① equalize subjective and objective expectations $\quad {}_tP^e_{t+1} = E_t P_{t+1}$

② general form of REE $\quad p_t = a + b_1 p_{t-1} + b_2 p_{t-2} + c m_t + d\eta_t$

$$E_t P_{t+1} = E_t(a + b_1 p_t + b_2 p_{t-1} + c m_{t+1} + d\eta_{t+1})$$

$$m_{t+1} = u + \delta m_t + \varepsilon_{t+1}$$

$$= a + b_1 p_t + b_2 p_{t-1} + c E_t[(u + \delta m_t + \varepsilon_{t+1}) + d\eta_t]$$

$$= a + b_1 p_t + b_2 p_{t-1} + c(u + \delta m_t)$$

③ substitute in money demand eq.

$$m_t - p_t = -\gamma[a + b_1 p_t + b_2 p_{t-1} + c(u + \delta m_t) - p_t] + \eta_t$$

Substitute for p_t the REE form.

$$m_t - a - b_1 p_{t-1} - b_2 p_{t-2} - c m_t - d\eta_t = -\gamma[a + (b_1 - 1)(a + b_1 p_{t-1} + b_2 p_{t-2} + c m_t + d\eta_t)$$
$$+ b_2 p_{t-1} + c(u + \delta m_t)] + \eta_t$$

Method of undetermined coefficients:

	LHS		RHS
m_t:	$1 - c$	$=$	$-\gamma \cdot (b_1 - 1) \cdot c - \gamma c\delta = -\gamma(b_1 - 1 - \delta)c$
p_{t-1}:	$-b_1$	$=$	$-\gamma[(b_1 - 1) b_1 + b_2] = -\gamma(b_1^2 - b_1 + b_2)$
p_{t-2}:	$-b_2$	$=$	$-\gamma[(b_1 - 1) b_2] = -\gamma b_2 (b_1 - 1)$
η_t:	$-d$	$=$	$-\gamma[(b_1 - 1)d] + 1 = -\gamma d(b_1 - 1)d + 1$
const:	$-a$	$=$	$-\gamma[a + (b_1 - 1)a + cu]$

From ρ_{t-2} eq follow $\Rightarrow$ $b_2 = 0$
or $b_1 = \frac{1+\rho}{\rho}$ and $b_2 \neq 0$ -

examine $b_1 = \frac{1+\rho}{\rho}$

~~solve for equ...~~

for use n_t:
$$-d = -\rho \frac{1}{\rho} d + 1$$
$$0 \neq 1$$
~~...solution~~ $\Rightarrow b_1 = \frac{1+\rho}{\rho}$ can not be solution

$\Rightarrow$ examine $b_2 = 0$

from ρ_{t-1}: ~~follows~~ $b_1 = \rho(b_1 - 1) b_1$ $\Rightarrow$ $b_1 = 0$
or $b_1 = \frac{1+\rho}{\rho}$

the case $b_1 = \frac{1+\rho}{\rho}$ is not solution
because Eq. n_t: $-d = -\rho d + 1$
$0 \neq 1$

$\Rightarrow$ solve for a, c, d using $b_1 = 0$
$b_2 = 0$

n_t: $d = -\rho d + 1$ $\Rightarrow$ $d = \frac{1}{1+\rho}$

m_t $1 - c = -\rho c + \rho \delta c$
$1 = (1 - \rho + \rho\delta) c$ $\Rightarrow$ $c = \frac{1}{1 - \rho + \rho\delta}$

const $a = \rho a - \rho a + \rho c \mu = \frac{\rho \mu}{1 - \rho + \rho\delta}$

Therefore ③

REE is $p_t = a + b_1 p_{t-1} + b_2 p_{t-2} + c m_t + d\eta_t$

where $a = \frac{\rho u}{1+\rho\delta-\rho}$, $b_1 = 0$, $b_2 = 0$, $c = \frac{1}{1+\rho\delta-\rho}$, $d = \frac{1}{1+\rho}$

PLM: $p_t = \frac{\rho u}{1-\rho+\rho\delta} + \frac{1}{1+\rho\delta-\rho} m_t$

Solving for ALM

$p_{t+1} = a + c m_{t+1}$

plugin $m_{t+1} = u + \delta m_t + \varepsilon_t$

$p_{t+1} = a + c(u + \delta m_t + \varepsilon_t)$

$E_t p_{t+1} = a + cu + c\delta m_t$. plug this into real money demand eq equation

$m_t - p_t = -\rho(a + cu + c\delta m_t - p_t) + \eta_t$

$m_t + \rho c\delta m_t + \rho(a+cu) + \eta_t = (1+\rho)p_t$

ALM: $p_t = \frac{\rho}{1+\rho}(a+cu) + \frac{1+\rho c\delta}{1+\rho} m_t - \frac{1}{1+\rho}\eta_t$

Therefore the mapping T is given by $T(\phi) = \begin{pmatrix} \frac{\rho}{1+\rho}(a+cu) \\ \frac{1+\rho c\delta}{1+\rho} \end{pmatrix}$, $\phi = \begin{pmatrix} a \\ c \end{pmatrix}$

To obtain ~~exp~~ expectational stability conditions differential equation $\frac{d\phi}{d\tau} = T(\phi) - \phi$ needs to be assessed (see p. 472 Handbook, ch. 7 Learning dynamics, eq. (22))

Calculate Jacobian and search for eigenvalues.

$$J^* = \begin{bmatrix} \frac{\rho}{1+\rho} - 1 & \frac{\rho}{1-\rho}\mu \\ 0 & \frac{\rho}{1+\rho}\delta - 1 \end{bmatrix} \Leftarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\rho}{1+\rho}(a+c\mu) - a \\ \frac{1+\rho c\delta}{1+\rho} - c \end{pmatrix}$$

This ~~system~~, system is locally stable if and only iff all eigenvalues have negative real parts.

because

$$\det(J) = \underset{<0}{\lambda_1} \cdot \underset{<0}{\lambda_2} \Rightarrow \frac{\rho}{1+\rho} - 1 < 0 \text{ and } \frac{\rho}{1+\rho}\delta - 1 < 0$$

and $\det(J^*) = \left(\frac{\rho}{1+\rho} - 1\right) \cdot \left(\frac{\rho}{1+\rho}\delta - 1\right)$

$$\frac{\rho}{1+\rho} < 1 \quad \text{holds for } \forall\ \rho > 0$$

$$\frac{\rho}{1+\rho}\delta < 1 \quad \text{holds } \forall\ \delta \text{ such that } \delta < \frac{1+\rho}{\rho} \quad *$$

$$\delta < \frac{1+\rho}{\rho}$$

if $\delta \in \langle 0,1 \rangle$ condition (*) is satisfied.

to evaluate strong E-stability we use the PLM that have more parameters than the REE of interest $\Rightarrow$ overparametrization.

therefore $\bar{\Phi} = (ac \ 0 \ 0 \ \ldots)$ and the

evaluate strong. E stability the system $\frac{\partial \bar{\Phi}}{\partial \tau} = T(\bar{\Phi}) - \bar{\Phi}$ need to be locally E-stable.

see Handbook, Ch?. learning dynamics p 473.