Essays on Mathematical Methods for Economics

Thesis defense

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[1] The views expressed here are my own and do not necessarily represent the views of the CNB.

Outline

1. Data Envelopment Analysis in Development Economics
2. Models for Stochastic Data Envelopment Analysis
3. Announced Change of Monetary Regime

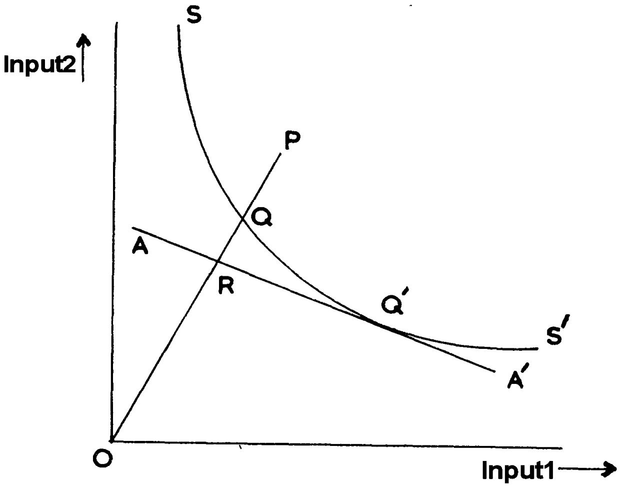
Methods of productivity analysis

Competing Methods for Efficiency Measurement:

1. SFA approach
   - Parametric method
   - Specification of production function
   - $y_i = f(x_i, \beta) + \varepsilon_i - v_i$; where ε_i is error term and v_i is positive inefficiency term
2. DEA approach
   - Non–parametric
   - Properties of production possibility set
   - $y_i = f(x_i) - u_i$; where u_i positive inefficiency term

Efficiency concept

OE=0R/0P, TE=0Q/0P, AE=0R/0Q



DEA Methodology

- n homogenous DMUs: m inputs and s outputs
- $T \subset \mathbf{R}_{+}^{m+s}$ is general a production possibility set, where $T = \{(x, y) \mid \text{using inputs } x \text{ outputs } y \text{ are produced}\}$
- Properties of production possibility set:
  - Convexity
  - Inefficiency property – free disposal
  - Minimum extrapolation
  - No free lunch
- Efficiency dominance: DMU is dominated when there exist a DMU that can produce the same levels of outputs with less intensive use of inputs

CERGE-EI

DEA Input Oriented Model

- Input oriented model:

$$
\begin{aligned}
\min_{\lambda_j,\theta_j,e_j,s_j} \quad & \theta_j \\
s.t. \quad & \theta_j x_{ij} - {}_{i}x\lambda_j - e_{ij} = 0, \quad & i = 1,\ldots,m; \\
& {}_{r}y\lambda_j - s_{rj} = y_{rj}, \quad & r = 1,\ldots,k; \\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi; \\
& \lambda_j, e_j, s_j \geq 0,
\end{aligned}
$$

- θ proportional reduction of inputs
- e_j, s_j non-proportional slacks
- λ intensity variable
- $\varphi = 1$ variable returns to scale
- $\varphi = 0$ constant returns to scale

Presentation outline

Motivation

- Motivation:
  - Unique data set
  - Success of "Green Revolution"
    - Growth of Indonesian rice production over 1950–1980 period
- Goals:
  - Test farm size–productivity relation
    - Townsend, Kirsten and Vink (1998), Helfand and Levine (2004): farm size–productivity relationship reconsideration
  - Evaluate impact of intensification program and other factors on farm's efficiency
    - Farm specific factors: labor, fertilizers, etc.
    - Economic factors: prices of inputs
    - Environmental factors: location, wet-dry period, etc.

Methodology

- Stage 1: DEA
    - Price distortions: Input oriented model
    - Time invariant production frontier
    - Time varying production frontier
- Stage 2: Tobit
    - Efficiency scores – censored variable
    - Efficiency model estimation
        - Random effect model
        - Mundlak's correction: Handling Correlation of individual characteristics and unobserved heterogeneity

Efficiency scores

- Stage 1:
  - High correlation of average DEA score ranking with SFA rankings: 0.7127 – 0.8214
  - Average technical efficiency scores range from 0.60 to 0.77
  - High average scale efficiency 0.90
  - Approximately 70% of farms are located in DRS region of production possibility set
  - Efficiency scores are consistent across models
  - No significant technological change over considered period – Malmquist index
  - Production growth was mainly driven by expansion of area used for production
  - High degree of heterogeneity in scores

Production Factors

- Stage 2:
  - HYV employment and sharecropping positively related with efficiency score
  - No significant efficiency benefit from intensification program participation
  - No significant effect of wet period: inefficient irrigation systems
  - Positive effect of family labor share: quality of labor
  - Size–efficiency relation:
    - "U" shaped relation – quadratic
    - Threshold – apx. 1.41 ha and apx. 1.9 (using Mundlak's correction)
    - Threshold coincides with farm size on other islands

Conclusions

- Adopt "best–practice" production mixes: 23%–42% proportional reduction of all inputs
- Positive returns of switch to HYV
- Adjust farm size: Pooling plots
- Reform of subsidies system to avoid overuse of inputs: Pesticides prices
- Personalization of intensification program

Presentation outline

1. Data Envelopment Analysis in Development Economics
2. Models for Stochastic Data Envelopment Analysis
3. Announced Change of Monetary Regime

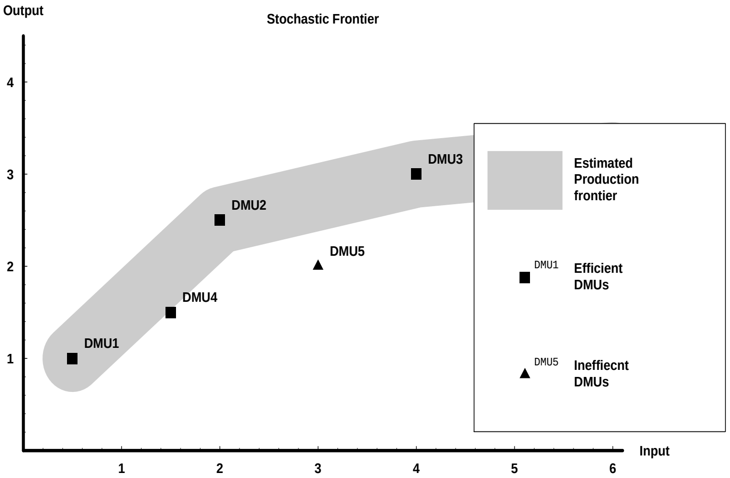
Introduction

- Goals of Productivity analysis:
  - Estimate production function
  - Measure distance between observation and production possibility frontier
  - Evaluate efficiency of observed production points
- Goals:
  - Develop the SDEA oriented models
  - Compare efficiency rankings

Motivation

- Problem:
  - DEA - extreme point method
    - Invalid efficiency evaluation
    - Robustness of results
- Solution:
  - SDEA - inputs and outputs are random variables

SDEA approach



Methodology

- Theoretical Work:
  - Oriented models
  - Models with variable returns to scale
  - Linearized models
- Applications:
  - Solver: fast; large size problems; solutions with low number of zero elements
  - Study: Indonesian rice farms efficiency
  - Comparing results with parametric methods

SDEA Methodology

- Shock structure:

$$
\begin{aligned}
\tilde{x}_{ij} &= \bar{x}_{ij} + a_{ij}\varepsilon \\
\tilde{y}_{ij} &= \bar{y}_{ij} + b_{ij}\varepsilon
\end{aligned}
$$

- Model:

$$
\begin{aligned}
\max_{\lambda} \quad & Prob(e^T(\tilde{X}\lambda - \tilde{x}_j) + e^T(\tilde{y}_j - \tilde{Y}\lambda) < 0) - \alpha \\
s.t. \quad & Prob({}_i\tilde{x}\lambda < \tilde{x}_{ij}) \geq 1 - \epsilon, \; i = 1, \ldots, m; \\
& Prob({}_r\tilde{y}\lambda > \tilde{y}_{rj}) \geq 1 - \epsilon, \; r = 1, \ldots, s; \\
& \lambda \geq 0,
\end{aligned}
$$

SDEA problem

- Input oriented model:

$$
\begin{aligned}
\min_{\lambda_j,\theta_j} \quad & \theta_j - \epsilon(Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \theta_j\tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda_j) < 0) - \alpha) \\
s.t. \quad & Prob({}_i\tilde{x}\lambda_j < \theta_j\tilde{x}_{ij}) \geq 1-\epsilon, i = 1, \dots, m; \\
& Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) \geq 1-\epsilon, r = 1, \dots, s; \\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi; \\
& \lambda_j \geq 0.
\end{aligned}
$$

- θ proportional reduction of inputs
- e_j, s_j non-proportional slacks
- λ intensity variable
- $\varphi = 1$ variable returns to scale
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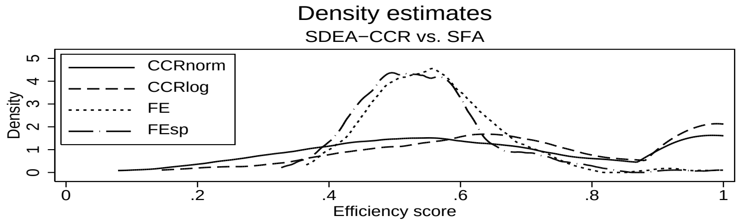
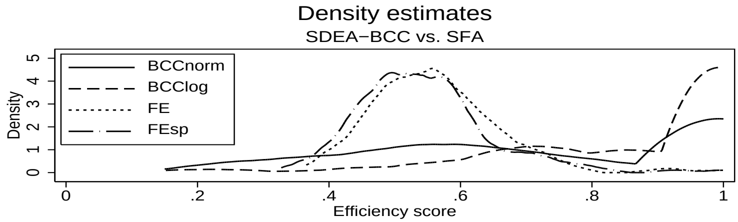
Efficiency scores distribution





Ranking consistence

Spearman correlation coefficient

	SDEA CCR_N	BCC_N	CCR_{LN}	BCC_{LN}
SFA				
FE	0.2534**	0.2448**	-0.0224	-0.0292
FE_{sp}	0.2115**	0.2399	-0.0835**	-0.0762*

Note: ** and * means significance at 1%, 5% respectively

- Low values of ranking correlation coefficients
- Data with high degree of variation

Presentation outline

Motivation

- Czech Republic is considering monetary union entry
- Macroeconomic stability in small open economy environment: Collard & Dellas (2002)
    - variance of series
    - evolution of variance
- Currency peg regime can support macroeconomic stability:
    - Cuche-Curti et al. (2008): rigidity in the goods market
    - Dellas and Tavlas (2003): presence of nominal rigidities
- Small open economy: Behavior of after the announcement of switch toward the exchange rate stability rule (unilateral peg)

Models of regime switch

Questions:

- How will the response to shocks of interest rates change over the transition period?
- What monetary regime is optimal for transition?
- Are business cycles getting synchronized over the transition period?

Goal:

- Modeling a monetary regime switch in DSGE model
- Introduce new theoretical framework for regime switch modeling
- Farmer, Waggoner and Zha (2007): Recent works rely on Markov switching processes

Model I

Justiniano and Preston (2004) framework:

- Two countries:
  - Home – small economy
    - Optimizing agents: households and firms
  - Foreign – large economy (monetary union)
    - Exogenous processes
- Domestic agents:
  - Households: habit formation
  - Firms: domestic producers, importers, and final good producer

Model II

- Model features:
  - No capital
  - All goods are tradable
  - Complete markets: Symmetric equilibrium
  - Zero inflation steady state
  - Nominal rigidities: Monopolistic competition
    - Monopolistic competition: Intermediate good
    - Inflation indexation of good prices
    - Importers: Law of one price gap
    - Final good aggregation: Dixit-Stiglitz form

Model III

- Domestic monetary policy rules:
  - Pre-transition:
    Targeting of inflation, output gap or change in nominal exchange rate
  - Transition:
    Follow pre-transition rule with knowledge of regime switch
  - Post-transition:
    Rule of offsetting foreseen changes in the nominal exchange rate

Monetary policy rules

Generalization of monetary regimes:

- Pre-transition regime (independent monetary policy):
  $i_t^I = \rho_i i_{t-1} + (1-\rho_i)(\rho_\pi \pi_t^{CPI} + \rho_y y_t + \rho_e \Delta e_t)$
- Post-transition regime (stability of exchange rate):
  $i_t^U = \widehat{\rho_e} \sum_{j=t}^{\infty} \left(\frac{1}{2}\right)^{t-j} \Delta E_t[e_j]$
- Transition regime:
  $i_t^T = regime_t \; i_t^I + (1 - regime_t) \; i_t^U$, where $regime_t \in \{0, 1\}$
  - where $0 \leq \rho_i < 1,\ \rho_\pi > 1,\ \rho_y > 0$ and $\rho_e \geq 0$
  - and $\widehat{\rho_e} = 2.0$

Information buffer I

- Future information is added to the state space
- Agents foresee the future changes of monetary regime
- Regime indicator:

$$
\begin{aligned}
regime_t &= inf_{t,1} \\
inf_{t,1} &= inf_{t-1,2} + \nu_{t,1} \\
inf_{t,2} &= inf_{t-1,3} + \nu_{t,2} \\
&\vdots \\
inf_{t,N-1} &= inf_{t-1,N} + \nu_{t,N-1} \\
inf_{t,N} &= \nu_{t,N},
\end{aligned} \tag{1}
$$

Information buffer II

- $inf_{t,i}, \ i \in 1, \ldots, N$ are new endogenous variables, $\nu_{t,i}, \ i \in 1, \ldots, N$ are information shocks in the period t.
- Announcement is modeled as a series of information shocks realization
- $$\nu_{k,i} = \begin{cases} 1, & i \leq T; \\ 0, & i > T, \end{cases}$$
- $\nu_{l,i} = 0, \ \forall i$ and in the all subsequent periods $l, \ l > k$
- $\nu_{l,i}$ is zero mean and zero variance random variable

Solution

Three models:

- Model of independent policy: linear
- Transition period model: quadratic
- Final period model: linear

1. Solve model:
    - Easy for independent a final period model
    - Transition period: Second order approximation of the monetary policy rule
    - Dynare++: fast solver for large problems
2. Estimate model
    - Dynare: Bayesian estimation
3. Define scenarios:
    - Evaluate information shocks
    - Simulate the linear model

Estimation results

- High value of the openness parameter: 0.35
- Inverse elasticity of labor supply: 1.08
- Monetary policy rule: high interest rate smoothing, inflation stability is almost 3 times more preferred than output stability
- Slightly more rigidity in domestic good sector than in imported good
- Inflation indexation: 0.56

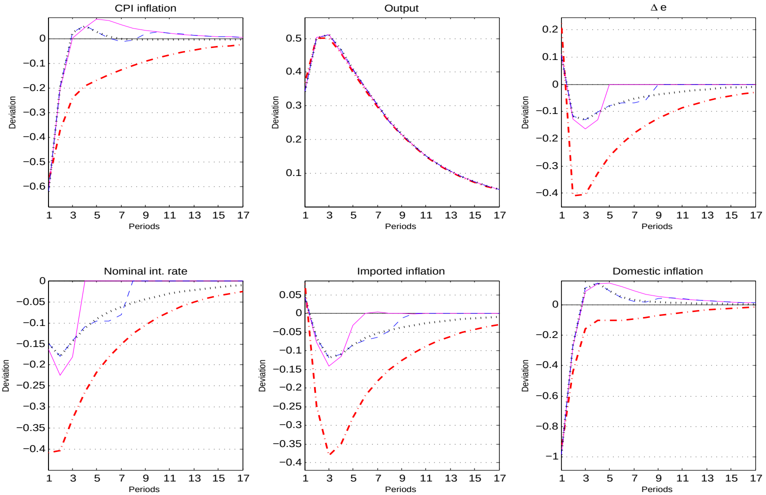
Irfs

How will the response to shocks of interest rates change over the transition period?

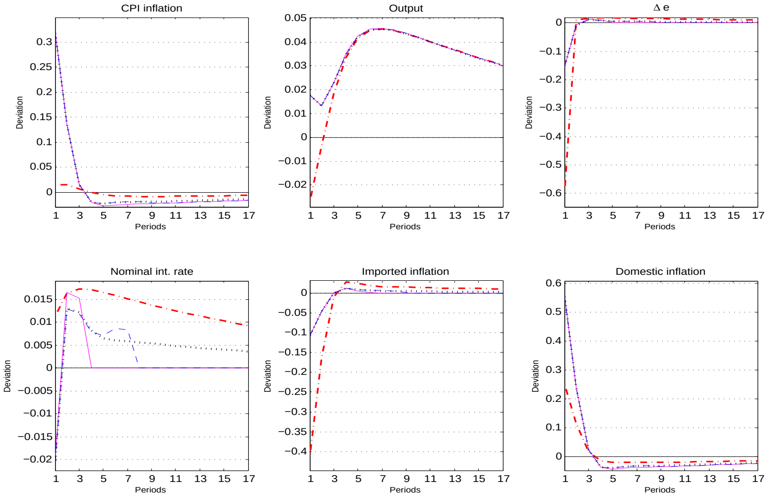
Compare responses:

- Examine the effect of the transition period length
- Examine the effects of choice of the transition period regime
    - Choice of weights in the monetary policy rule to reflect standard regimes

Irf (Transition length): Technology shock



Irf (Transition length): Preference shock



Transition period: Welfare evaluation I

What monetary regime is optimal for the transition?
Assumptions:

- Pre-transition period: estimated regime
- Transition period: Optimal regime

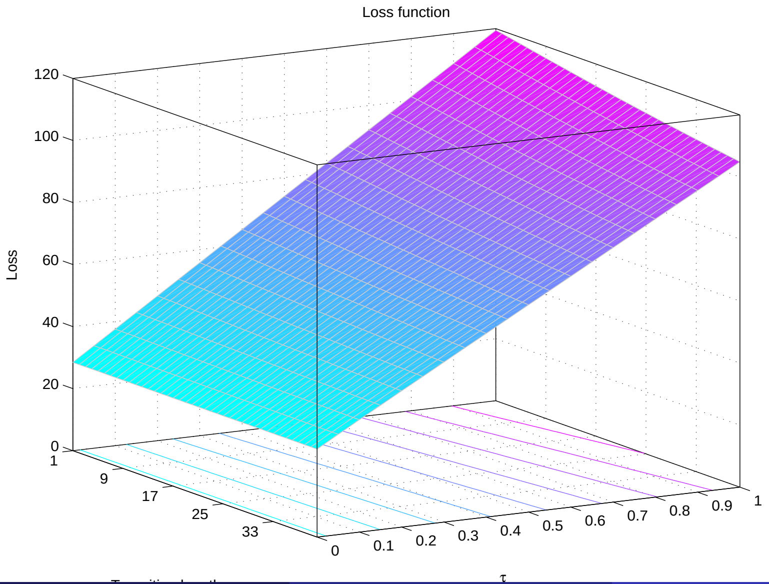
Welfare evaluation:

- Santacreu (2005):

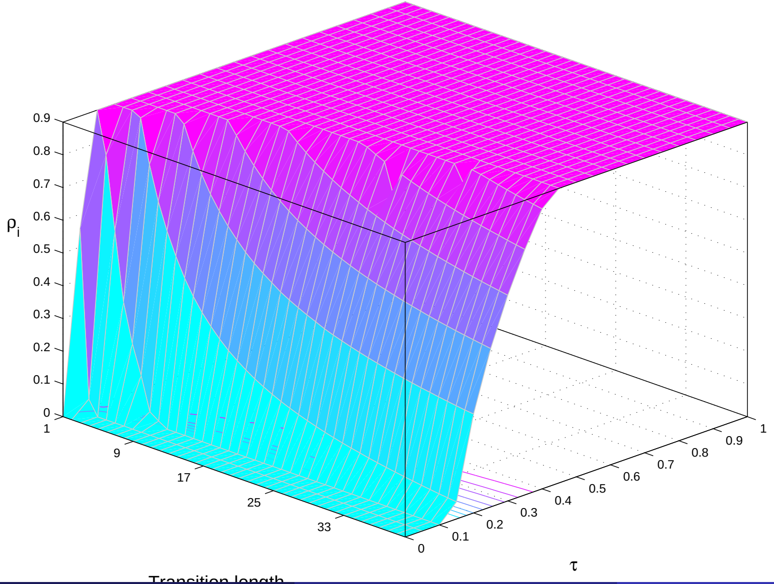
$$L_t = \tau Var(\pi_t) + (1-\tau) Var(y_t) + \frac{\tau}{4}(\Delta i_t),$$

where $\tau \in < 0, 1 >$

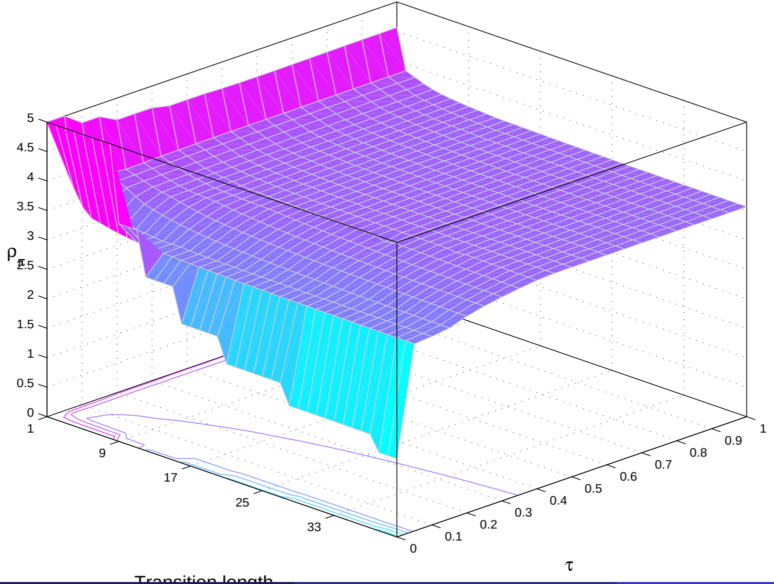
Loss function evaluation



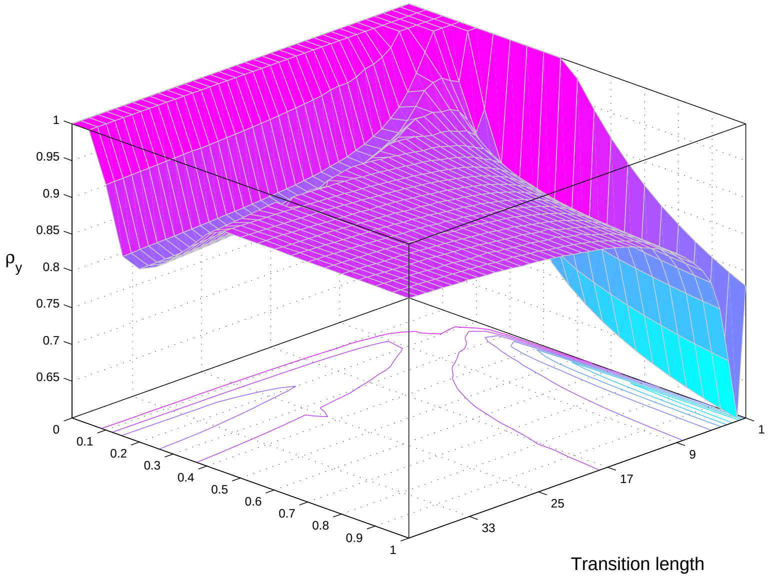
Optimal function for the transition: ρ_i



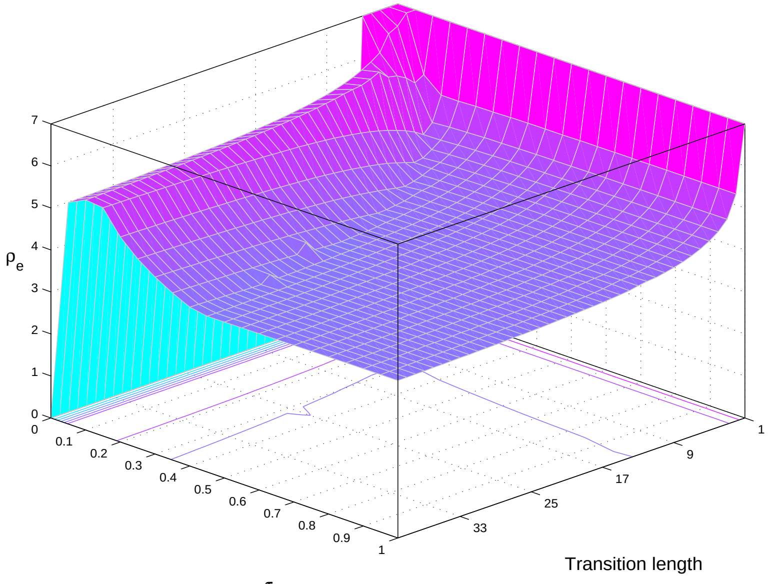
Optimal function for the transition: ρ_π



Optimal function for the transition: ρ_y



Optimal function for the transition: ρ_e

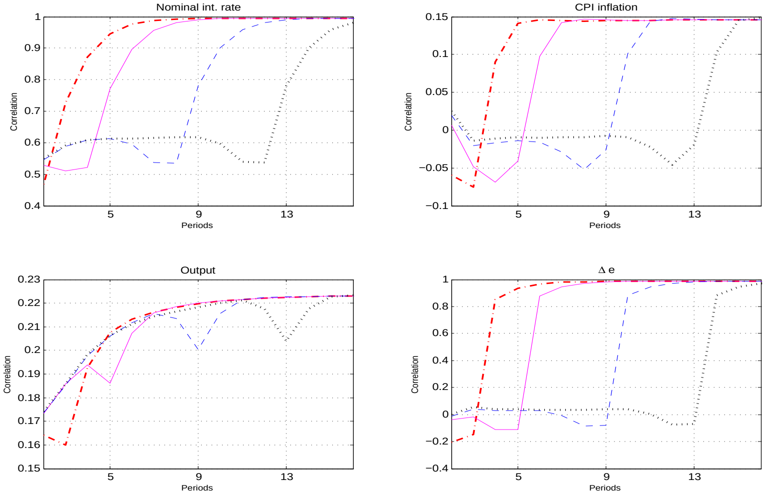


Business cycles correlations

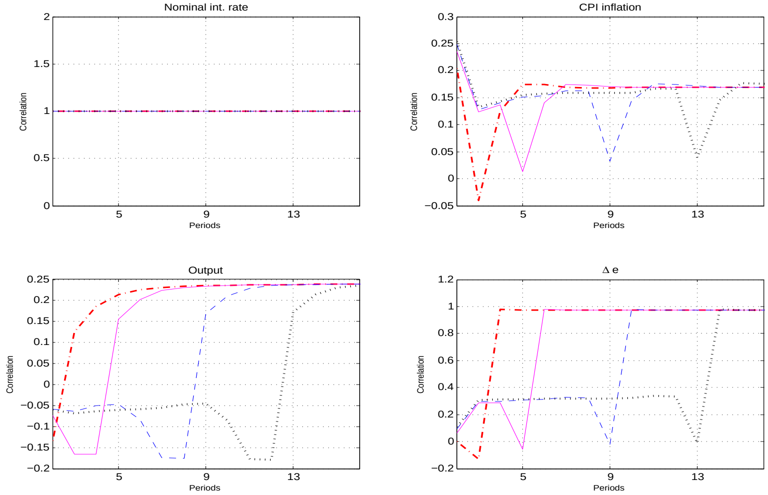
Are business cycles getting synchronized over the transition period?

- Exchange rate stabilization vs. lost of monetary policy influence on inflation
- Interest rate gets more correlated with the changes in the exchange rate over the transition period

Correlation: Foreign interest rate



Correlation: Domestic interest rate



Policy implications

- The inflation-interest rate correlation drops mainly in the initial and late phase of the transition.
- Consistently with the experiment design the interest rate - exchange rate correlation increases
- Influence of monetary policy on inflation and output is altered from anti-cyclical to pro-cyclical
  - Initial loss: Increase in interest rate signals to depreciation under the post-transition regime

Response I

1. Chapter 1:
   - Link between convexity type and returns to scale is drawn
   - The relations between results of optimization problem take form of theorem
   - The use of IPM method is reasoned by readiness of IPM solver code
   - Relative measure statement was corrected

Response II

2. Chapter 2:
   - Input orientation is used while strong distortions to inputs prices are present
   - Farms were delivering close to self-sufficiency levels while some inputs were wasted
   - For the robustness check different types of efficiency measure are used
   - $\chi_j = \left(\theta_j^* - \frac{\mathbf{1}^T e_j^*}{\mathbf{1}^T x_j}\right) \frac{\mathbf{1}^T y_j}{\mathbf{1}^T Y \lambda_j^*}$ (page 53)
   - Harvest cost is fraction of crop received by workers: Output composition
   - Labor is measured as man hours

Response III

3. Chapter 3:
   - Mark-up shocks may take form of demand shocks
   - The extension by wage mark-up shock may help to explain the variance in employment
   - Fixing may be beneficial for countries in stress or small countries
   - In the future work on this topic higher order approximation of underlying model is going to be considered

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S. M. Helfand and E. S. Levine
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  National Bureau of Economic Research, Working Paper Series, 12965,

- Harris Dellas and G. S. Tavlas
  Wage rigidity and monetary union
  CEPR Discussion Papers, Jan, 2003

- Nicolas A. Cuche-Curti and Harris Dellas and Jean-Marc Natal
  Inflation Targeting in a Small Open Economy
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A theory of optimum currency areas
The American Economic Review 51 (4), 1961

Chapter 2: Second stage

- Tobit:
  - $\chi^*_{ij} = \beta^T x + \nu_i + \epsilon_{ij}$, where χ_{it} is censored variable
  - random effects, ν_i, are iid $N(0, \sigma^2_\nu)$ and ϵ_{it} are iid $N(0, \sigma^2_\epsilon)$ independently of ν_i
- Unobserved heterogeneity modeling:
  - Mundlak (1978): unobserved heterogeneity can be modeled as a function of means of included regressors
  - $\nu_i = \bar{\beta}\bar{x}_i + \alpha_i$
    - α_i is a part of farm's unobserved heterogeneity and uncorrelated with regressors
    - $\bar{x}_i$ is vector of farm i means for individual regressors x_i over the observed period

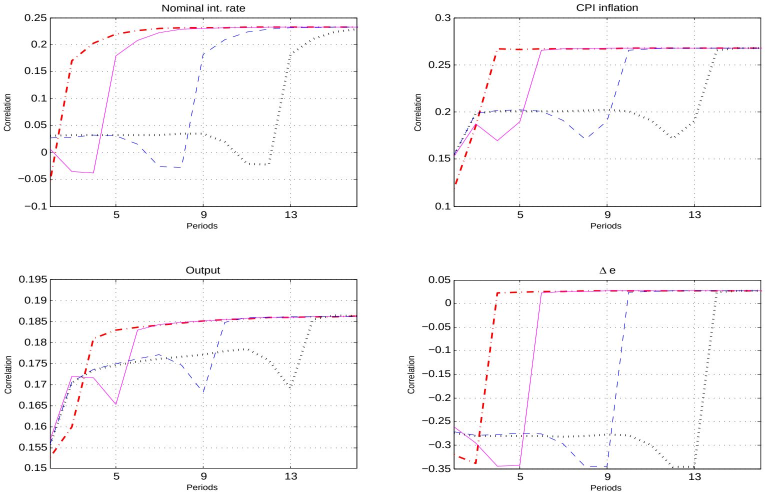
Chapter 1: Linearized Model

- Linearized input oriented model:

$$\begin{gathered}\min_{\lambda_j, q_{kr}, h_{ki}, \theta_j} \ \theta_j + \epsilon[\mathbf{1}^T(\bar{X}\lambda_j - \theta_j \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) + \\ +\delta(\mathbf{1}^T(A\lambda_j - \theta_j a_j) + \mathbf{1}^T(b_j - B\lambda_j))\sigma_\varepsilon \Phi^{-1}(\alpha)] + \\ +\epsilon(\textstyle\sum_{r=1}^{s}(q_{1r} + q_{2r}) + \sum_{i=1}^{m}(h_{1i} + h_{2i}))\end{gathered}$$

$$\begin{aligned} s.t. \qquad & {}_i\bar{x}\lambda_j \leq \theta_j \bar{x}_{ij} + (h_{1i} + h_{2i})\sigma_\varepsilon \Phi^{-1}(\epsilon), \\ & {}_i a\lambda_j - \theta_j a_{ij} = h_{1i} - h_{2i}, \quad i = 1, \ldots, m, \\ & \bar{y}_j \lambda_j \leq {}_r\bar{y} + (q_{1r} + q_{2r})\sigma_\varepsilon \Phi^{-1}(\epsilon), \\ & b_{rj} - {}_r b\lambda_j = q_{1r} - q_{2r}, \quad r = 1, \ldots, s, \\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi, \\ & \lambda_j \geq 0, q_{kr} \geq 0, h_{ki} \geq 0, \quad k = 1, 2 \end{aligned}$$

Chapter 3: Correlation with foreign inflation rate



Chapter 3: Correlation with foreign output