MACRO III
(Summer 2005)
Sketch of solution to HW 4

- It seems to me that this was just a simple computational exercise even though the setup was little unclear and suggested that there could be some tricks.
- I think that the role of adjustment costs here is to give rise of multiple solutions only.

a)

$$\mathcal{H} = e^{-rt}\left\{AK - I\left[1 + \frac{b}{2}\,\frac{I}{K}\right]\right\} + q\left[I - \delta K\right]$$

FOC: $\frac{\partial \mathcal{H}}{\partial I} = 0$: $q = \left(1 + b\,\frac{I}{K}\right)e^{-rt}$

$$\dot{q} = e^{-rt}\left[-r\left(1 + b\,\frac{I}{K}\right) + b\,\dot{\left(\frac{I}{K}\right)}\right.$$

$$\dot{q} = -\frac{\partial \mathcal{H}}{\partial K} = -e^{-rt}\left[A + \frac{b}{2}\left(\frac{I}{K}\right)^2\right] + \delta q$$

- use the law of motion to substitute $\frac{I}{K} = \gamma_K + \delta$ and $\dot{\left(\frac{I}{K}\right)} = \dot{\gamma}_K$ (zero in SS)

$$-\cancel{e^{-rt}}\left[-r(1+b(\gamma_K+\delta))+b\overset{0}{\dot{\gamma}_K}\right]=\cancel{e^{-rt}}\left[\overset{r+\delta}{A}+\frac{b}{2}(\gamma_K+\delta)^2-\delta(1+b(\gamma_K+\delta))\right]$$

$$r\left[1+b(\gamma_K+\delta)\right]=r+\delta+\frac{b}{2}(\gamma_K+\delta)^2-\delta\left[1+b(\gamma_K+\delta)\right]$$

$$(r+\delta)\left[1+b(\gamma_K+\delta)-1\right]=\frac{b}{2}(\gamma_K+\delta)^2$$

$$(r+\delta)(\gamma_K+\delta)=\frac{1}{2}(\gamma_K+\delta)^2$$

i) $\gamma_K^*=-\delta$

ii) $\gamma_K+\delta=2(r+\delta)$

$\gamma_K^*=2r+\delta$

- the firm's problem gives us two solutions, to discriminate between them check TVC

TVC: $\lim_{t\to\infty} q_t K_t=0$, use $K_t=K_0 e^{\gamma_K t}$

$$\left.\begin{array}{l} \text{i) } \lim_{t\to\infty}\left(1+b\overset{0}{\frac{I}{K}}\right)K_0 e^{-(r+\delta)t}=0 \\ \text{ii) } \lim_{t\to\infty}\left(1+b\underset{2(r+\delta)}{\frac{I}{K}}\right)K_0 e^{(r+\delta)t}>0 \end{array}\right\}\ \text{only } \gamma_K^*=-\delta \text{ is the equilibrium growth rate}$$

- the relationship between r and γ_K

→ we have got already

$$r = -\delta + \frac{A + \frac{b}{2}(\gamma_K + \delta)^2}{1 + b(\gamma_K + \delta)}$$

$$\frac{dr}{d\gamma_K} = \frac{b(\gamma_K+\delta)\left[1+b(\gamma_K+\delta)\right] - bA - \frac{b^2}{2}(\gamma_K+\delta)^2}{\left[1+b(\gamma_K+\delta)\right]^2} =$$

$$= \frac{b(\gamma_K+\delta)\left[1+\frac{b}{2}(\gamma_K+\delta)\right] - bA}{\left[1+b(\gamma_K+\delta)\right]^2}$$

so that the relationship is quadratic

- However, when evaluated at the steady state ($\gamma_K = -\delta$)

$$\frac{dr}{d\gamma_K} = -bA$$

, so that in the [redacted] steady state the relationship is negative

b) we have two conditions

$$\gamma_K = \gamma_C = \frac{1}{\theta}[r-\rho] \rightarrow r = \theta\gamma_K + \rho$$

$$r = -\delta + \frac{A + \frac{b}{2}(\gamma_K+\delta)^2}{1+b(\gamma_K+\delta)}$$

- solving these two equations for γ_K gives the quadratic equation

$$(\theta\gamma_K + \rho)[1+b(\gamma_K+\delta)] = -\delta + A + \frac{b}{2}(\gamma_K+\delta)^2$$

$$(\theta b - \tfrac{b}{2})\gamma_K^2 + (\theta + \rho b + \theta\delta - b\delta)\gamma_K + \rho + \rho b\delta + \delta - A - \tfrac{b}{2}\delta^2 = 0$$

- it has two roots so that we again need TVC to choose the right one

c) from the law of motion in the household's problem follows

$$\dot{k} = (r-n)k - c \rightarrow \gamma_K = (r-n) - \frac{c}{k}$$

more precise way is in Barro, Sala-i-Martin p. 142-3

again using the fact that γ_K is constant in SS we can differentiate this equation to get

$\dot{\gamma}_K = 0 = -\gamma_C + \gamma_K$, therefore, $\gamma_C = \gamma_K$ and from the production function $Y = AK \Rightarrow \gamma_Y = \gamma_K$

(or from the feasibility constraint $Y = C + I = C + \dot{K} + \delta K$
$\rightarrow \frac{Y}{K} = \frac{C}{K} + \gamma_K + \delta \Rightarrow \gamma_Y - \gamma_K = \gamma_C - \gamma_K + \dot{\gamma}_K \rightarrow \gamma_Y = \gamma_C = \gamma_K$)