**Microeconomics 3** **Andreas Ortmann, Ph.D.**
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***Week of June 2, lecture 8:***
***Game theory:***
**Wrapping up the previous week's material.**
**(What is a game? Examples of classic games. Forms of representation: extensive and normal (strategic). Dominant and dominated strategies. Nash equilibrium. Subgame perfect Nash equilibrium. Mixed strategy equilibrium. Reaction correspondences. Rationalizability.)**

***Key readings:***
MWG 7A-E, MWG 8A - D

Andreoni and Miller (2002), Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism. Econometrica 70, 737 - 752.

Harbaugh, Krause, and Berry (2001), GARP for Kids: On the Development of Rational Choice Behavior. American Economic Review 91, 1539 - 1545.

***Assignments:***

**[for June 3]** Problem set # 4 ~ downloadable from home.cerge-ei.cz/babicky/micro3.

**[for June 10]** MWG 8E (Bayesian Nash equilibria) and
MWG 8F (trembling-hand perfection)

**[for June 12]** Problem set # 5 ~ downloadable from home.cerge-ei.cz/babicky/micro3. (This is not a typo. The exercise session next week will be Thursday during classtime. The second lecture will be Tuesday during the time where normally is exercise session.)

**[for June 10]** Johnson, Camerer, Sen, and Rymon (2002), Detecting Failures of Backward Induction: Monitoring Information Search in Sequential Bargaining. Journal of Economic Theory, 1 - 32.

**[for June 10]** McCabe, Smith, and LePore (2000), Intentionality detection and “mindreading”: Why does game form matter? Proceedings of the National Academy of Sciences 97.8., 4404 - 4409.

“Midterm” coming up: June 5 (Please be there a few minutes earlier.)

***Please note the following correction:***

*Ten commandments for the 4-page paper:*

9. The deadline is June 17, 2003, 22.00.

Talking about corrections, ... please send me an e-mail (or stop by) if you find a mistake in the lecture notes or problem sets or solutions sheets. The micro III home page there will have an errata section before long.

***Remark (rehash):***

Solving
(predicting what the – likely – outcome of the game will be if ... ):
[For now we ignore the possibility that players will randomize in their action choices.]

Normal (strategic) form

-> Dominance (strict, weak)

- Weak dominance could cause problems; strict doesn't.

-> Iterated dominance

- Requires rationality and common.knowledge (also of other players' rationality).
- Leads to rationalizability (Bernheim, Pearce).

-> Nash equilibrium

- Requires rationality and common knowledge and mutually correct expectations ("best response" property).
- Is silent on selection of "the best" among several equilibria.
- Leads to refinements (selection theories) and evolutionary models.

Extensive form

-> Iterated dominance (the principle of sequential rationality)

- Requires rationality and common.knowledge (also of other players' rationality).
- Leads to backward induction.
- Leads to subgame perfection (Nash equilibria that are credible).

***Decision questionnaire 3:*** Your name:........................

1. You will be randomly matched with **one other member [two other members]** of the class.

2. The two of you [three of you] will be referred to as group from here on.

3. Each member of your group has to choose either 0 or 1 [0, 1, or 2].

4. The person in your group who is closest to ½ of **the average of the numbers chosen by all members** of the group will win 300 korunas. In case there are ties, this prize will be split.

5. One group will be randomly selected and the winner(s) will be paid off.

**6. Document the complete reasoning process leading to your choice below.**

7. Make your decision.

8. Also answer the question at the bottom of the page.

Decision:
I choose (please circle) 0 1 [0 1 2]

Question: What number do you expect the other member of your subgroup to choose?
Please circle: 0 1 [0 1 2]

***Note: There were three versions.***
***How to tackle this problem best?***
***Representation as extensive form game?***
***Representation as normal form game?***
***Other ways?***

[Discussion/Jiri's well-taken argument motivates logit-equilibrium approach etc.]

***Definition*** 8.D.1 ***(Nash equilibrium, pure strategies)***:
A strategy profile $s = (s_1,\ldots, s_I)$ constitutes a Nash equilibrium of game $\Gamma_N = [I, \{S_i\}, \{u_i(\cdot)\}]$ if for every $i = 1, \ldots, I$, $u_i(s_i,s_{-i}) \geq u_i(s'_i,s_{-i})$ for all $s_i \in S_i$.

***Note:*** In a Nash equilibrium each player's strategy choice is a best response to the strategies actually played by the other players. [We'll get back to the notion of best response presently when discussing *rationalizable strategies*.]

[How to determine a pure strategy Nash equilibrium: Illustrated in class with several examples]

*Definition* 7.E.1 *(mixed strategy)*:
Given player i's (finite) pure strategy set $S_i$, a mixed strategy for player i, $\sigma_i$, $S_i \to [0,1]$, assigns to each pure strategy $s_i \in S_i$ a probability $\sigma_i(s_i) \geq 0$ that it will be played, where $\Sigma\ \sigma_i(s_i) = 1$.

*Note:* A mixed strategy represents the convexification, or mixed extension of $S_i$. In fact, this mixed extension spans a simplex whose vertices are the pure strategies that support the mixed strategy. Note that this implies that pure strategies can be thought of as degenerate mixed strategies. [For more formal details see MWG p. 232.]

*Definition* 8.D.2 ***(Nash equilibrium, mixed strategies)***:
A mixed strategy profile $\sigma = (\sigma_1,\ldots, \sigma_I)$ constitutes a Nash equilibrium of game $\Gamma_N = [I, \{\Delta S_i\}, \{u_i(\cdot)\}]$ if for every $i = 1, \ldots, I$,
$u_i(\sigma_i,\sigma_{-i}) \geq u_i(\sigma'_i,\sigma_{-i})$ for all $\sigma'_i \in \Delta S_i$.

[How to compute a mixed strategy Nash equilibrium:
Bishop & Cannings (1978).
Illustrated in class with Chicken, Matching Pennies, and BoS.]

*Note 1*:
Mixed strategies can be dominant strategies. Analyze the following decision situation determine a payoff maximizing strategy:

| | | | | |
|---|---|---|---|---|
| | 15,15 | 75, 0 | 0,75 | 5,25 |
| PRSD | 0,75 | 15,15 | 75,0 | 5,25 |
| | 75, 0 | 0,75 | 15,15 | 5,25 |
| | 25, 5 | 25, 5 | 25, 5 | 0, 0 |

Your strategy? ...............................................
*[Note: You may choose mixtures of pure strategies.]*

Let M in PRSD denote the mixture {1/3, 1/3, 1/3, 0}. Verify that the resulting compound lottery is strategically equivalent to the outcome MM in the following game MD:

| | |
|---|---|
| 30, 30 | 5,25 |
| 25,5 | 0,0 |

Note that D is strictly dominated by M and therefore there are no beliefs that can "rationalize" playing D when the numbers in the bi-matrix denote utility.

*Note 2:*
Computing mixed strategies is also a necessary condition to compute the reaction curves (reaction correspondences).

[Illustrated in class for game of Matching Pennies and one of the PDG parameterizations . See problem set # 4 for Chicken and Battle of the Sexes.]

*Definition* 8.C.1 (*best response, never best response*):
In game $'_N$ = [I, {Î$S_i$ }, {$u_i$ (@)}] strategy $F_i$ is the best response for player i to his rivals' strategies $F_{-i}$ if

$$u_i\ (F_i,F_{-i})\ \ \$\ u_i\ (F'_i,F_{-i})\ \text{for all}\ F'_i\ 0^a S_i.$$

Strategy profile $F_i$ is never a best response if there is no $F_{-i}$ for which $F_i$ is the best response.

*Definition* 8.C.2 (*rationalizable strategies*):
In game $'_N$ = [I, {Î$S_i$ }, {$u_i$ (@)}], the strategies in Î($S_i$) that survive the iterated removal of strategies that are never a best response are known as player i's rationalizable strategies.

*Note 1:* A strategy that is strictly dominated is never a best response. (Unfortunately, a strategy that might never be a best response may not be strictly dominated. So, the set of rationalizable strategies and the set remaining after iterative deletion of strictly dominated strategies do not necessarily coincide, with the ? potentially being larger than the ? but not the other way round.)

*Note 2:* For the set of two-player games (I = 2) the set of rationalizable strategies and the set remaining after iterative deletion of strictly dominated strategies are identical.

*Note 3:* Note that both an assessment of what constitutes (never) a best response requires a belief on a player's part on what strategies are in the choice set, given  common knowledge of rationality and the structure of the game.

[Example 8.C.1.]

*Note 4:* The set of rationalizable strategies can be rather large (because the set of strategies that's never a best response, given common knowledge of rationality and the structure of the game, tends to be rather small.) The Nash equilibrium concept imposes as condition that players have to be correct in their expectations of their opponents' choices. Put differently, Nash strategies are those rationalizable strategies which, <u>if actually played</u>, confirm the expectations on which they were based. (Hence, "self-confiming" or "self-enforcing" strategies.) Nash equilibrium, to introduce other notions, require that players' beliefs are "consistently aligned" or "mutually consistent".

*Note 5:* Every strategy that is part of a Nash equilibrium is rationalizable.

***Definition*** 9.B.1 ***(subgame)***:
A subgame of an extensive form game $\Gamma_E$ is a subset of the game having the following properties:

(i) It begins with an information set containing a single decision node, contains all the decision nodes that are successors (both immediate and later) of this node, and contains *only* these nodes.
(ii) If decision node x is in the subgame, then every x’ $\in$ H(x) is also, where H(x) is the information set that contains decision node x.

***Note 1:*** The game as a whole is a subgame, as may be some strict subsets of the game.

***Note 2:*** In a finite game of perfect information (i.e., when information sets are singletons), every decision node initiates a subgame.

***Definition*** 9.B.1 ***(subgame perfect Nash equilibrium)***:
A strategy profile $s = (s_1, \ldots, s_I)$ in an I-player extensive form game $\Gamma_E$ is a subgame perfect Nash equilibrium (SPNE) if it induces a Nash equilibrium in every subgame of $\Gamma_E$.

[Application of these concepts to some of the sample games, e.g., chain store game, Stackelberg game, alternating offer game, AoN, ... ]

***Note:***

Alternating offer game:

Three-round two-person alternating-offer game starts with player I making a proposal of how to split a “pie” of 10 dollars (=v). Each round, the respective other player can either accept or reject. Each round half of the pie melts away ($* = .5$).

Apply the following *principle*
(applicable to bargaining games that involve proposers and responders):
In equilibrium, a proposer always plans to offer the responder an amount that will make the responder indifferent between accepting and refusing. In equilibrium, the responder always plans to accept such an offer or better, and to refuse anything worse.
[Rationalize via finite version of the ultimatum game,
with smallest unit of currency approaching zero.]

Period T = 3: $(*^{T-1}v, 0) = (*^{2}10, 0) = (2.50, 0)$
Period T - 1 = 2: $(*^{T-1}v, *^{T-}v - *^{T-1}v) = (*^{2}10, *10 - *^{2}10) = (2.50, 5.00 - 2.50)$
Period T - 2 = 1: $v_I^* (T) = v_I^* (3) = v[1 - * + *^2] = 10[1 - .5 + .25]$ and

$$v_{II}^* (T) = v - v_I^* (3) = 10 - 10[1 - .5 + .25]$$

Does this game have a SPNE? Is it unique? What exactly is it?

Assume that the game has no finite ending but otherwise the same holds as before, i.e., two-person alternating-offer game starts with player I making a proposal of how to split a “pie” of 10 dollars (=v). Each round, the respective other player can either accept or reject. Each round half of the pie melts away ($* = .5$).

Does this game have a SPNE? Is it unique? What exactly is it?

(Rubinstein bargaining solution)

*Existence propositions and classification results.*
*And rationalizations.*

*Proposition* 8.D.2 *(existence)*:
Every game $\Gamma_N = [I, \{\in S_i\}, \{u_i(\cdot)\}]$ in which the sets $S_1, \ldots, S_I$ have a finite number of elements has a mixed strategy Nash equilibrium.

*Note 1:* This result can be generalized to games in which strategies can be modeled as continuous variables. See for details proposition 8.D.3 in the textbook. For proofs of both propositions, see MWG 260 - 1.

*Note 2:* For 2x2 normal form games it can be shown (e.g., Eichberger, Haller, & Milne, Journal of Economic Behavior and Organization 1993) that there are essentially three large classes of games according to their equilibrium configuration:
- the first contains those games with no pure and one mixed Nash equilibrium
- the second contains those games with two pure and one mixed Nash equilibrium
- the third contains those games with exactly one pure and no mixed Nash equilibrium

There is a residual class of games that may have either two pure Nash equilibria (one of which contains dominated strategies) or a continuum of mixed strategy equilibria containing at least one pure strategy equilibrium.

This residual class is "non-generic in the space of payoff parameters", i.e. such games you are very unlikely to encounter.

*Proposition* 9.B.2 *(existence)*:
Every finite game of perfect information $\Gamma_E$ has a pure strategy subgame perfect Nash equilibrium (computable through backward induction). Moreover, if no player has the same payoffs at any two terminal nodes, then there is a unique subgame perfect Nash equilibrium.

*Note:* MWG discuss five “rationalizations” of Nash equilibrium. Essentially, there are two competing explanations: First (MWG iv), Nash eq. as self-enforcing outcome of an unspecified negotiation process. Second (MWG v), Nash eq. as stable outcome of unspecified learning process. [This rationalization of particular interest as it has motivated evolutive approach.]

**Guiding questions on Andreoni & Miller (2002), Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism**

*What's the basic goal of the authors?*
- To study whether people are consistent in their choices when they are altruistic. Consistency is the rationality criterion. Consistency in the sense of the generalized axiom of revealed preferences (GARP):

I a bundle A is indirectly revealed preferred to another bundle B [transitivity argument], then B is not strictly directly revealed preferred to A.

A bundle A being "DRP" to another bundle B means that B was in the choice set when A was chosen.

That is,
if A IRP to B, then A is not in strictly within the budget set when B is chosen.

*What's the basic idea of how to test consistency of preferences for altruism?*
- Let people make dictator decisions under various income and price combinations.
[See Table 1. See Figure 1. Discuss budgets 7 - 9. Discuss Hold Value and Pass Value. Relative Price of Giving. Identify the various budget constraints in the Figure.
Note the aggregate non-monotonicity in giving behavior in 7, 9, 8. Probably small-sample effect. Discuss a couple of additional budget lines but focus on downward sloping budget lines.]
[Note that the basic idea can be extended to strategic environments - see section 8.]
- See whether they make inconsistent choices.
- Infer the utility function of those that make consistent choices.

*Experimental set-up and implementation?*
- Low-technology ("paper & pencil"; yet got published in Econometrica)
- 5 sessions with 34 - 38 subjects, for total of 176 subjects.
- Decision problems in random order;
    four sessions with 8 problems, one with 11.
- Each session between 34 - 38 subjects, for a total of 176 subjects.
- Subjects were prompted: "Divide m tokens:
    Hold ... at ... ($p_s$) points each, and Pass ... at ... ($p_o$) each."
- *Task for subjects:* To choose under various budget constraints (endowments) and price ratios, allocations of payments to self and other (i.e. dictator game). So, max $U_s = u_s (B_s, B_o)$ s to $p_s B_s + p_o B_o = m$. Resultant selection could be checked for violations of WARP, SARP, and GARP: See Figure 2.a.
- Subjects get to keep $0.10 per point.
- "Random lottery procedure"

*Experimental results?*
- Looking at budget 7 - 9, subject gave away on average about 23 percent. This is in line with previously reported results in literature.
- With the exception of 3 subjects (1.7 percent), observed behavior can be rationalized by a quasi-concave utility function.
- What shape exactly would such a utility function take? Depends.
- selfishness: 22.7 percent -> $U(B_s, B_o) = B_s$
- inequality aversion: 14.2 percent -> $U(B_s, B_o) = \min\{B_s, B_o\}$ [Leontief technology]
- perfect substitutes: 6.2 percent -> $U(B_s, B_o) = B_s + B_o$ [perfect substitutes]
- if one uses a classification which clusters subjects into groups that minimizes the distance to choices from on of the three utility functions just described, then a full classification can be given with
    - selfishness: 47.2 percent
    - inequality aversion: 30.4 percent
    - perfect substitutes: 22.4 percent

(See Table III)

That's all very slick. At least two obvious problems here. Anyone sees then?
[Discussed in class]

*Definitions:*

*DRP:* A is directly revealed preferred to B if B was in the choice set when A was chosen.
*IRP:* A is indirectly revealed preferred to Z if A DRP to B, B DRP to C, . etc.

*Revealed preference axioms:*
*WARP:* A DRP to B, then B not DRP to A (nec condition)
*SARP:* A IRP to B, then B not DRP to A (nec and suff condition)

*GARP* (to allow for indifference curves that are not strictly convex):
A IRP to B, then B not strictly DRP to A (A is not strictly within the budget set when B is chosen).

**Guiding questions on Harbaugh, Krause, & Berry (2002), GARP for Kids: On the Development of Rational Choice Behavior**

*Experimental design and implementation:*

- Similar in design to Andreoni & Miller (2002)
  So question is again that of whether choices are consistent, i.e. whether they obey transitivity.
- Testing GARP with 7-, 11-, and ~ 21- year olds (31, 42, 55, respectively)
- Choices made from finite sets of allowable bundles
  [Discuss Figures 1 and 2]
- Subjects paid with bundles of "small bags of potato chips" and "fruit juice"
- Subjects chose from a stapled packet ; each page in the packet listed the bundles for a single choice set. Three chances to express preferences.

*Results:*

- See Table 1.