

The Quarterly Projection Model

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Outline

- 1 Trend and cycles
- 2 Structure of the Quarterly Projection Model
- 3 Parameters setup
- 4 Properties of the Model

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Time series analysis

- Analysis of time series data is based on smoothing past data in order to separate the underlying pattern in the data series from randomness.
- The underlying pattern then can be projected into the future and used as the forecast.
- The underlying pattern can also be broken down into sub patterns to identify the component factors that influence each of the values in a series: decomposition
- Decomposition methods: identify separate components of the basic underlying pattern that tend to characterize economics and business series.

In search for trends



Decomposition Techniques

- Goal: separation of data into several unobservable components, generally in an additive or multiplicative form.
- Components: trend, seasonal pattern, cycle, and residual or irregular pattern
- Seasonal component: the periodic fluctuations of constant length
- Trend-cycle component: long term changes in the level of series

Detrending

- Trend Component: The tendency of a variable to grow over time, either positively or negatively.
- Basic forces in trend: population change, price change, technological change, productivity change, product life cycles
- The long term movements or trend in a series can be described by a straight line or a smooth curve.
- The long-term trend is estimated from the seasonally adjusted data for the variable of interest
- Interpretation:
 - ▶ Trends: long run equilibrium
 - ▶ Gaps: cyclical fluctuations

Trend analysis

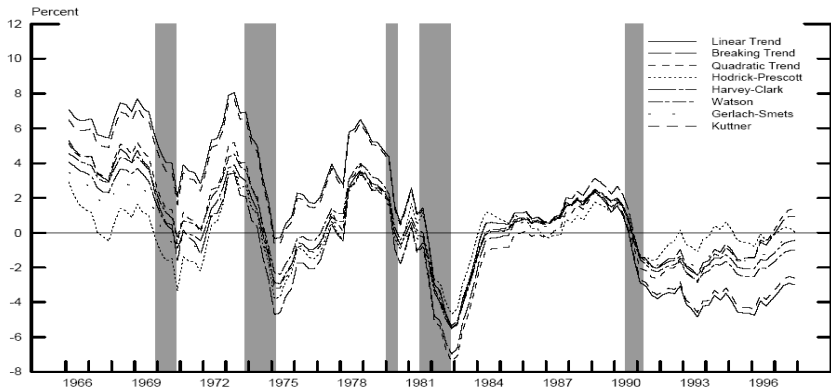
- Assume seasonally adjusted data
- Trend-Cycle decomposition: $\text{Series} = \text{Trend} + \text{Cycle} + \text{Noise}$
- No general-automatic techniques for detrending
- Simple techniques: Smoothing
 - ▶ Moving average: The average eliminate some higher frequency noise in the data, and leaves a smooth trend-cycle component. What order to use?
 - ▶ Simple centered moving average: can be defined for any odd order. A moving average of order k , is defined as the average consisting of an observation and the $m = (k-1)/2$ points on either side.
 - ▶ Centered moving average: take the simple centered moving average, assign weights and create weighted average
- Advanced techniques of detrending:
 - ▶ Fitting a polynomial
 - ▶ Using a structural model

Detrending techniques overview I

- Watson detrending: greater business cycle persistence; trend component follows a random walk with drift and cyclical component is a stationary finite order AR process.
- Harvey-Clark detrending: local linear trend model
- Hodrick-Prescott filter: univariate method
- Kalman filter: multivariate method, structural method
- Bandpass filter: not widely used, frequency domain analysis

Detrending techniques overview II

- Detrending comparison: US GDP gap



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Motivation for QPM

- Separate econometric methods: Inconsistencies
- Short experience with FPAS: Forecasting and Policy Analysis System
- State:
 - ▶ Insufficient data and experience to support advanced model
 - ▶ Need to increase participation of other departments and bank board
 - ▶ Communication of results: support for decision
 - ▶ Need for research tool
- The further step on the way to complex structural models: DSGE

Features of QPM

- Reflects inflation targeting regime:
 - ▶ In December 1997: after an exchange rate crisis
 - ▶ CNB adopted a series of end-year inflation targets
 - ▶ Regime proved very effective in combating inflation and anchoring
 - ▶ Evolution toward a more transparent inflation targeting regime where monetary policy is anchored by a medium-term perspective
 - ▶ Change to point inflation target: Inflation target band
 - ▶ The character of the regime was further enhanced by publication of unconditional forecasts
- Linked to quarterly data
- Small open-economy gap model

Model of trends and cycle

- Two separate blocks:
 - ▶ Long run equilibrium trends
 - ▶ Cyclical fluctuations - gaps
 - ▶ These blocks are separable
 - ▶ Super-neutrality: no long-run trade-off between output and inflation
- 85 equations at start
- Further extensions

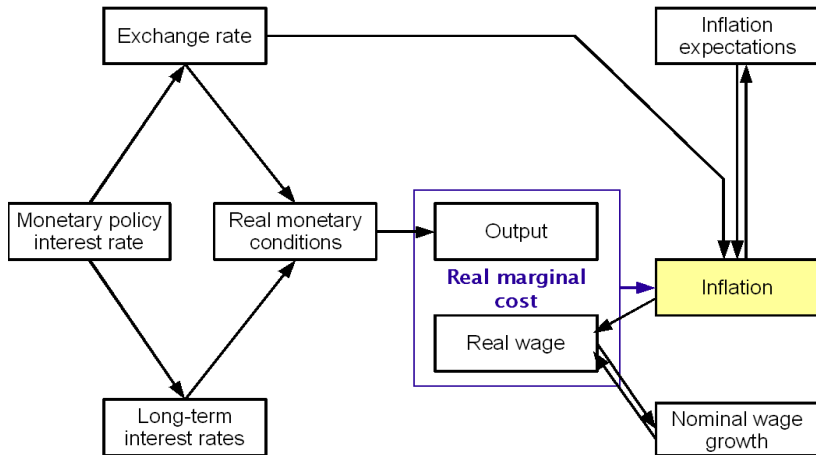
Long Run Trends

- First step: filter trend series
 - ▶ History - estimated by a simple statistical model (Kalman filter) and expert judgement
 - ▶ Forecast - exogenous (expert judgement), respecting steady state properties of QPM
- Important equilibrium values:
 - ▶ Real output growth
 - ▶ Real wage growth
 - ▶ Real exchange rate appreciation
 - ▶ Real interest rate
 - ▶ Stationarity is required: growth rates in focus
- Monetary decisions have small impact on long term real trends

Cyclical Part of QPM

- Description of the position of the Czech economy
- Monetary policy characteristics:
 - ▶ Inflation targeting regime
 - ▶ Forward looking policy
 - ▶ Focus on deviations from the target → reaction to expected inflation a year ahead
 - ▶ Floating exchange rate - endogenous
- Description of behavior economic agents includes forward looking components
- Price frictions:
 - ▶ Wage stickiness
 - ▶ Final price stickiness
 - ▶ Expectation stickiness

Scheme of model



Real Economy I

IS curve (Aggregate demand):

- Output: function of lagged output, the real interest rate, the real exchange rate and foreign demand and interest rate
- Includes impact of a change in interest rates with longer maturity on aggregate demand and take into account expectations about yield-curve on the dynamic properties of the model
- Real impact of monetary policy in a sticky-price model of a small open economy
- Marginal costs: cost of producing additional unit of a good

Real Economy II

Real Marginal Costs Gap:

- Approximation of inflationary pressures from the real economy.
- Marginal costs consist of the costs arising from the increasing volume of production (the "output gap") and wage costs (the "real wage gap").
- A positive real marginal cost gap implies an inflationary effect of the real economy

$$\widehat{mc}_t = \lambda \widehat{y}_t + \widehat{wr}_t$$

Real Economy III

Output Gap:

- Standard economic theory: higher real interest rate reduce aggregate demand by increasing the reward to saving
- Output gap: responds negatively to the difference between the real interest rate and its equilibrium value
- Open economy: the exchange rate matters
- Currency appreciation will, all else equal, make domestic goods more expensive in foreign markets and reduce demand for domestic goods abroad; cheaper imports may displace domestic goods

$$\begin{aligned}\widehat{y}_t &= \alpha_1 \widehat{y}_{t-1} - \widehat{\text{rmci}}_{t-1} + \alpha_2 \widehat{y}_t^f + \varepsilon_t^{\widehat{y}} \\ \widehat{\text{rmci}}_t &= \beta_1 \left(\beta_3 \widehat{\text{rc}}_t + \beta_4 \widehat{\text{r4}}_t + (1 - \beta_3 - \beta_4) \widehat{\text{r4}}_t^f \right) + \beta_2 \widehat{z}_t\end{aligned}$$

Real Economy IV

Real Wage Gap:

- Introduced in January 2007
- Wage costs are above their equilibrium level, they have an inflationary effect
- The effect of a deviation of the current level of the average real wage from its equilibrium level, which in the long run rises at the same rate as equilibrium real output (non-accelerating inflation real output)

$$\widehat{wR}_t = \widehat{wR}_{t-1} + \frac{w_t}{4} - \frac{\pi_t}{4} - \frac{\Delta \overline{wR}_t}{4} + \varepsilon_t^{\widehat{wR}}$$

Real Economy V

Unemployment:

- Okun law
- Unemployment gap depends on its lag output gap.

Phillips Curves I

Price Inflation:

- Standard Phillips curve has been modified for a small open economy
- Blocks for various goods
- Import price effects
- Wage setters derive their nominal wage demand real consumer wage

- x for fuel, food, or adjusted excl. fuel inflation
- Administered prices are exogenous in baseline

Phillips Curves II

$$\begin{aligned}\pi_t^x &= \gamma_1^x \left(\pi 4_t^{Mx} + \Delta_4 \bar{Z}_t^x \right) + \gamma_2^x \left(E \pi 4_t + \Delta_4 \bar{Z}_t^x - \Delta_4 \bar{Z}_t \right) \\ &+ (1 - \gamma_1^x - \gamma_2^x) \pi_{t-1}^x + \gamma_3^x \widehat{mc}_t + \varepsilon_t^{\pi^x}\end{aligned}$$

Wage Inflation:

- Erceg, C.J., Henderson, D.W., Levin A.T.: Optimal monetary policy with staggered wage and price contracts, 2000

$$w_t = \delta_1 E w 4_t + (1 - \delta_1) w_{t-1} - \delta_2 \left(\widehat{w}r_t - \delta_3 \widehat{y}_t \right) + \varepsilon_t^w$$

Expectations I

Price Inflation Expectations:

- Expected inflation: a weighted combination of a backward-looking and a forward-looking component (the expected value of overall CPI inflation over the next four quarters)
- Overall CPI: an explicit link between changes in administered and energy prices and pressures on the rate of inflation for market prices

$$E\pi_4_t = \lambda_1 \pi_{t+1} + (1 - \lambda_1) \pi_{t-1} + \varepsilon_t^{E4}$$

Wage Inflation Expectations:

$$Ew_4_t = \lambda_2 w_{t+1} + (1 - \lambda_2) w_{t-1} + \varepsilon_t^{Ew4}$$

Uncovered interest rate parity

Nominal Exchange Rate:

- UIP condition: arbitrage condition; international investors will equalize effective rates of return on investments in different currencies, allowing for any country-specific risk premiums
- foreign investor expecting a depreciation (appreciation) of the koruna will demand a higher (lower) return from Czech assets
- Moving average form

$$s_t = \phi s_{t+1} + (1 - \phi) \left(s_{t-1} + 2 \left(\frac{E_t \pi}{4} - \frac{E_t \pi^f}{4} \right) + 2 \Delta \bar{z}_t \right) + \frac{i_t}{4} - \frac{i_t^f}{4} - \text{prem}_t + \varepsilon_t^s$$

Reaction Function

Nominal Interest Rate:

- Forward-looking reaction function
- CPI inflation expected to be above the target rate: central bank push up the short-term
- Excess demand: the central bank increases short-term interest rate
- Long-term level for rates and some additional dynamic structure
- Interest rate inertia: interest rate smoothing

$$\begin{aligned}
 i_t &= \psi i_{t-1} + (1 - \psi) \left(i_t^{neutral} + \Pi_t \right) + \varepsilon_t^i \\
 i_t^{neutral} &= \bar{r}_t + \pi 4_{t+4} + \varepsilon_t^i \\
 \Pi_t &= \kappa_1 \left(\pi 4_{t+4} - \pi 4_{t+4}^{target} \right) + \kappa_2 \hat{y}_t
 \end{aligned}$$

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Calibration vs. Estimation

- QPM is calibrated, partially estimated
- Problems in estimation:
 - ▶ Short data sample
 - ▶ Structural changes in economy
 - ▶ Changes of monetary policy regime
 - ▶ It is impossible to estimate some parameters: identification problems

Calibration of QPM

Parameters setup:

- Restrictions on parameters originating from economic theory
- Parameters are set to match the properties of data
- Responses to structural shocks

Parameters checks:

- Reactions to shocks
- Residuals
- In-sample simulations
- Curve-fitting estimates

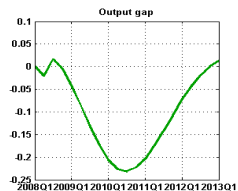
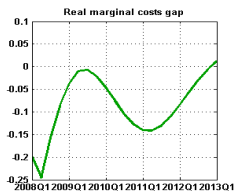
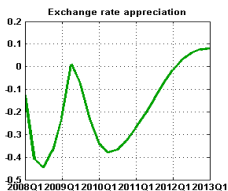
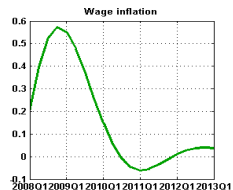
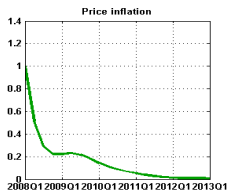
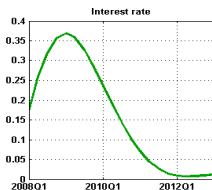
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Price shock I

- Positive shock to the output gap
- Upward pressure on inflation
- Currency depreciation
- Central bank increases interest rate
- Cumulative effect on output is very close to zero: feature of linear models;
- Offsetting of excess supply to counteract the effects of shocks that create excess demand

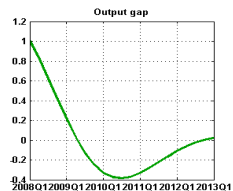
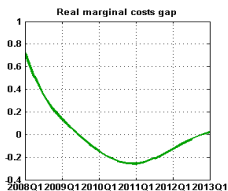
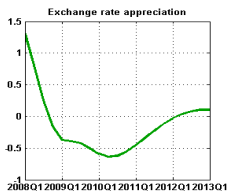
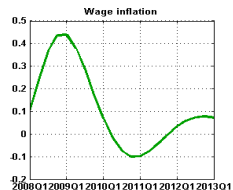
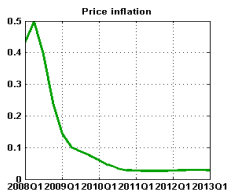
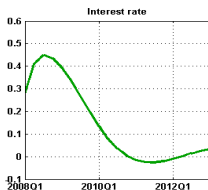
Price shock II



Aggregate demand shock I

- Positive shock to the output gap
- Upward pressure on inflation
- Currency depreciation
- Central bank increases interest rate
- Cumulative effect on output is very close to zero: feature of linear models;
- Offsetting of excess supply to counteract the effects of shocks that create excess demand

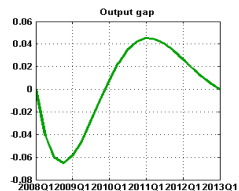
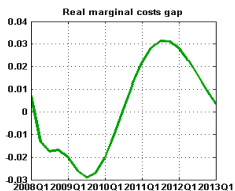
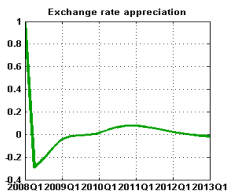
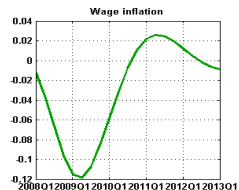
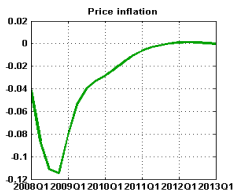
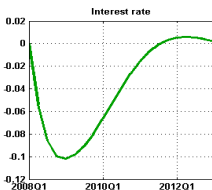
Aggregate demand shock II



Exchange rate shock I

- Depreciation acts to increase aggregate demand, opening a positive output gap

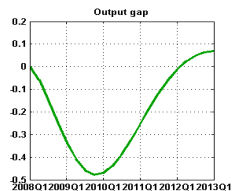
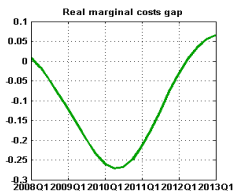
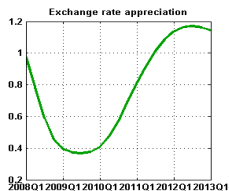
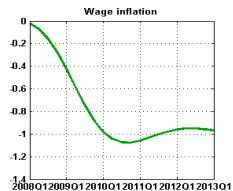
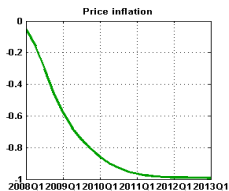
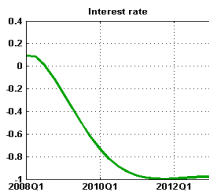
Exchange rate shock II



Inflation target change I

- Lower the target rate of inflation by one percentage point
- To achieve disinflation: raise the short rate
- Appreciation: Import prices fall
- The combined effect of the import price decline and the excess supply gap works to gradually pull down the rate of inflation
- Note: purely nominal shock, and since the model is super-neutral, there is no change to any real equilibrium in this shock, including the real exchange rate. The nominal exchange rate changes, of course, with the cumulative
- Cumulative effects on output and employment
- Sacrifice ratio: a cumulative loss of output vs. lower inflation by a percentage point

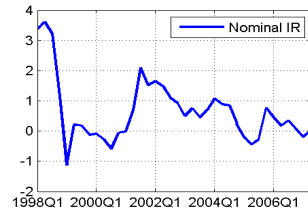
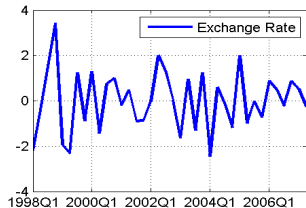
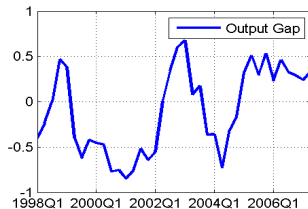
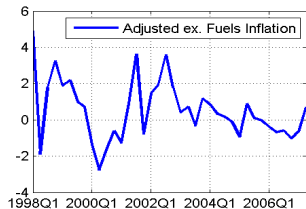
Inflation target change II



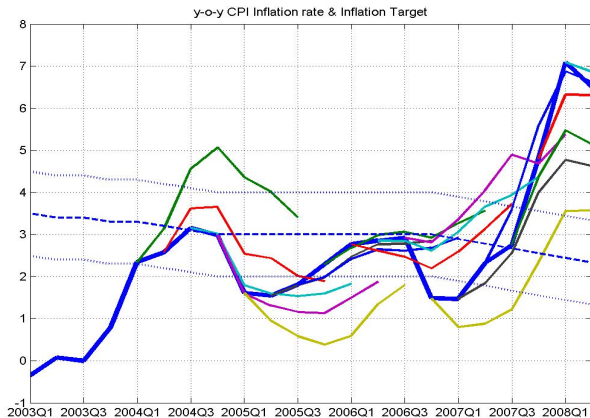
Residuals I

- Conflict between estimated parameters and calibrated
- The parameters have to be chosen so as to give reasonable model behavior
- Examined how well the model performs over the historical sample
- Identify systematic biases

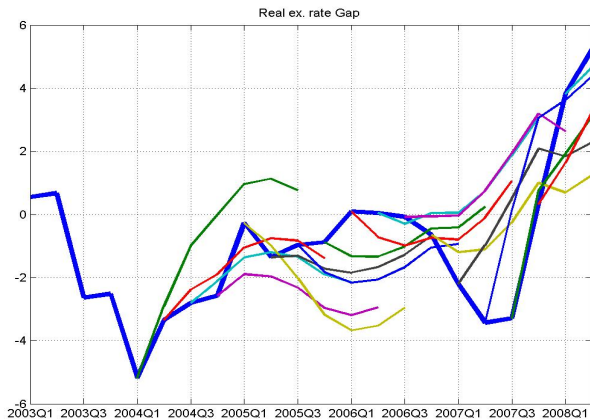
Residuals II



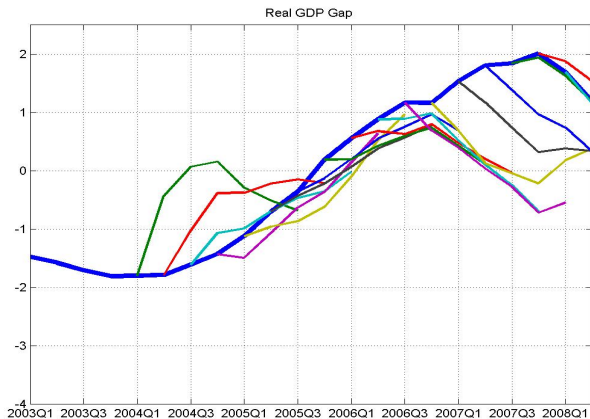
In-Sample Simulations: CPI



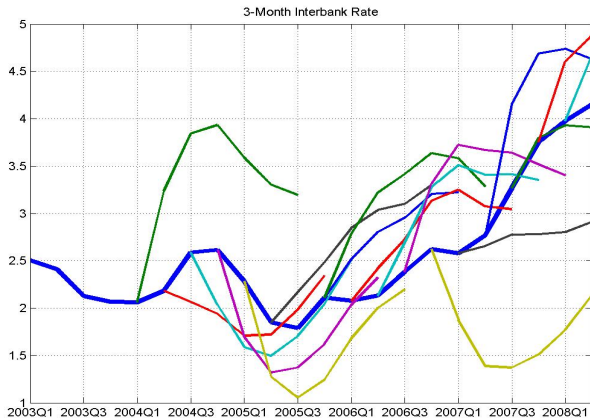
In-Sample Simulations: Ex. rate



In-Sample Simulations: GDP



In-Sample Simulations



Modeling tools

- Implementation in Matlab
- IRIS by Jaromír Beneš

Univariate filtering I

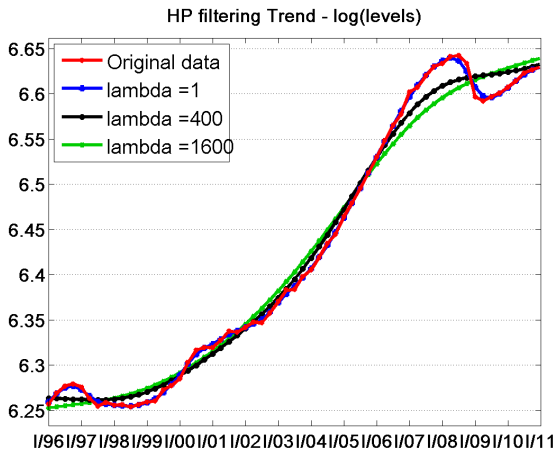
- Hodrick-Prescott filter: optimally extracts a trend which is stochastic but moves smoothly over time and is uncorrelated with the cyclical component
- Mathematics of HP filter:
 - ▶ Decomposition: $y_t = \tau_t + c_t$
 - ▶ Solve:
$$\min \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda * \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$$
 - ▶ $\lambda = 100 * (\text{number of periods in a year})^2$
- Assumption that the trend is smooth is imposed by assuming that the sum of squares of the second differences of τ_t is small
- Sensitivity of the trend to short-term fluctuations is achieved by modifying a multiplier λ

Univariate filtering II

- Drawbacks:
 - ▶ One-time permanent shock, split growth rates present: Filter identifies non-existing shifts in the trend
 - ▶ It pushes noise in data to Normal distribution
 - ▶ Misleading predictive outcome: Analysis is purely historical and static

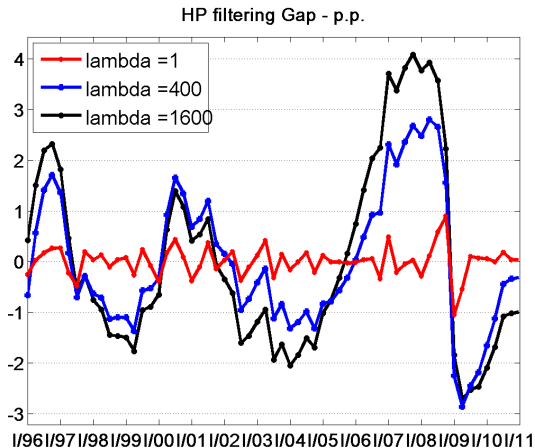
Univariate filtering III

- Trend:



Univariate filtering IV

- Gap:



Kalman filter I

- Separate the cyclical component of a time series from raw data
- Can handle more series and exploit relations between them
- Kalman filter is a powerful tool for:
 - ▶ Estimation
 - ▶ Prediction
 - ▶ Smoothing

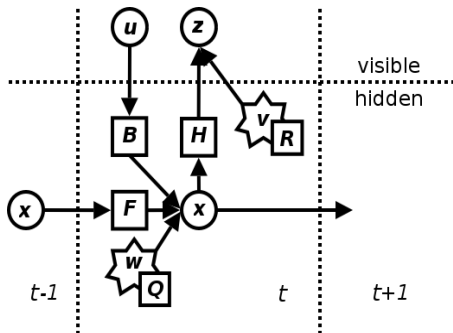
Kalman filter:

- ▶ Online estimation procedure
- ▶ States are estimated, when the new observations are coming in

Kalman smoother:

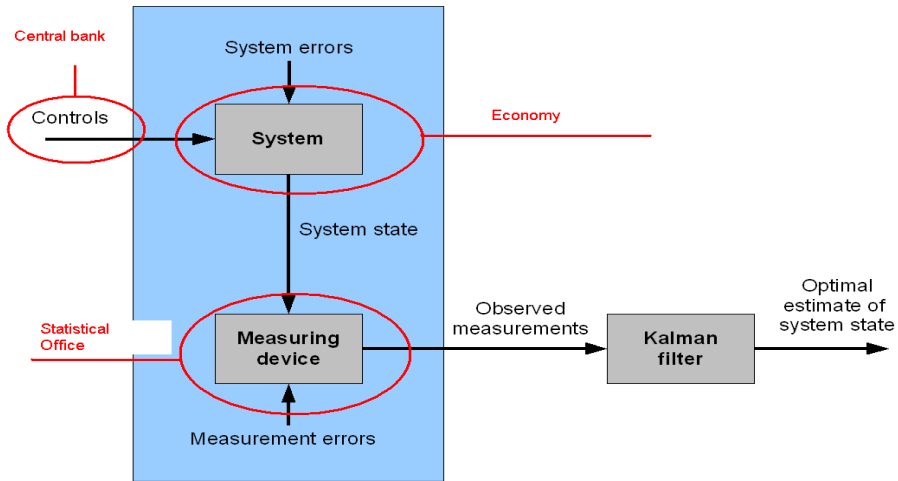
- ▶ Off-line estimation procedure
- ▶ The state estimation of is not only based on all previous observations, but also on all later observations

Kalman filter II



- F is the state transition model
- B is the control-input model
- H is the observation model
- w is the process noise
- z is the measurement
- v is the measurement error
- u is the exogenous control

Kalman filter structure



Description of variables

- Measurement variables: ΔEU_LGDP , $EU_LGDPGAP_EXPERT$
- State variables: ΔEU_LGDP_EQ , MU , $EU_LGDPGAP$
- Exogenous-variables: $EU_RMCIGAP$
- Shocks: ν 's
- Coefficients: a_1 , a_2 , a_3 and μ_{SS}
- Variance: $\sigma_1, \sigma_2, \sigma_3, \sigma_4$
- Remark: In the following slides the filtering is actually smoothing

Description of model

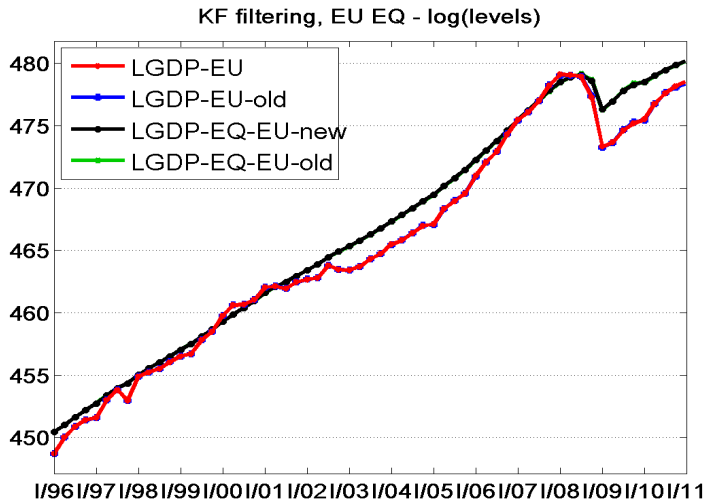
- Measurement equations:

$$\begin{aligned} \Delta EU_LGDP &= \Delta EU_LGDP_EQ + \\ &+ 4 * (EU_LGDPGAP - EU_LGDPGAP\{-1\}) \\ EU_LGDPGAP &= EU_LGDPGAP_EXPERT + \sigma_4 * \nu_4 \end{aligned}$$

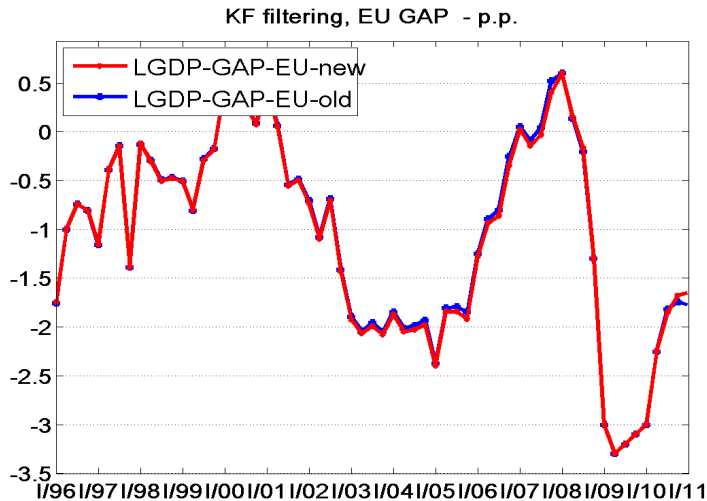
- State equations:

$$\begin{aligned} \Delta EU_LGDP_EQ &= \mu + \sigma_1 * \nu_1 \\ \mu &= (1 - a_3) * \mu_{SS} + a_3 * \mu\{-1\} + \sigma_3 * \nu_3 \\ EU_LGDPGAP &= a_1 * EU_LGDPGAP\{-1\} + \\ &+ a_2 * EU_RMCIGAP\{-1\} + \sigma_2 * \nu_2 \end{aligned}$$

Filtering results: EU Eq. trajectories

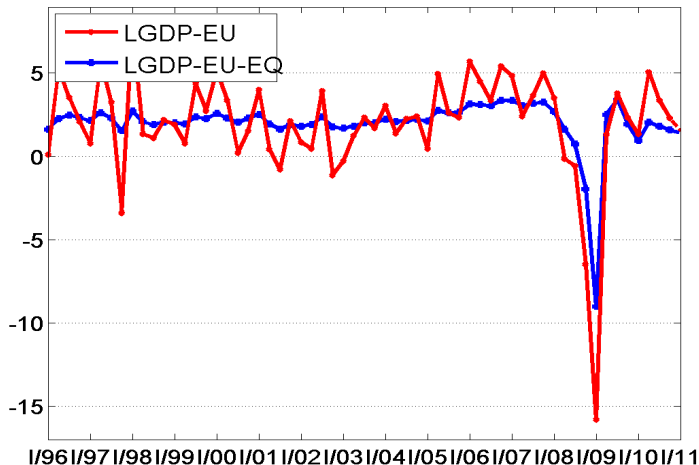


Filtering results: EU Gap estimate

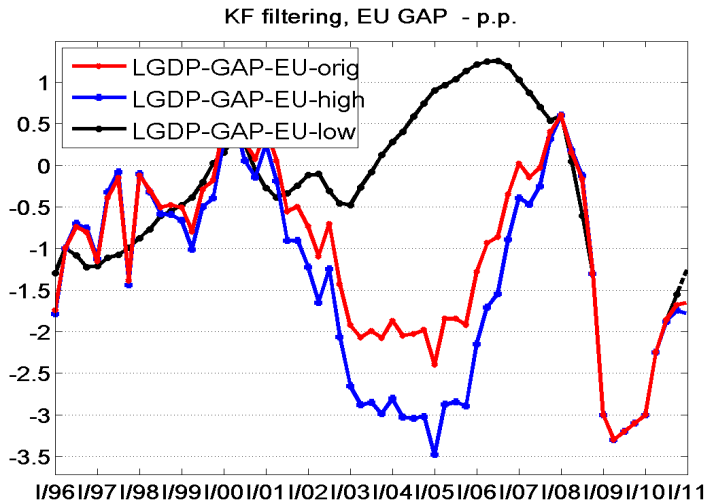


Filtering results: Removing volatility

KF filtering, EU EQ - q-o-q growth ann.



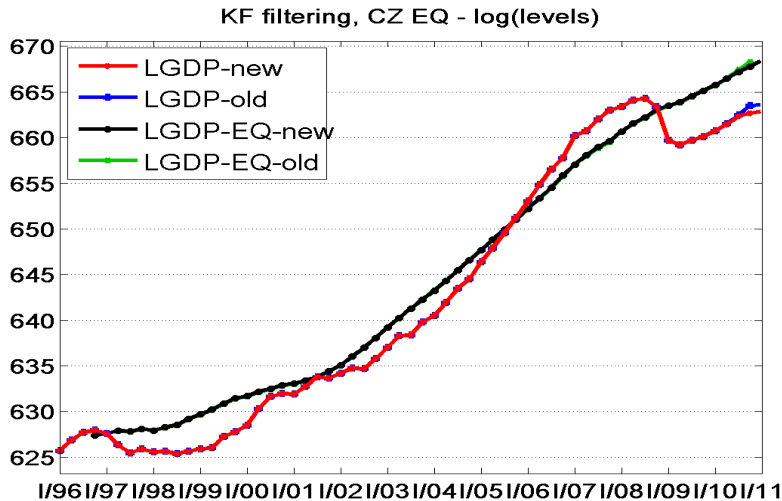
Model setting: Changes in volatility of gap σ_2



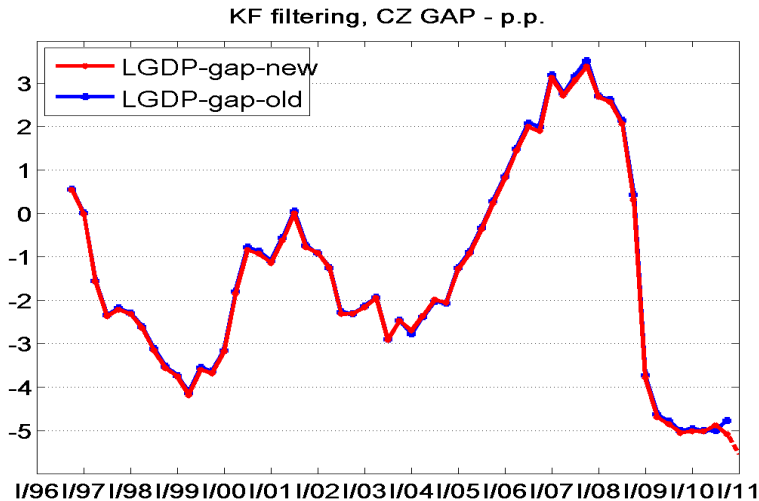
Filtering domestic variables

- First step:
 - ▶ Decompose real variables: trend and cycle
 - ▶ Simple model for: Real interest rate, Real exchange rate, Exchange risk premium
- Second step:
 - ▶ Utilize measurement of inflation and wage growth
 - ▶ Fit simple backward-looking Phillips curves: relation between inflation and output gap
 - ▶ Fit IS curve: relation between output gap and gaps in real interest and exchange rate
 - ▶ Decompose: domestic output, real wage, unemployment

Filtering results: Domestic Eq. trajectory



Filtering results: Domestic output gap



Description: Second step model

- Measurement variables: $DOT_LGDP, DOT_UNR, PIE_CORE, PIE_W, DOT_LWR, LWR_GAP_EXPERT, LGDP_GAP_EXPERT, UNR_GAP_EXPERT$
- State variables: $DOT_LGDP_EQ, MU, LGDP_GAP, DOT_UNR_EQ, UNR_GAP, PIE_CORE_S, PIE_W_S, DOT_LWR_EQ, LWR_GAP$
- Exogenous-variables: $RRC_GAP, RR4_GAP, EU_RR4_GAP, LZ_GAP, EU_LGDP_GAP, PIE_M_XENERGY4, DOT_LZ_CORE_EQ4, DOT_LZ_EQ4, E0_CORE4, E0_PIE_W4, DOT_LWR_PRIOR, E0_PIE4$
- Shocks: ν_s
- Variance: σ_s

Model I

- Measurement equations:

$$DOT_LGDP = DOT_LGDP_EQ + 4 * (LGDP_GAP - LGDP_GAP\{-1\})$$

$$DOT_UNR = DOT_UNR_EQ - 4 * (UNR_GAP - UNR_GAP\{-1\})$$

$$PIE_CORE = PIE_CORE_S$$

$$PIE_W = PIE_W_S$$

$$DOT_LWR = DOT_LWR_EQ + 4 * (LWR_GAP - LWR_GAP\{-1\})$$

$$LWR_GAP = LWR_GAP_EXPERT + std_w3 * \nu_LWR_GAP_EXPERT$$

$$LGDP_GAP = LGDP_GAP_EXPERT + std_w1 * \nu_LGDP_GAP_EXPERT$$

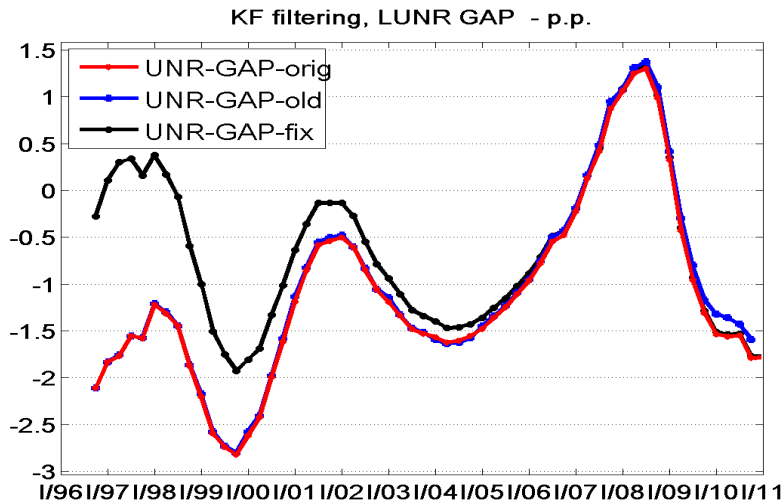
$$UNR_GAP = UNR_GAP_EXPERT + std_w2 * \nu_UNR_GAP_EXPERT$$

Model II

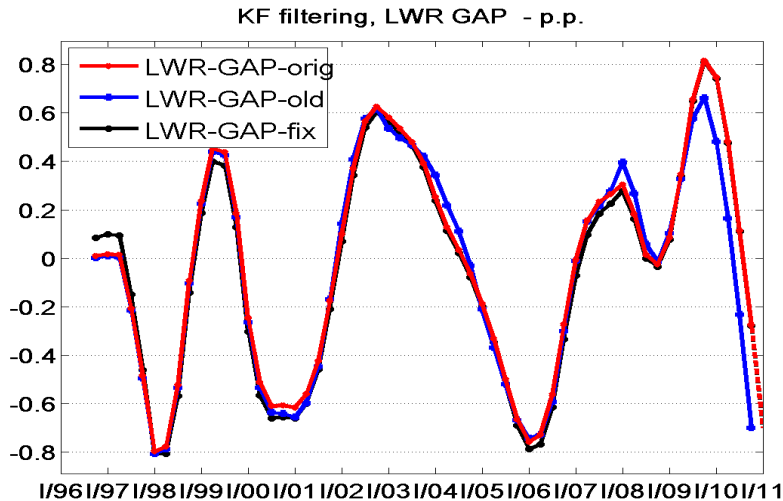
- State equations:

$$\begin{aligned}
 \text{DOT_LGDP_EQ} &= \text{MU}\{-1\} + a1 * \text{DOT_UNR_EQ} + \text{std_v1} * \nu_ \text{DOT_LGDP_EQ} \\
 \text{LGDP_GAP} &= \text{LGDP_GAP_C01} * \text{LGDP_GAP}\{-1\} - \text{RMCI_GAP_C02} * (b2 * \text{RRC_GAP}\{-1\} \\
 &+ b3 * \text{RR4_GAP}\{-1\} + b4 * \text{EU_RR4_GAP}\{-1\}) \\
 &\quad - \text{RMCI_GAP_C01} * \text{LZ_GAP}\{-1\} + \\
 &\quad \text{LGDP_GAP_C02} * \text{EU_LGDP_GAP} + \text{std_v2} * \nu_ \text{LGDP_GAP} \\
 \text{MU} &= (1 - a3) * \text{MU_SS} + a3 * \text{MU}\{-1\} + \text{std_v3} * \nu_ \text{MU} \\
 \text{DOT_UNR_EQ} &= \text{std_v4} * \nu_ \text{DOT_UNR_EQ} \\
 \text{UNR_GAP} &= \text{UNR_GAP_C01} * \text{UNR_GAP}\{-1\} \\
 &+ \text{UNR_GAP_C02} * \text{LGDP_GAP} + \text{std_v5} * \nu_ \text{UNR_GAP} \\
 \text{PIE_CORE_S} &= \text{PIE_CORE_C01} * (\text{PIE_M_XENERGY4} + \text{DOT_LZ_CORE_EQ4}) \\
 &+ \text{PIE_CORE_C02} * (\text{PIE_CORE_C05} * \text{E0_CORE4} \\
 &+ (1 - \text{PIE_CORE_C05}) * \text{E0_PIE4}) \\
 &+ (1 - \text{PIE_CORE_C01} - \text{PIE_CORE_C02}) * \text{PIE_CORE_S}\{-1\} \\
 &+ \text{RMC_GAP_C01} * \text{PIE_CORE_C03} * \text{LGDP_GAP} \\
 &+ \text{PIE_CORE_C03} * \text{LWR_GAP} \\
 &+ \text{std_v6} * \nu_ \text{PIE_CORE} \\
 \text{PIE_W_S} &= \text{PIE_W_C01} * \text{E0_PIE_W4} + (1 - \text{PIE_W_C01}) * \text{PIE_W_S}\{-1\} \\
 &+ \text{PIE_W_C02} * (\text{LWR_GAP} - \text{PIE_W_C03} * \text{LGDP_GAP}) + \text{std_v7} * \nu_ \text{PIE_W} \\
 \text{DOT_LWR_EQ} &= \text{DOT_LGDP_EQ} + \text{DOT_LWR_PRIOR} + \text{std_v8} * \nu_ \text{DOT_LWR_EQ} \\
 \text{LWR_GAP} &= f1 * \text{LWR_GAP}\{-1\} + \text{std_v9} * \nu_ \text{LWR_GAP}
 \end{aligned}$$

Filtering results: Expert judgement



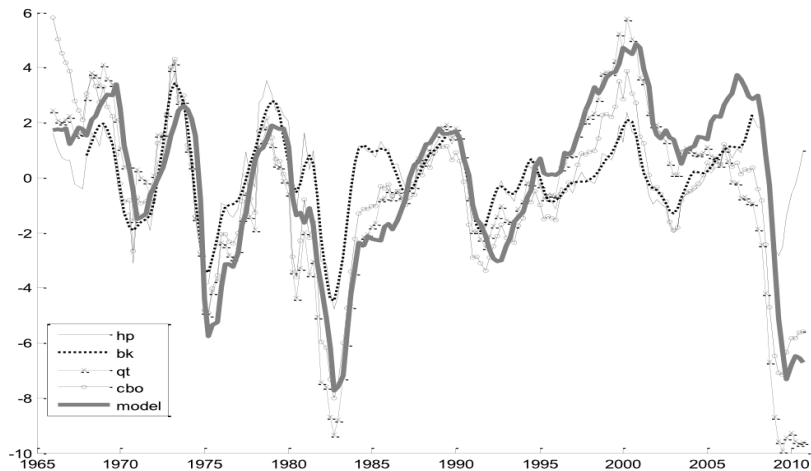
Filtering results: Expert judgement



Advanced filtering

- Criticism of simple models: lack of reference to unemployment
- J. Galí, F. Smets and R. Wouters (2011):
 - ▶ Address this issue in an extended model
 - ▶ Conclusion: Model-based output gap resembles conventional measures of the cyclical component of log GDP.
 - ▶ Comparison of a variety of statistical detrending methods
 - ▶ HP filter, band-pass filter, quadratic detrending, and the Congressional Budget Office's measure

Advanced filtering



In search for future trends



List of Variables I

\hat{a}	gap of the variable a
\bar{a}	trend (equilibrium) value of the variable a
a^f	variable a for the foreign country
ε^a	residual in the equation for the variable a
mc	real marginal costs
y	real output
rw	real wage
rmci	real monetary condition index
r4	real 1Y interbank rate
r	real 3M interbank rate
rc	real rate of newly-issued bank loans
z	real exchange rate

List of Variables II

$\pi 4^{target}$	inflation target (y-o-y)
π	price inflation (q-o-q)
$\pi 4$	price inflation (y-o-y)
w	wage inflation (q-o-q)
w4	wage inflation (y-o-y)
$\pi 4^M$	imported inflation (y-o-y)
s	nominal exchange rate
prem	risk premium
i	nominal short-term interest rate
$i^{neutral}$	policy neutral short-term interest rate
$\alpha, \beta, \gamma, \delta, \phi, \psi, \kappa, \lambda$	parameters

For Further Reading I



Cbo'S Method For Estimating Potential Output: An Update,
<http://www.cbo.gov/doc.cfm?index=3020&type=0>



Jordi Galí and Frank Smets and Rafael Wouters
Unemployment In An Estimated New Keynesian Model,
National Bureau Of Economic Research, vol. 17084, 2011



Peter K. Clark
The Cyclical Component of U.S. Economic Activity,
The Quarterly Journal of Economics, vol. 102, 1987

For Further Reading II



Rudolph E. Kalman

A New Approach to Linear Filtering and Prediction Problems
Transactions of the ASME–Journal of Basic Engineering, vol. 82,
Series D, 1960



Greg Welch and Gary Bishop

An introduction to the Kalman filter.
University of North Carolina, July, 2006; 2000.



Harvey, Andrew C, 1985

Trends and Cycles in Macroeconomic Time Series
Journal of Business and Economic Statistics, Vol. 3 p. 216

For Further Reading III



Watson, Mark M, 1986

Univariate Detrending Methods with Stochastic Trends

Journal of Monetary Economics, Vol. 18, p. 49



Athanasios Orphanides and Simon van Norden, 2002

The Unreliability of Output-Gap Estimates in Real Time

The Review of Economics and Statistics, Vol. 84, Num. 4

Additional one ...

