Guessing Games with Homogeneous and Heterogeneous Players: An Experimental Reconsideration *

Eugen Kováč Martin Vojtek
Andreas Ortmann† František Brázdik

Center for Economic Research and Graduate Education,
Charles University, Prague, Czech Republic

Abstract

We replicate an experiment previously reported in this journal (Güth, Kocher and Sutter 2002). Our results are at variance with their results, but confirm their key hypothesis that heterogeneous players guess closer to the equilibrium than homogeneous players.

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†Corresponding author: Politických vězňů 7, 111 21 Prague 1, Czech Republic; telephone: +420 224005117; fax: +420 22421137; e-mail: `andreas.ortmann@cerge-ei.cz`

1 Introduction

In the Guessing Game (Beauty-contest Game) participants are asked to choose a number from a closed interval. The winner is the person who picks the number closest to a given proportion of the average of all chosen numbers. The simplicity and flexibility of this game have made it a frequent topic of experimental studies of depth of reasoning, e.g., Nagel (1995), Duffy and Nagel (1997), Ho, Camerer and Weigelt (1998).

Recently, Güth et al. (2002) have considered a modification of the Beauty-contest Game that allows for continuous payoffs, homogeneous and heterogeneous players, interior and boundary equilibria.

Güth et al. (2002) hypothesize, first, that an interior equilibrium of a Guessing Game yields smaller deviations of the guesses from the game-theoretic equilibrium than from a boundary equilibrium. They hypothesize, second, that players in heterogeneous groups guess closer to equilibrium than those in homogeneous groups. In their experiment they found evidence only for the first hypothesis. Contrary to their second hypothesis, the convergence in homogeneous groups was faster than in heterogeneous ones. The motivation for carrying out our experiment was to check the robustness of this latter result. Güth et al. (2002) argue that the result is counter-intuitive since the heterogeneity of players should induce the players to consider each others' strategies more thoroughly.

2 The game and main hypothesis

Let n (where $n > 2$) be the number of players participating in the game. Each player $i \in \{1, \ldots, n\}$ chooses a real number $s_i \in S_i = [0, 100]$[1].

Such a choice is a pure strategy, where the interval $[0, 100]$ is the set of all possible strategies for each player. For any strategy vector $s = (s_1, \ldots, s_n)$ denote

$$\overline{s} = \frac{1}{n} \sum_{i=1}^{n} s_i$$

the average of numbers chosen by all players. Let the general payoff function $u_i(s)$ be given by

$$u_i(s) = C - c|s_i - q_i \overline{s}|,$$

where $q_i \in (0, 1)$ is the proportion of $\overline{s}$ which determines player i's best guess. If player i guesses exactly the number $q_i \overline{s}$, he receives the payoff C. If he does not, the amount c (where $c > 0$) is deducted from C for each unit by which the numbers s_i and $q_i \overline{s}$ differ[2].

Following the parametrization in Güth et al. (2002) we set $n = 4$, $C = 50$, and $c = 1$, in two treatments (call them HOM and HET). In the HOM treatment, all four players are given $q_i = \frac{1}{2}$ (for $i = 1, 2, 3, 4$). We will call the players in that treatment, somewhat simplifying, homogeneous. In the HET treatment, two of the participants are given $q_i = \frac{1}{3}$ (for $i = 1, 2$) and two of the participants are given $q_i = \frac{2}{3}$ (for $i = 3, 4$). We will call the players

[1] The instructions specified that the participants had to choose only numbers with up to two decimals.

[2] Here $|x|$ denotes the absolute value.

in that treatment heterogeneous. Güth et al. (2002) prove that the equilibria for both treatments are on the boundary with $s_i^* = 0$ and they are unique.

Following Güth et al. (2002), our main hypothesis is: Heterogeneous players think more deeply about the strategies of players of the other type and hence will make better guesses. Homogeneous players perceive the game as less complex and they will hence not consider as deeply what other players will do. Therefore, we expect that heterogeneous players guess closer to equilibrium than do homogeneous players.

3 Design and implementation

The experiments were run in June 2002 at two camps for young mathematicians, organized by the students of the Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava, Slovakia. The participants of these camps were chosen according to their performance in two independent national correspondence competitions in mathematics. Nearly all of them were students of secondary schools, aged 14–18; it was their first experience with experimental economics. None of the participants had taken a course on game theory before.

In both camps, we first ran an experiment with the 'Choose Game'[3] (Guessing Game redux), and paid all participants their payoffs. In one camp we then ran the HOM treatment with seven groups of four homogeneous players; in the other camp we ran the HET treatment with seven groups of

[3]Reported in Ortmann and Ostatnický (in prep).

four heterogeneous players.

At every camp, the participants were read the instructions (see Appendix). The instructions specified that in each of five rounds the subjects would be matched randomly so as to yield 7 four-player groups. In the treatment with heterogeneous agents, subjects were told that in each round there would be two subjects of each type in each group.

After each round we collected the record sheets, calculated averages for each group, and also publicly gave information about all averages. After the experiment, for each round two groups were randomly selected and earnings were paid out to every selected participant. This payment mode had been announced as a part of the instructions.

The maximum amount that participants could earn in each round was 50 SKK[4]. For every unit of difference from the target number they lost 1 SKK. However, they were informed in the instructions that the minimal outcome is 0 SKK. So the actual payoff function was $u_i(s) = \max\{0,\ 50 - |s_i - q_i\bar{s}|\}$.

4 Results

Table 1 shows average payoffs and Table 2 shows means, standard deviations, as well as the minimum and the maximum of guesses, separately for each round and each treatment at math camps. We observe that average payoffs

[4]The amount of 50 SKK was, according to the official exchange rate from the end of June 2002, about 1.13 EURO. In Slovakia, for 50 SKK, it is possible to buy 2–3 beers or 3 loaves of bread. Given the age of the participants, the payoffs were not insignificant.

increased continually with the number of rounds. The increase in average payoff was the smallest for homogeneous players. Also we can see that the standard deviation of guesses, as well as the maximum of guesses, dropped substantially from round to round.

— *Table 1 and Table 2 about here* —

Figure 1 shows the distribution of first round guesses[5]. Figure 2 shows the average guesses in the course of the experiment for both treatments. We split the asymmetric treatment data into two clusters, with $q_i = \frac{1}{3}$ and $q_i = \frac{2}{3}$. Clearly, all guesses converge to the equilibrium.

— *Figure 1 about here* —

In an informal post-experimental debriefing, participants told us that the difference in the payoff did not matter very much when the guesses were low. There were altogether nine participants whose guesses in each round were not higher than 5, three students who guessed 0 in each round, and 6 students who guessed 0 in the first round but increased their guess in some later round.

— *Figure 2 about here* —

As to the hypothesis, namely that the heterogeneity of players induces guessing closer to the equilibrium, heterogeneous agents guessed indeed closer

[5]Note, that the total number of participants for treatment with homogeneous players is twice as large as the total number of participants for each heterogeneous treatment.

to the equilibrium in each round[6] (Figure 2). The only exception was the second round, where the guesses of players with $q_i = \frac{2}{3}$ were a bit higher on average than those of homogeneous players. As expected, session averages for players with $q_i = \frac{1}{3}$ were lower than averages for players with $q_i = \frac{2}{3}$.

Our results confirm our first hypothesis and contrast with the findings of Güth et al. (2002) who found that session averages with homogenous groups were significantly lower than average guesses in heterogeneous groups.

Comparing our results to those of Güth et al. (2002), we see also faster convergence to the equilibrium for our data. In our treatments, the averages were well below 5 in the fifth round while in Güth et al. (2002), the averages in the fifth round were approximately 2.5, 7, and 12 for players with q_i equal to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{2}{3}$, respectively.

This difference in speed of convergence may be due to one or several of the following four factors. First, the public information may have sped up the process of learning. Players could see what happened in the other groups and thus they could eliminate dominated strategies. Second, our participants were the best mathematical talents in the Slovak Republic for their age category. We can assume that their reasoning process, and process

[6]We tested our hypothesis using the Mann-Whitney test (one tailed test with the null hypothesis that the distributions of guesses for both homogeneous and heterogeneous players are the same against the alternative that they differ and heterogeneous players had lower guesses) for each round. Starting from the second round, the distributions were significantly different and heterogeneous players' guesses were closer to the equilibrium (with $p < 0.1$ in round 2; $p < 0.05$ in round 3, and $p < 0.01$ in rounds 4 and 5).

of learning, is quicker than is typical for subjects. (Using the Choose Game data from the experiment that preceded the one we report here, Ortmann and Ostatnický (in prep) find that increasing mathematical ability[7] results in more equilibrium play.) Third, the results may in fact have been influenced by the Choose Game experiment. In this experiment, participants could see that the optimal strategy is to bid 0 and could learn that a similar strategy can be useful in the case of the Guessing Game. Finally, the experiment was not computerized and the calculation of averages for participants took about 3–5 minutes. The subjects could use this time for deeper thinking about the game. While the speed of convergence is an interesting issue, it was not our focus in this study. Hence we do not pursue the issue here further.

5 Conclusion

Repetition is the hallmark of experimental economics. We have replicated an experimental test of a recent version of the Guessing Game. Our subjects were mathematically talented youths and the experiments confirmed our main hypothesis that heterogeneous players guess closer to the (unique) equilibrium. It is unclear why our results do not replicate those by Güth et al. (2002). It would be interesting, indeed to model more formally the persuasive intuition in Güth et al. (2002) that seems to be confirmed by our data.

[7]Mathematical ability was measured by subjects' performance in the mathematical competition that was the basis for their selection for the camps (see section 3).

6 Appendix: instructions

Instructions were written in Slovak. Based on the instructions by Güth et al. (2002), we first created an English version which was later translated into Slovak. The instructions in the Appendix are for heterogeneous players with $q = \frac{1}{3}$. The instructions for heterogeneous players with $q = \frac{2}{3}$ differ only in the last sentence of paragraph 4 and in the formal expression for the payoff. In the instructions for homogeneous players (i.e. $q = \frac{1}{2}$) the whole of paragraph 4 was replaced by: "The target number for you (and everyone else in your group) is one-half of the average of all 4 chosen numbers in your group." Additionally, the formal expression for the earnings contained $\frac{1}{2}$ instead of $\frac{1}{3}$.

Sample instructions

Welcome to our experiment and thank you for participating. From now on please stop talking to your neighbor(s). If you have a question, please raise your hand.

You will be randomly divided into groups of 4 persons. Each person in your group chooses a number between zero (0) and one hundred (100). Zero and 100 are also possible. It is not necessary to choose an integer. However, numbers with more than two decimals are excluded.

Your potential earnings depend on how close your chosen number is to a target number. The closer your chosen number is to the target number, the higher are your earnings.

Your group consists of two participants of type A and two participants of type B. Target numbers of type A and type B participants are different. If you are a type A, your target number is one-third of the average of all 4 numbers chosen in the group. If you are type B, your target number is two-thirds of the average of all 4 numbers chosen in the group. **You are type A**, so the target number in your case is **one-third of the average of all 4 chosen numbers** in your group.

The potential earnings in each round depend on the difference between your chosen number and the target number. If your chosen number in that round is identical with the target number, your earnings will be 50 crowns. If the two numbers differ, their distance will be deducted from the 50 crowns. Formally, your potential earnings per round are calculated as follows:

$$\text{earnings (per round)} = 50 - \left| x - \frac{1}{3}\,\text{average} \right|.$$

If your earnings are negative, we will treat them as zero.

The experiment will last 5 rounds. Groups are rematched in each round. (You can see from the Table at the bottom of these Instructions the number of the group to which you belong in a particular round.) In each round you will write the chosen number on one of the attached Record Sheets and we will collect it.

After each round, we will write on the "blackboard" the average of each group. We advise you to enter in the Table at the end of these Instructions your chosen number; we also urge you to keep track of the average in your group. We recommend that you calculate your earnings after each round (using the above formula).

After the experiment proper, we will collect these Instructions (and the Table) and then will draw randomly one quarter of all participants to pay them off. All earnings will be paid in cash and privately at the end of the experiment.

— *Table 3 here* —

References

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Quotas		Round 1	Round 2	Round 3	Round 4	Round 5
$q_i = \frac{2}{3}$	average	34.82	44.14	45.38	47.98	48.11
$q_i = \frac{1}{3}$	average	38.41	42.04	45.57	48.21	49.10
$q_i = \frac{1}{2}$	average	37.63	42.54	44.10	47.17	47.54

Table 1: Averages of payoffs in the experiment (in SKK)

Quotas		Round 1	Round 2	Round 3	Round 4	Round 5
$q_i = \frac{2}{3}$	mean	20.05	13	5.51	2.7	1.89
	std. deviation	22.46	14.08	7.32	4.54	4.14
	max	66	47	26	15	15
	min	0	0	0	0	0
$q_i = \frac{1}{3}$	mean	17.39	7.35	4.76	1.85	0.72
	std. deviation	13.67	6.22	3.65	1.98	1.61
	max	54.50	21	12	7	6
	min	0	1	0	0	0
$q_i = \frac{1}{2}$	mean	20.28	12.55	9.24	4.36	2.99
	std. deviation	22.56	9.70	8.04	4.15	4.77
	max	100	38	35	19.73	24
	min	0	0	0	0	0

Table 2: Descriptive statistics for the experiment

Round	**1**	**2**	**3**	**4**	**5**
Group					
Chosen number					
Average					
Earnings					

Table 3: Table included in the Instructions

Figure 1: Distribution of first round guesses

Figure 2: Treatment averages