SKETCH OF SOLUTION TO FINAL EXAMINATION

1)

a) Household solves maximization problem

$$\max_{c_t} \int_0^\infty \frac{(c_t+g_t)^{1-\theta}}{1-\theta} e^{-(\rho-n)} dt$$

Subject to: budget constraint

$$\dot{a}_t = r_t a_t + w_t - n a_t - (1+\tau_c) c_t$$

and No Ponzi Game constraint

$$\lim_{t\to\infty} a_t e^{-\int_0^t (r(s)-n)\, ds} \geq 0$$

$$a_0 > 0$$

and $\quad a_t = k_t + b_t$

b_t: loans to finance consumption.

b) Set up Hamiltonian

$$H = \frac{(c_t+g_t)^{1-\theta}}{1-\theta} e^{-(\rho-n)t} + \lambda_t\left[w_t + (r_t-n)a_t - (1+\tau_c)c_t\right]$$

where a_t is state variable and c_t is control variable.

F.O.C.

1) $\frac{\partial H}{\partial c_t} = 0 \quad \ldots \quad (c_t+g_t)^{-\theta} e^{-(\rho-n)t} = \lambda_t(1+\tau_c)$

2) $\frac{\partial H}{\partial a_t} = -\dot{\lambda}_t \quad \ldots \quad (r_t-n)\lambda_t = -\dot{\lambda}_t$

TVC:

$$\lim_{t\to\infty} \lambda_t a_t = 0$$

c) government's flow budget constraint

$$g_t = \tau_t c_t$$

There is no need for specification of NPG condition for government because government needs to maintain balanced budget $\forall t$ therefore it follows that $g_t = \tau_t c_t \ \forall t$ and government is not using borrowings to finance its consumption.

d)

because $g_t = \tau_t c_t \ \forall t \Rightarrow$ F.O.C. 1) from part b)

becomes $\left((1+\tau_t)c_t\right)^{-\theta} e^{-(\rho-n)t} = \lambda_t (1+\tau_t) \qquad / \frac{1}{1+\tau_t}$

and differentiate w.r.t. time

$$\dot{\lambda}_t = (1+\tau_t)^{-\theta-1} \cdot \left[-\theta c_t^{-\theta-1} \cdot \dot{c}_t e^{-(\rho-n)t} - (\rho-n) c_t^{-\theta} \cdot e^{-(\rho-n)t} \right]$$

from F.O.C 2) $\Rightarrow \quad \dfrac{-\dot{\lambda}_t}{\lambda_t} = r_t - n$ plug in for $\dot{\lambda}_t$ and λ_t [from F.O.C 1)]

$$-\frac{(1+\tau_t)^{-\theta-1} \cdot e^{-(\rho-n)t} \cdot c_t^{-\theta} \cdot \left[-\theta \cdot \dot{c}_t \cdot c_t^{-1} - (\rho - n) \right]}{(1+\tau_t)^{-\theta-1} \, c_t^{-\theta} \, e^{-(\rho-n)t}} = r_t - n$$

rearrange $\quad +\theta \dfrac{\dot{c}_t}{c_t} + \rho - n = r_t - n \Rightarrow r_t = \rho + \frac{1}{\theta}\frac{\dot{c}_t}{c_t}$

Euler equation $\quad \dfrac{\dot{c}_t}{c_t} = \dfrac{1}{\theta}\left[r_t - \rho\right]$

Explanation:
Households choose consumption to equate the rate of

of return r_t to rate of time preference + the rate of decrease of ~~[scribbled]~~ marginal utility of consumption $\left(\frac{1}{\theta}\frac{\dot{c}}{c}\right)$ due to change in per capita consumption.

in Optimizing environment Euler equation says that households equate rates of return (return on assets, return on shifting future consumption to present period) so households are indifferent between ~~consumptions in~~ consuming and saving.

e) Firm's profit maximization problem

$$\max_{k_t} \pi_i = A k_{ti}^{\alpha} - (r_t+\delta) k_{ti} - w_i$$

representative firm omit index "i"

use F.O.C to get $r_t + \delta = \alpha A k_t^{\alpha-1}$

$w_t = (1-\alpha) A k_t^{\alpha}$

competitive market equilibrium

$a_t = k_t \quad (b_t = 0) \quad \forall t$ — assuming homogeneity of HH

plug in to HH's budget constraint

to get $\dot{k}_t = A k_t^{\alpha} - (n+\delta) k_t - (1+\tau_c) c_t \quad (*)$

+ Euler equation $\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}\left[\alpha A k_t^{\alpha-1} - \delta - \rho\right] \quad (\square)$

competitive market equilibrium is characterized by $(*)$, $(\square)$ + the TVC : $\lim_{t\to\infty} k_t \cdot e^{-\int_0^t (r(s)-n)\,ds} = 0$

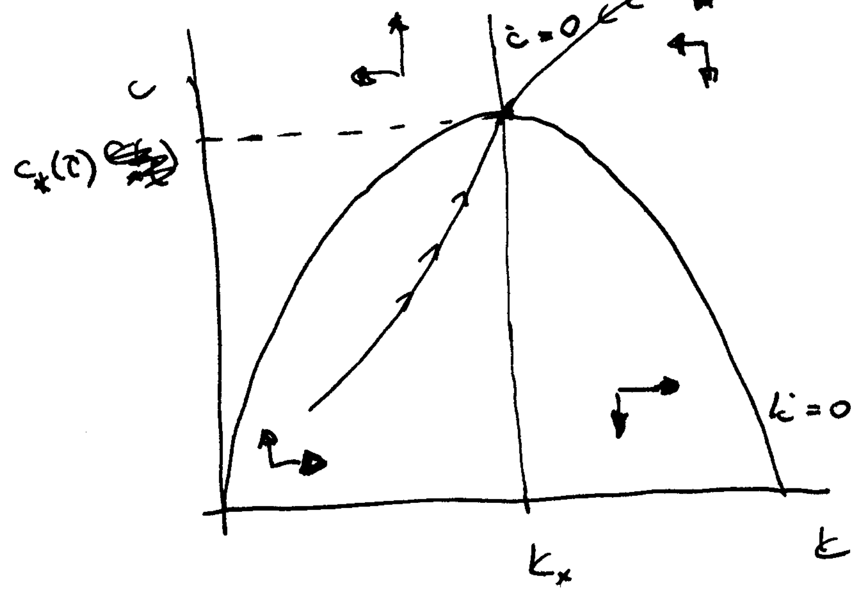
$k_0 > 0$

f) at steady state $\dot{c}_t = 0$

$\dot{k}_t = 0$

therefore $\alpha A k_*^{\alpha-1} - \delta - \rho = 0 \Rightarrow k_* = \left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$

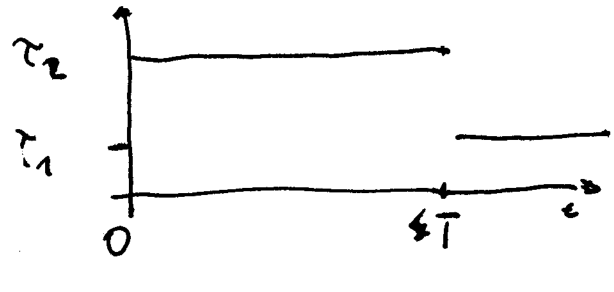
$A k_*^{\alpha} - (n+\delta)k_* - (1+\tau_c)c_* = 0 \Rightarrow c_* = \frac{A k_*^{\alpha} - (n+\delta)k_*}{1+\tau_c}$



g) because $c_*(\tau_c) = \frac{1}{1+\tau_c}\left(A k_*^{\alpha} - (n+\delta)k_*\right)$ the shifts in τ just affect the "height" of $\dot{k}_t = 0$ locus

unanticipated change.

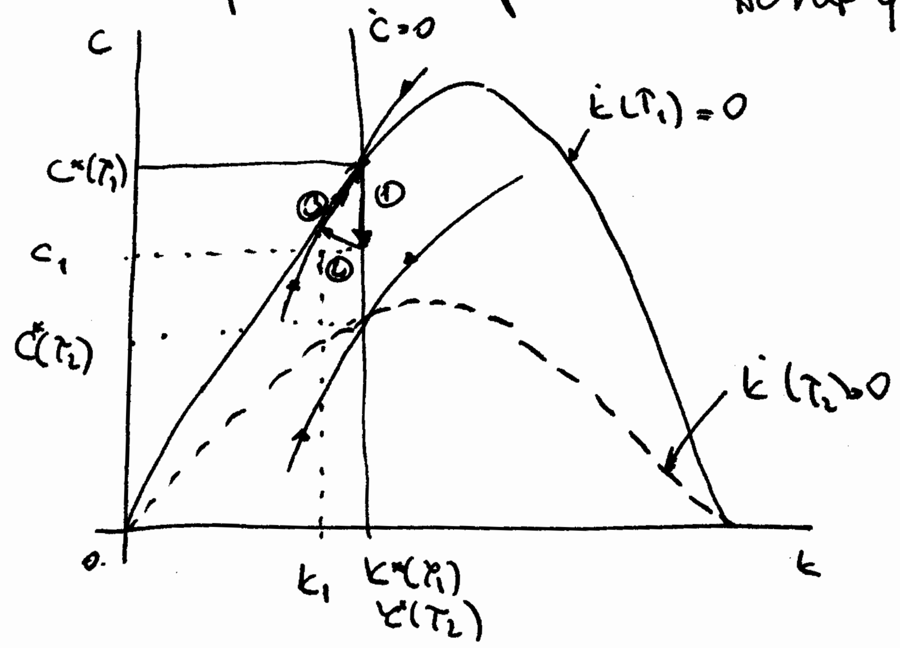
$\tau_1 = 0$

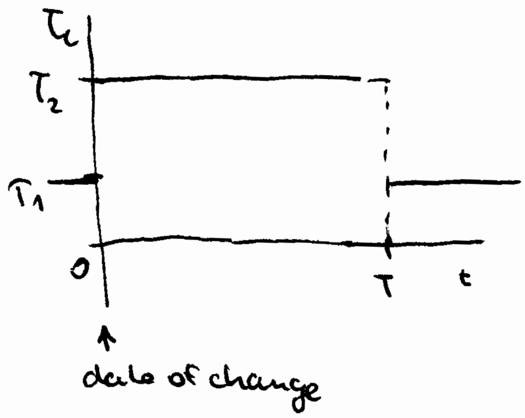


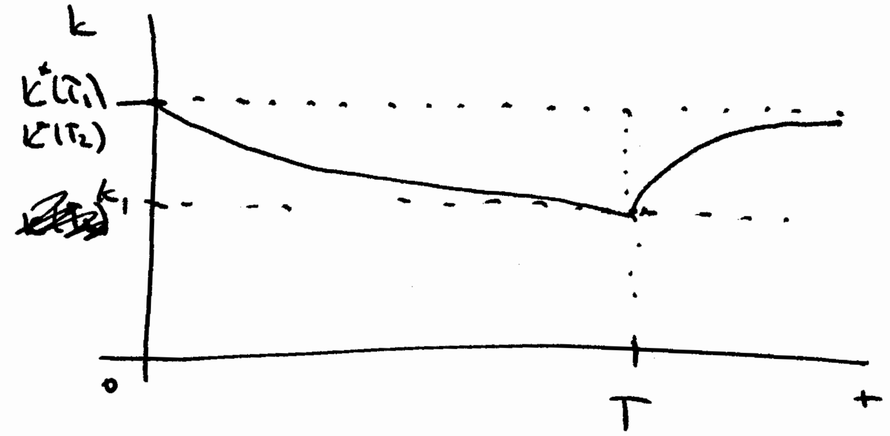
see next page.

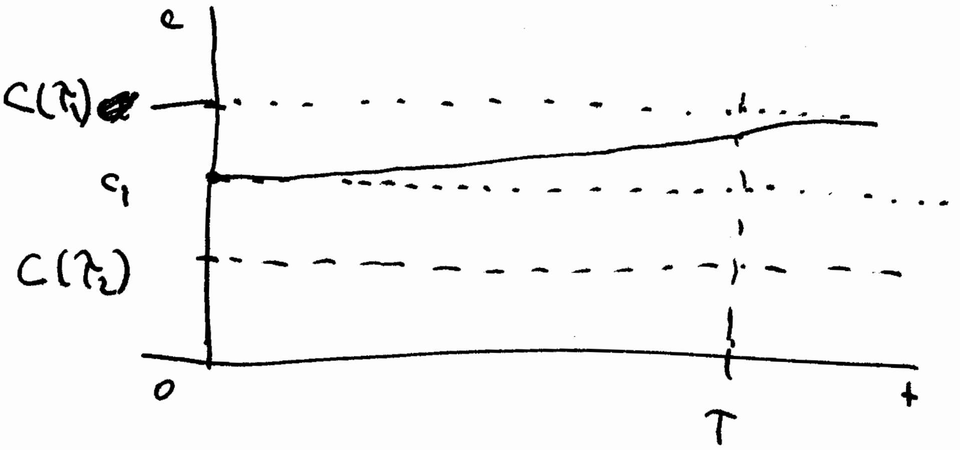
?

unanticipated change – temporary









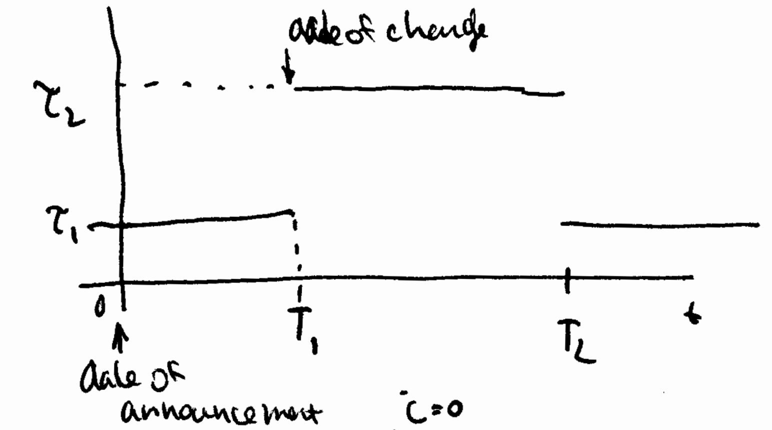
1) c jumps down in time 0 to c_1

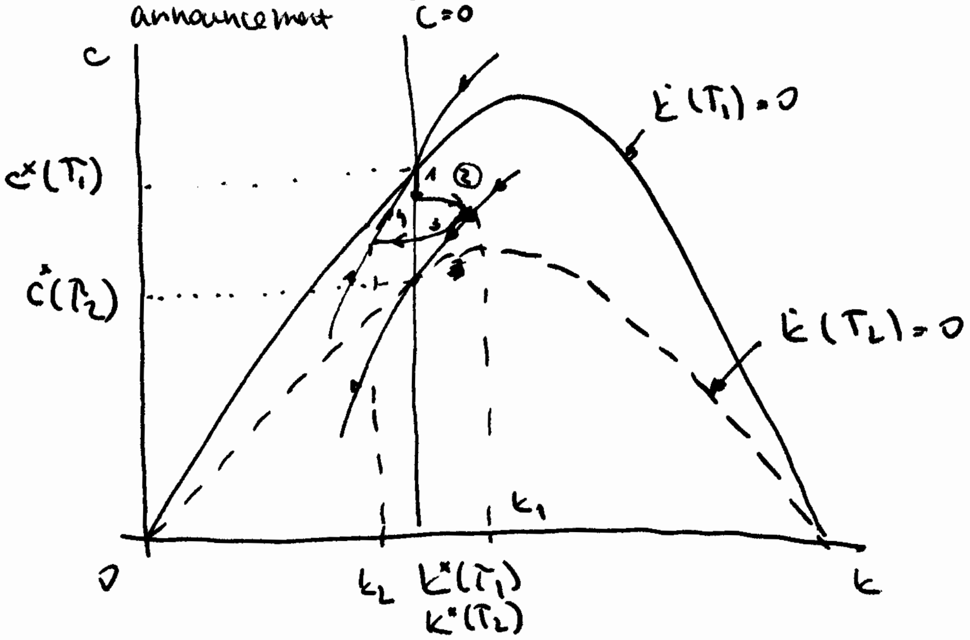
2) paths under taxation τ_2 are now driving behaviour of c_t, k_t during this period c_t, k_t

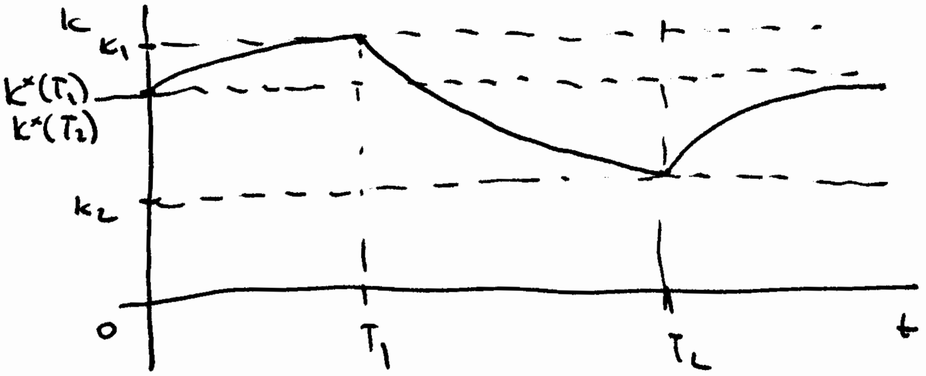
3) at time T, c_t and k_t are on path under τ_1 in this period c_t, k_t

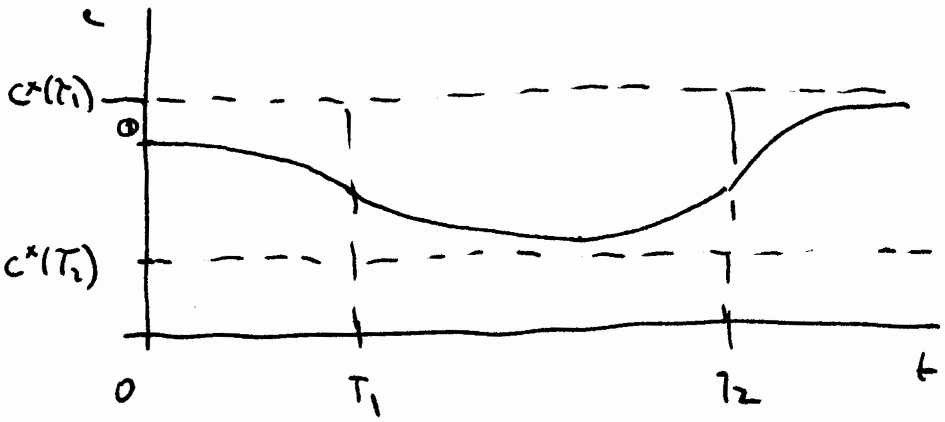
anticipated change - announced

$T_2 > T_1$









1) c jumps down
k does not change at the time of announcement

2) during $(0, T_1)$
paths from ~~...~~ under τ_1 are used for transition in such way that at T_1 c, k are approaching path ~~on path~~ toward $c^*(T_2)$, $k^*(T_2)$ under τ_2
c↓, k↑

3) during $\langle T_1, T_2 \rangle$
c, k use path under tax rate τ_2
in this period
c↓ and k↓

4) at T_2 c, k are back on path toward $c^*(T_1)$, $k^*(T_1)$ under τ_1
during this period
c↑, k↑

h) Planner solves ~~[illegible]~~

$$\max_{c_t, g_t} \int_0^\infty \frac{(c_t+g_t)^{1-\theta}}{1-\theta} e^{-(\rho-n)t} dt$$

$$\text{s.t.} \quad Y_t = C_t + I_t + G_t$$

rewrite Budget constraint

$$\dot{k}_t = Ak_t^\alpha - (n+\delta)k_t - c_t - g_t$$

k_0 given

$c_t \geq 0$

Hamiltonian

$$H = \frac{(c_t+g_t)^{1-\theta}}{1-\theta} e^{-(\rho-n)t} + \lambda_t [Ak_t^\alpha - (n+\delta)k_t - c_t - g_t]$$

F.O.C

1) $\frac{\partial H}{\partial c_t} = 0 \qquad (c_t+g_t)^{-\theta} e^{-(\rho-n)t} = \lambda_t \quad (0)$

2) $\frac{\partial H}{\partial g_t} = 0 \qquad (c_t+g_t)^{-\theta} e^{-(\rho-n)t} = \lambda_t$

3) $\frac{\partial H}{\partial k_t} = -\dot{\lambda}_t \qquad (r_t - n)\lambda_t = -\dot{\lambda}_t$

TVC $\quad \lim_{t\to\infty} \lambda_t k_t = 0$

→ indeterminacy between choice of c_t and g_t (3 unknowns, but only 2 eqn)

~~[illegible]~~

perfect substitutability + (c_t+g_t) satisfy (0) ⇒ that any choice ~~[illegible]~~ of g_t such that $g_t \leq c_t^*$ delivers the same consumption as in case without taxes.

when considering path with $g_t = 0 \ \forall t$ ~~we~~ this problem collapses to standard problem without distortions.

Therefore this result support our view that any equilibrium with time path g_t ($g_t \leq c_t^* \ \forall t$) is socially optimal.

From this follows that for any plausible level of taxation the competitive market equilibrium (decentralized) is socially optimal.

Change to lump sum taxation will not lead to change in social optimality of solution, because the consumption taxation already delivers social optimal solution.

perfect substitutability of private ~~and~~ consumption and government purchase plays crucial role, because it makes households indifferent between ~~private~~ utility from private consumption

and utility from government good.

i) change will occur in F.O.C.s

$$u(c,g) = \frac{(c^{\gamma} g^{1-\gamma})^{1-\theta}}{1-\theta}$$

therefore $u_c \, e^{-(\rho-n)t} = \lambda_t$ changes to

$$\gamma c_t^{\gamma(1-\theta)-1} \cdot g_t^{(1-\gamma)(1-\theta)} \cdot e^{-(\rho-n)t} = \lambda_t (1+\tau_c)$$

using $g_t = \tau_c c_t$ we get

$$\gamma \tau_c^{(1-\gamma)(1-\theta)} \cdot c_t^{\gamma(1-\theta)-1+(1-\gamma)(1-\theta)} \cdot e^{-(\rho-n)t} = \lambda_t (1+\tau_c)$$

$$\gamma \tau_c^{(1-\gamma)(1-\theta)} \cdot c_t^{-\theta} \cdot e^{-(\rho-n)t} = \lambda_t (1+\tau_c)$$

differentiate w.r.t. time

$$-\theta \frac{\gamma \tau_c^{(1-\gamma)(1-\theta)}}{1+\tau_c} c_t^{-\theta-1} \dot{c}_t \, e^{-(\rho-n)t} = \dot{\lambda}_t$$

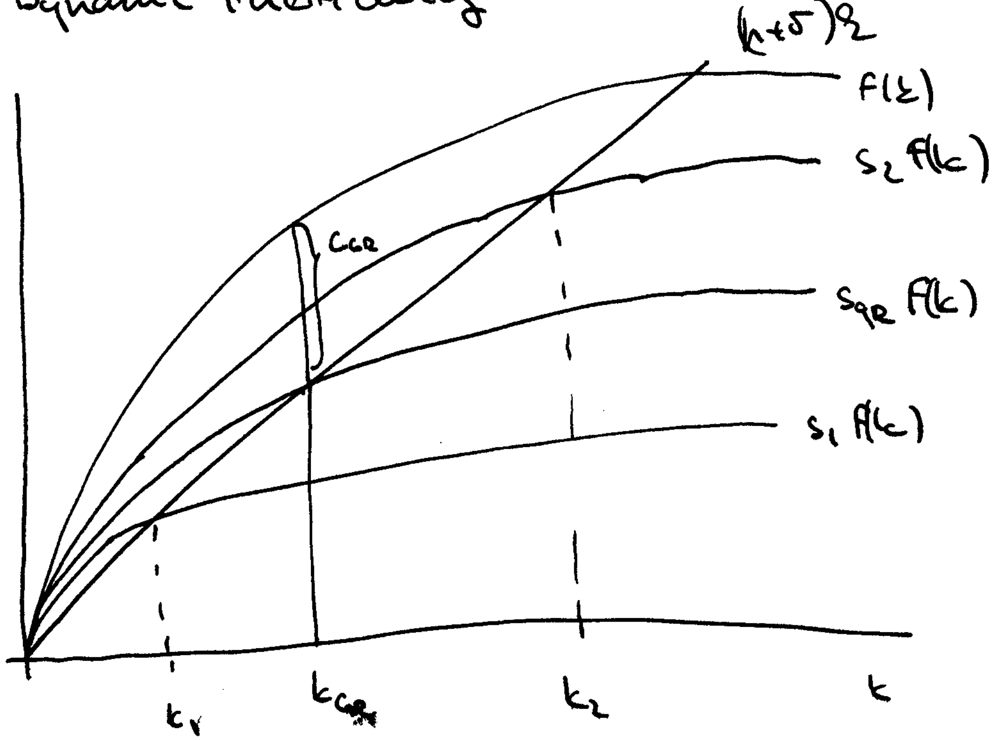
plug in $(r_t - n) = -\frac{\dot{\lambda}_t}{\lambda_t}$

$$r_t - n = \frac{\theta \gamma \frac{\tau_c^{(1-\gamma)(1-\theta)}}{1+\tau_c} c_t^{-\theta-1} \dot{c}_t \, e^{-(\rho-n)t}}{\gamma \frac{\tau_c^{(1-\gamma)(1-\theta)}}{1+\tau_c} c_t^{-\theta} e^{-(\rho-n)t}}$$

„new" Euler Equation

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r_t - n)$$

3) Dynamic inefficiency



Consider 3 savings rates low s_1

golden rule s_{GR} (consumption's maximized)

high s_2

When household are over-saving ~~and~~, at rate s_2 ($s_2 > s_{GR}$) then planner can decide to decrease saving rate to level s_{GR}. Along the transition path to lower saving rate (s_{GR}) the consumption is ~~~~ . higher at all points of time (per capita)

then on the path with saving rate s_2 therefore the economy with too high saving rate is dynamically inefficient.

When households ~~~~ save at rate $s_1 < s_{GR}$, the ~~~~ per capita consumption can be ~~~~ increased, but it requires lowering ~~consump~~

Consumption (temporary) and the overall outcome has to take into account the discounting of the future consumption

3)

Models of endogenous growth usually use capital accumulation ~~or~~ (also include human capital into production) or innovations.

In the view of models of ~~the~~ endogenous technological change capital accumulation and inovation are two aspects of the same process, because introduction of new forms of capital leads to accumulation of ~~these forms of~~ these new forms of capital.

Therefore the models of endogenous technological change introduce expansion of the number of varities of ~~[illegible]~~ goods.

~~[illegible]~~

These models ~~[illegible]~~ describe the production of final good, that produced from a large variety of intermediate goods, the expansion of number of varieties requires technological advance - invention and the assumption is that inventor maintains monopoly right on the production (and sale) of its goods for some period.

~~It can be shown that as economy get large and the ... input into research of new goods ... remain constant~~

For more details:

See chapter 6 3/2 (page 213) Barro and Sala-i-Martin in Economic growth