

Homework 3

Due July 4, 2005

1. *An "AK" Model with Physical and Human Capital.* Consider the following aggregate production function:

$$Y = K^\alpha (hL)^{1-\alpha},$$

where $h = H/L$ is human capital per person, Y is output, and K is physical capital. Accumulation of the capital inputs is given by

$$\begin{aligned}\dot{K} &= s_K Y - \delta K \\ \dot{H} &= s_H Y - \delta H\end{aligned}$$

where s_K and s_H are constant and exogenously given. The labor force, L , grows at the exogenous growth rate n .

- (a) What is the per capita growth rate of the economy along the balanced growth path?
 - (b) Show that along the balanced growth path the production function can be written as $Y = AK$ where A is constant. What is the value of A ?
 - (c) Discuss briefly the "story" one would tell to construct a decentralized, competitive equilibrium for this economy. Are there any externalities needed? Will the decentralized equilibrium be socially optimal if we endogenize the determination of s_K and s_H ?
 - (d) Discuss the effect of imposing the inequality restrictions $I_K \geq 0$ and $I_H \geq 0$.
2. *Capital Externalities in a Neoclassical Growth Model.* Consider the usual Ramsey-Cass-Koopmans model as discussed in class (i.e. with population growth at rate n and exogenous technological progress at rate g). Assume instantaneous utility is CRRA. Let final output be produced by a continuum of (identical) firms of mass

one (firms can be continuously indexed on the interval $i \in [0, 1]$), with each firm using the same production function

$$Y_i = \kappa^\beta K_i^\alpha (AL_i)^{1-\alpha},$$

where $\kappa = \int_0^1 K_i di$. Assume that there are positive externalities to capital accumulation. That is, firms take the aggregate capital stock, κ , as given at each point in time. Notice that because the measure of firms is one, $\kappa = K = K_i$. The rest of the model is identical to the Ramsey framework.

- (a) What is the per capita growth rate of the economy along the balanced growth path? If $g = 0$ is there per capita growth? Why?
- (b) Solve for the investment rate $s = I/Y$ along the balanced growth path for the decentralized economy.
- (c) What is the socially optimal investment rate along the balanced growth path, and why are these rates different?