~~Ma~~ HW3.

1) a)

$Y = K^{\alpha} (hL)^{1-\alpha} \Rightarrow y = k^{\alpha} h^{1-\alpha}$

$\dot{K} = s_k Y - \delta K \Rightarrow \dot{k} = s_k y - (\delta + n) k \Rightarrow \gamma_k = s_k \frac{y}{k} - (\delta + n)$

$\dot{H} = s_h Y - \delta H \Rightarrow \dot{h} = s_h y - (\delta + n) h \Rightarrow \gamma_h = s_h \frac{y}{h} - (\delta + n)$

on BGP γ_k and γ_h are constant. $\Rightarrow \frac{y}{k}$ and $\frac{y}{h}$ are constant.

and $\frac{\partial \gamma_k}{\partial t} = 0 \Rightarrow$ taking logs + diff. w. r. to time t

$\Rightarrow \frac{\dot{y}}{y} = \frac{\dot{k}}{k}$

the same follows for human capital $\Rightarrow$

$\Rightarrow \gamma_y = \gamma_k = \gamma_h$

$\Rightarrow s_k \frac{y}{k} - (\delta + n) = s_h \frac{y}{h} - (\delta + n) \Rightarrow \frac{h}{k} = \frac{s_h}{s_k}$ along BGP

$\gamma_y = \frac{\dot{y}}{y} = s_k \frac{y}{k} - (\delta + n) = s_k \left(\frac{h}{k}\right)^{1-\alpha} k / k = s_k^{\alpha} s_h^{1-\alpha} - (\delta + n)$

b) $y = k^{\alpha} h^{1-\alpha} k = \left(\frac{s_h}{s_k}\right)^{1-\alpha} k = Ak$

A

Using the ratio of $\frac{s_h}{s_k}$ along BGP from a) c)

To construct decentralized equilibrium we introduce households that will maximize their lifetime utility.

Because $\gamma_y = s_k^{\alpha} s_h^{1-\alpha} - (\delta + n)$, ~~How~~ that is non zero even in the case where $n = 0$, $\Rightarrow$ there is always some ~~g~~ growth even without introduction of externalities.

The decentralized equilibrium will be socially optimal because there are ~~no~~ no distortions present therefore it will the same as SP equilibrium.

d) When non negativity restrictions are imposed then there is no possibility of negative adjustment in capital (physical or human). Therefore the capital cannot adjust immediately and ~~this~~ gradual dissaving of excessive capital at rate of depreciation occur.

For complete discussion see section 5.12. Barro + Sala-i-Martin

2)

Firm i's maximization problem $\max_{k_i, L_i} k^{\beta} K_i^{\alpha} (AL_i)^{1-\alpha} - wL_i - (r+\delta)K_i$

to rearrange FOC's we use $k = K = K_i$; ~~[illegible]~~

+ we use $g = \frac{\dot{A}}{A}$, $n = \frac{\dot{L}_i}{L_i}$

F.O.C.

$$K_i: \quad \alpha k^{\beta} K_i^{\alpha-1} (AL_i)^{1-\alpha} - (r+\delta) = 0$$

substitute k

$$\alpha k^{\beta+\alpha-1} (AL_i)^{1-\alpha} - (r+\delta) = 0$$

here we solve for decentralized solution, therefore we take derivative with respect to K_i and substitute for K_i
in part c) we first substitute and the differentiate to find social planner solution

take logs

$$\ln \alpha + (\beta+\alpha-1) \ln k + (1-\alpha)[\ln A + \ln L_i] - \ln(r+\delta) = 0$$

we $\gamma_k = \frac{\dot{k}}{k} = \frac{d \ln k}{dt}$; differentiate

$$(\beta+\alpha-1) \frac{\dot{k}}{k} + (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{L}_i}{L_i}\right) = 0$$

$$(\beta+\alpha-1) \gamma_k + (1-\alpha)(g+n) = 0$$

$$\Rightarrow \gamma_K = -\frac{(1-\alpha)(g+n)}{\beta+\alpha-1}$$

a) to calculate ~~economy~~ growth of economy, we need to calculate γ_y

$$y \quad Y = k^{\beta} K_i^{\alpha} (AL_i)^{1-\alpha} = k^{\beta+\alpha} (AL_i)^{1-\alpha} \quad / \text{take logs and differentiate.}$$

we $k = K_i$

$$\ln y = \ln k (\beta+\alpha) + (1-\alpha)(\ln A + \ln L_i)$$

$$\gamma_Y = \gamma_K (\beta+\alpha) + (1-\alpha)(n+g)$$

$$\gamma_Y = -\frac{(1-\alpha)(g+n)(\alpha+\beta)}{\beta+\alpha-1} + (1-\alpha)(n+g) = \frac{(g+n)(1-\alpha)}{\beta+\alpha-1}(-\alpha-\beta+\beta+\alpha-1) =$$

$$= -\frac{(1-\alpha)(g+n)}{\beta+\alpha-1}$$

compare γ_k with γ_Y ⇒ both grow at same rate

now calculate per capita growth γ_y

$$\frac{\partial \ln \frac{Y}{L}}{\partial t} = \frac{\left(\dot{\frac{Y}{L}}\right)}{\frac{Y}{L}} = \frac{\dot{Y}L - Y\dot{L}}{YL} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \gamma_Y - n$$

therefore per capita growth of economy is $\gamma_y = \gamma_Y - n = \frac{-(1-\alpha)(g+n)}{\beta+\alpha-1} - n$

$$\gamma_y = \frac{-(1-\alpha)g + n\beta}{\beta+\alpha-1}$$

set $g=0$

$$\gamma_y = \frac{(\alpha-1)n - n(\alpha-1) - n\beta}{\beta+\alpha-1} = \frac{-n\beta}{\beta+\alpha-1} \neq 0 \text{ for } n \neq 0 \text{ and } \beta \neq 0$$

⇒ There is still non zero per capita growth, even without techology growth.

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b) Solve HH's problem $\max \int_0^\infty e^{-(\rho-n t)} u(c_t)\, dt$

$$\dot{a} = ra + w - c - na$$

+NPG condition

we get Euler equation $\frac{\dot{c}}{c} = \frac{1}{\theta}[r - \rho]$

plug from a) for r ⇒ $\frac{\dot{c}}{c} = \frac{1}{\theta}[\alpha K^{\beta+\alpha-1}(AL)^{1-\alpha} - \delta - \rho]$

$$\Rightarrow \frac{\dot{c}}{c} = \frac{1}{\theta}\left[\alpha \frac{y}{k} - \delta - \rho\right] = \gamma_c$$

because in Ramsey model $I = Y - C$, ⇒ $\dot{K} = Y - C + \delta K$

and $s = \frac{Y-C}{Y}$

⇒ in per capita terms $\frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k} - (n+\delta)$ case $\frac{\dot{k}}{k} = \gamma_k$

$$\Rightarrow \frac{c}{k} = \frac{y}{k} - (\gamma_k + n + \delta) \Rightarrow \frac{c}{y} = 1 - (\gamma_k + n + \delta)\cdot\frac{k}{y}$$

$$s = 1 - \frac{c}{y} = (\gamma_k + n + \delta)\frac{k}{y}$$

From EE. $\Rightarrow$

$$\frac{1}{\alpha}\left(\theta \mu_c + \delta + \rho\right) = \frac{y}{k}$$

$$\frac{k}{y} = \frac{\alpha}{(\theta \mu_c + \delta + \rho)}$$ on BGP $\mu_c = \mu_k$

therefore saving rate can ~~be written~~ rearranged as

$$s^{CE} = \frac{\alpha}{(\theta \mu_{c} + \delta + \rho)} \cdot (\mu_k + n + \delta)$$

c) The difference between social planner ~~solu~~ and decentralized solution (CE) is that the social planner recognizes that each firm's increase in ~~...~~ it's capital stock adds to aggregate capital stock.

So the SP solves $\max K^{\beta} k^{\alpha} (AL)^{1-\alpha} - Rk - wL$

$R = r + \delta$

and from F.O.C's follows that $r_t^{SP} = (\alpha+\beta) k^{\alpha+\beta-1} (AL)^{1-\alpha} - \delta$

! decentralized interest rate is $r_t^{CE} = \alpha k^{\alpha-1} K^{\beta} (AL)^{1-\alpha} - \delta$ (from part a)

Solve HH's problem and use Euler equation ~~...~~:

$$g_c = \frac{\dot{c}}{c} = \frac{1}{\theta}[r_t - \delta - \rho] \quad \text{plug in } r_t^{SP}$$

$$\Rightarrow g_c = \frac{1}{\theta}\left[(\alpha+\beta)\frac{y}{k} - \delta - \rho\right] \quad \Rightarrow \frac{k}{y} = \frac{\alpha+\beta}{(\theta g_c + \delta + \rho)}$$

on BGP $g_c = g_k$

$$s^{SP} = \frac{(\alpha+\beta)(g_c + n + \delta)}{(\theta g_c + \delta + \rho)}$$

Comparing with s_* from b $\Rightarrow s^{SP} > s^{CE}$
and $\beta > 0$

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