*}{\partial \tau}}\ \underset{(+)}{\left[1 - \underset{\text{share}}{\overset{\text{capital's}}{\text{pre-tax}}}\right]} < 0$$

ii) Do low τ, high k^* households have incentives to invest in low k^* countries

For low τ country assume $\tau = 0$. Then the return to capital is $f'(k_1^*) = \rho + \theta g$ $\boxed{(1-\tau) f'(k_2^*) = \rho + \theta g}$

In country 2 return on capital is ~~also~~ $\cancel{f'(k_2^*) = \rho + \theta g}$

Hence no incentives to invest in low k^* countries.

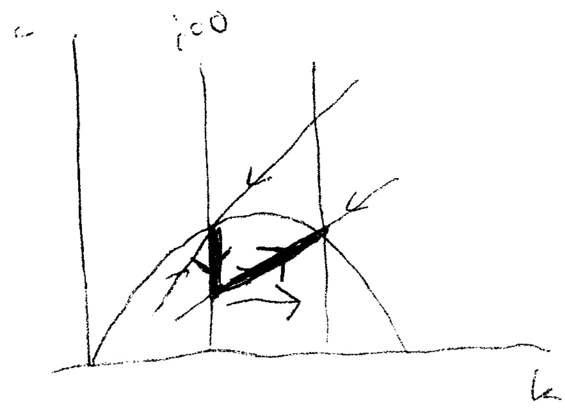
Intuition is that even though country 2 has less capital hence a higher pre-tax return, the difference is offset by the tax

e) Is subsidizing investment ($\tau < 0$) welfare improving.

<u>No.</u> With $\tau = 0$ the economy is already at the <u>modified</u> golden rule of k $\rightarrow$ Hence at a Pareto optimum

$\tau < 0$ will cause too much saving, less consumption and an overall welfare loss.

For completness w/ $\tau < 0$ we have



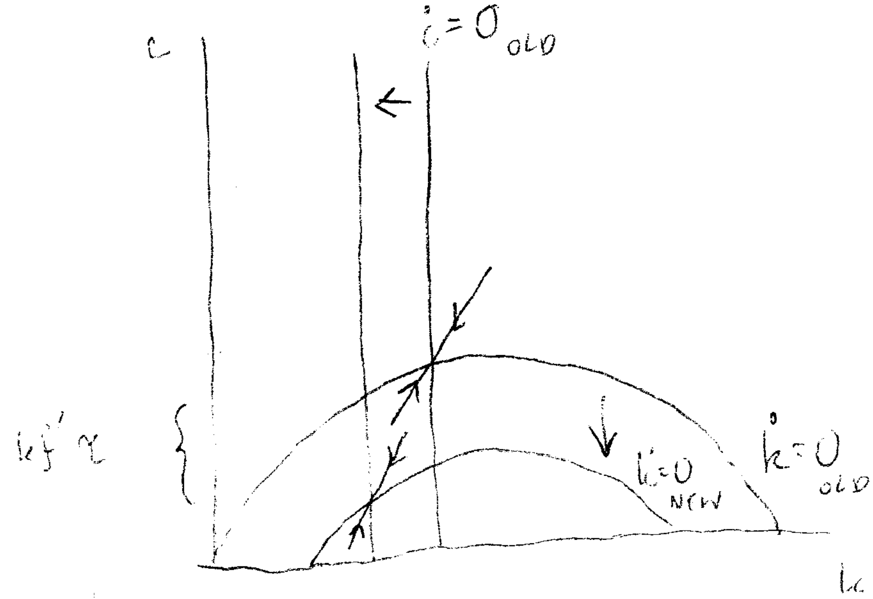
f) How do things change if the government spends the money.

For $\dot{k}=0$. Now part of output is unavailable for either consumption or private investment. Here we will also assume (as in Romer) that the government does not spend the money on public investment. So the modified $\dot{k}=0$ is:

$$\dot{k} = f(k) - c - G - (n+g)k$$

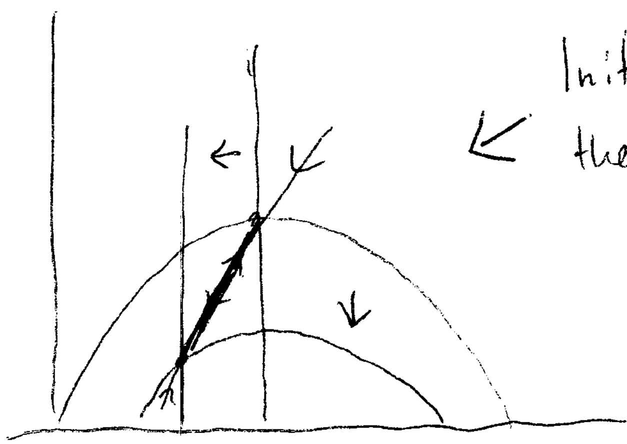
which means the $\dot{k}=0$ locus shifts down.

For $\dot{c}=0$ it depends on whether government spending provides utility for the households. Here the assumption is that G does not enter the utility function. (see prob. 2.12 for other case). So effect on $\dot{c}=0$ is as before; it shifts left.

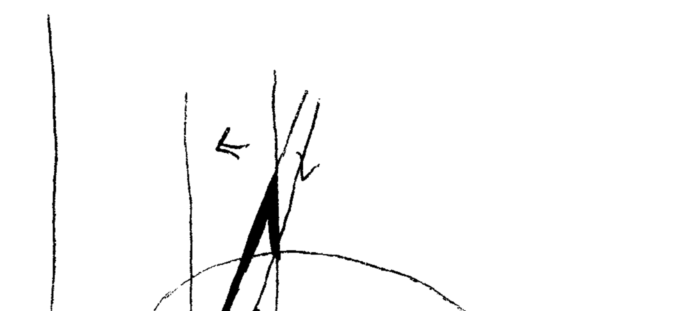


__What we know for sure__: In new BGP both k and c will be lower than in no tax case. c will also be lower than in the tax-with-rebate case since $\dot{k}=0$ shifted down. The amount of tax collected is $\tau f'(k)k$.

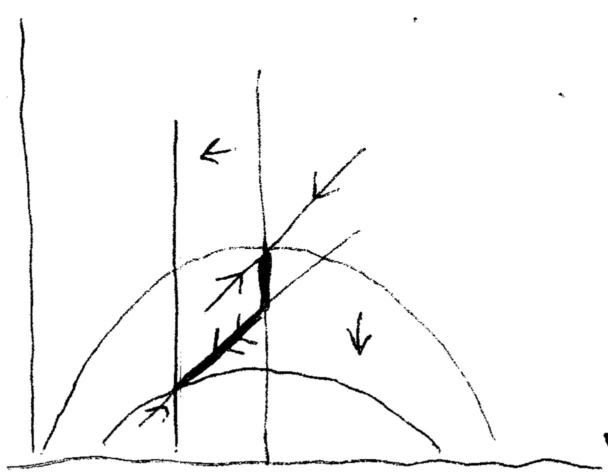
__What we don't know for sure__: What happens to c immidiately after imposition of the tax. 3 cases:



Initially no change in c but then $c \downarrow$ smoothly to new c^*



Initially c jumps up but then $c \downarrow$ smoothly to new c^*



Initially c jumps down and then $c \downarrow$ smoothly to new c^*

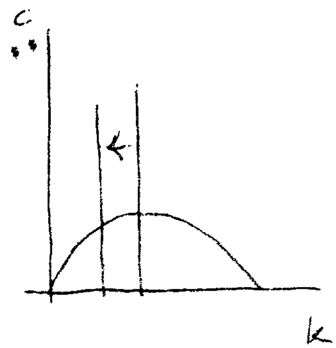
2.10 Anticipated changes

Announce future tax at $t=0$

Implement tax at $t=t_1$

a) Like in 2.9

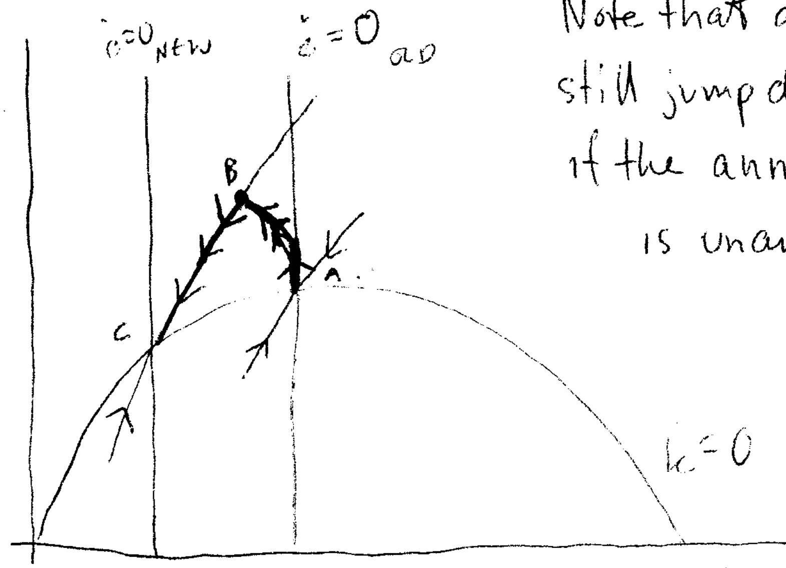
$$\frac{\dot{c}}{c}=\frac{(1-\tau)f'(k(t\geq t_1))-(\rho+\theta g)}{\theta}=0$$



No change in $\dot{k}=0$

b) c CANNOT change discontinously at t_1 because now the policy is anticipated and utility maximization requires consumption smoothing.

c) Putting a) and c) together



Note that at $t=0$ c can still jump discontinously if the annauncment itself is unanticipated

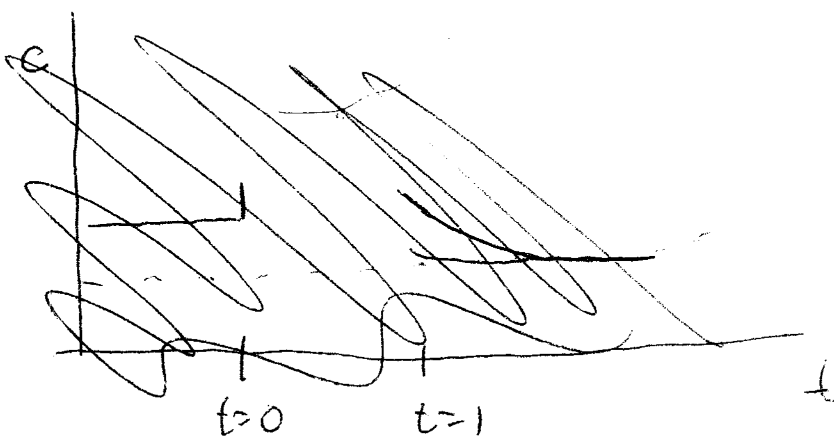
We also know that at $t=t_1$ the economy must be on the new stable path

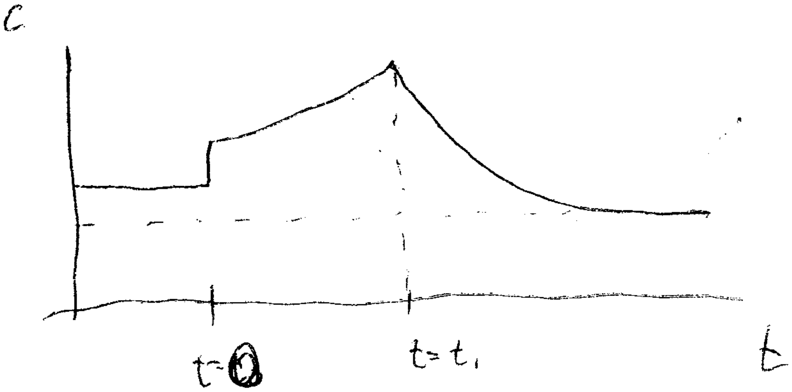
So at $t=0$ c will jump to some point between the new and old stable paths, adjust to new stable path (point B) between $t=0$ and $t=t_1$ and then adjust to new c^*. A possible transition path is illustrated above

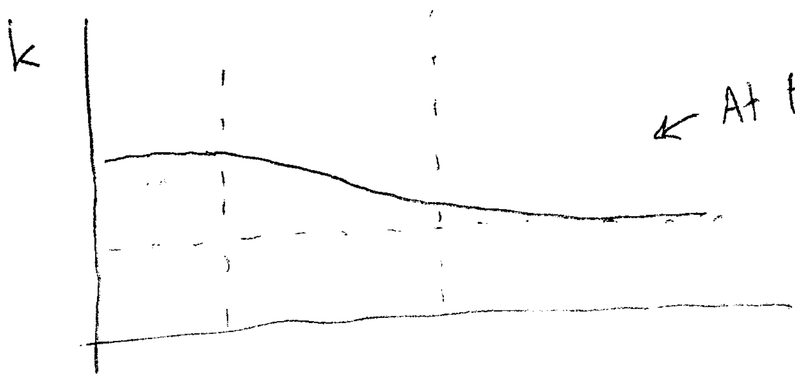
d) As discussed above at $t=0$ c will jump to some point between the old and new stable paths ~~at~~ on the original $\dot{c}=0$ locus.

How much c jumps at $t=0$ will be determined by the parameters of the model and perhaps, specific assumptions about how expectations are formed.

e)







← At $t=t_1$ k is still falling

2.12 Now G affects utility

$$U = B\int_0^{\infty} e^{-\beta t}\frac{[c(t)+G(t)]^{1-\theta}}{1-\theta}$$

Here the increase in G is temporary . . . .

As before $\dot{k}=0$ locus is modified as follows:

$$\dot{k} = f(k) - c - G - (n+g)k = 0$$ (still assuming gov. purchases are not investment)

So $\dot{k}=0$ shifts down

What happens to $\dot{c}=0$? Assume, WLOG that initially $G=0$. Since G and c are perfect substitutes define total consumption as (public & private)

as $C_{TOTAL} = c + G$

Note that households do not get to choose G - they take it as given in their maximization problem.

Hence the equation for $\dot{c}$ becomes

$$\frac{\dot{c}(t)}{c+G} = \frac{f'(k)-(\rho+\theta g)}{\theta}$$

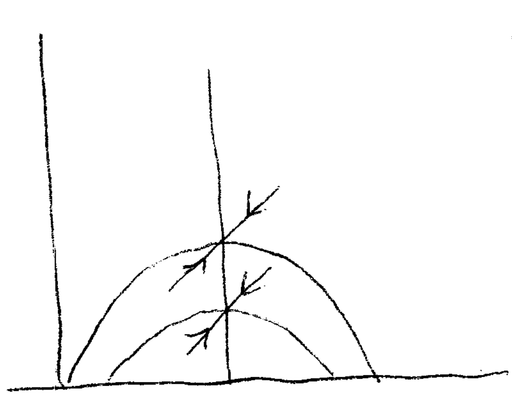
$\Rightarrow$ for $\dot{c}=0$ we still have $f'(k) = \rho + \theta g$

$\Rightarrow$ Nothing happens to $\dot{c}$

$\rightarrow$

Suppose the increase in G is unexpected

Then



When G↑, c↓ by exactly the same amount. This is because utility maximization requires smoothing marginal utility

Hence smoothing of C_{TOTAL}, not just c. When G↓ again, c↑ to its previous level. So ~~the affect of~~ G↑ has no effect on $\dot{c}$, i.e. ~~ø~~ (hence on the interest rate) only on the level of c. Notice that ~~if~~ the analysis w/ G ~~&~~ & c as perfect substitutes and G↑ temporary, is exactly the same as the case in the text where G↑ permanently but G is not in utility function.

If the increase in G is expected, same analysis applies. This is because in order to keep marginal utility constant households do not have to do anything prior to the time when G actually goes up.