Microeconomics 3 **Andreas Ortmann, Ph.D.**
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Week of June 23, lecture 13:

(More on) on beliefs and sequential rationality, including weak perfect Bayesian and sequential equilibria. Also, forward induction. Also, indefinitely repeated games.

Key readings:

[for June 24] MWG 9C (Beliefs and sequential rationality)
[for June 24] MWG 9D (Reasonable beliefs and forward induction)

[for June 24] Cachon and Camerer (1996), Loss-Avoidance and Forward Induction in Experimental Coordination Games. Quarterly Journal of Economics 111.1., 165 - 194.

[for June 24] MWG 12D and 12App (Repeated Interaction)

[for June 24] Problem set # 7 ~ downloadable from home.cerge-ei.cz/babicky/micro3.

Assignments:

[for June 26 - 29] Consultations on experimental paper, see follow-up e-mail message Thursday

[for July 1] Problem set # 8 ~ downloadable from home.cerge-ei.cz/babicky/micro3.

[for July 1] Aliprantis & Chakrabarti (2000), Games and Decision Making, chapter 6. [handout]

[for July 3] Jehle & Reny (2001), Advanced Microeconomic Theory. 2nd edition, chapter 9, pp. 373 - 383. [handout]

[for July 8] Ivanova-Stenzel & Salmon (2002), Bidder Preferences among Auction Institutions. Manuscript.

[for July 10] Goeree and Holt (2001), Ten Little Treasures and Ten Intuitive Contradictions. American Economic Review 91.5., 1402 - 1422.

[Commandments for experimental paper, see handout. You can also find them on the course website at home.cerge-ei.cz/babicky/micro3]

Four page papers were written on and by

Games and Phone Numbers: Do Short-Term Memory Bounds Affect Strategic Behavior ? (Devetag &Warglien)
- Zuzanna Bakasova & Alexandra Putzova
- Zvezda Dermendzhieva & Plamen Doudov
- Anush Hakobayan
- Aliya Kurmanbaeva & Galina Dvoretskina

“Crime” in the Lab - Detecting Social Interaction (Falk & Fischbacher)
- Volha Baranava & Vitaliy Strohush
- Vilem Semerak
- Lucie Soukenikova & Bohdan Vrazda

Words, Deeds and Lies (Duffy & Feltovich)
- Ivo Burger & Peter Skelnar
- Elena Savushkina & Oleksiy Skorokhod
- Viorel Roscovan & Andriy Kuznetzov

Monty Hall’s Three Doors: Construction and Deconstruction of a Choice Anomaly (Friedman)
- Volha Belush & Svetlana Tashchilova
- Juraj Kopecsni

The Electronic Mail Game: Strategic Behavior Under ‘Almost Common Knowledge’ (Rubinstein)
- Dorota Kowalczyk & Yuliya Rychalovska

Players’ Models of Other Players: Theory and Experimental Evidence (Stahl & Wilson)/A Cognitive Hierarchy Model of One-Shot Games (Camerer, Ho, & Chong)
- Aelita Belyaeva & Meruyert Kenshimova

Pay enough or don’t pay at all (Gneezy & Rustichini)
- Ainura Uzagaliewa

The Subsidence of Preference Reversals in Simplified and Marketlike Experimental Settings: A note (Chu & Chu)
- Lubos Briatka & Marian Baranec
- Nina Nikoleva

Class Struggle inside the Firm: A Study of German Codetermination (Gorton & Schmid)
- Maksim Tumilovich & Taras Kulyk

Do People Optimize Their Self-Confidence? (Camerer & Lovallo)
- Jakub Steiner & Jozef Zubricky

Incorporating Fairness into Game Theory and Economics
- Rustem Zhanisbayev

Note 1: Problems with the weak PBE:

- specified beliefs may not be sensible, e.g. MWG Example 9.C.4

- they may not specify a Nash equilibrium in the post-entry subgame (hence weak PBE strategies may not constitute a subgame perfect equilibrium), e.g. MWG Example 9.C.5

Note 2: Extra consistency requirements generate PBE and sequential equilibrium.

Definition 9.C.4 *(Sequential equilibrium):*
A profile of strategies and systems of beliefs (F, :) is a SE of extensive form game $'_E$ if it has the following properties:
(i) The strategy profile F is sequentially rational given belief system : .
(ii) There exists a sequence of completely mixed strategies $\{F^k\}_{k=1, ..., 4}$,
with $\lim_{k\text{-}> 4}$ F^k-> F, such that : = $\lim_{k\text{-}> 4}$ $:^k$, where $:^k$ denotes the beliefs derived from strategic profile F^k using Bayes' rule.

Proposition 9.C.4 *(Sequential equilibrium):*
In every SE of $'_E$ the equilibrium strategy profile constitutes a SPNE of $'_E$.

[Discussion MWG Example 9.C.4 and MWG Example 9.C.5]

Note 1: Subgame perfection often captures the principle of sequential rationality but not always. Hence, to capture the spirit of subgame perfection, the *weak Perfect Bayesian equilibrium* concept (F is sequentially rational given belief system :; belief system : is derived from F through Bayes' rule whenever possible.) Unfortunately, the weak PBE concept carries us only so far as it fails in those situations where the probability of reaching an information set is zero, making the straightforward application of Bayes' rule impossible and leading to strategies that are not subgame perfect (e.g. example 9.C.5). This fact, in turn, motivated us to impose further restrictions on beliefs (namely the consistency requirements of the *sequential equilibrium* concept.) This concept, like the weak PBE has two parts. In fact, part 1 (F is sequentially rational given belief system :) is identical to part 1 of the definition of a *weak PBE*. Part 2 circumvents the problem of the non-applicability of Bayes' rule by generating sequences of completely (strictly) mixed strategy profiles F_n and sequences of completely (strictly) mixed belief systems $:_n$ which are [and now can be] derived from the strategy profiles F_n through Bayes' rule.

Note 2: Worse, sequential equilibria may be sustained with beliefs that are not at all intuitively sensible. This fact leads to the definition of "intuitive" and "divine" criteria - further restrictions on beliefs in order to generate reasonable solutions.

[Discussion of

Remark 1: Figure 9.D.1 in MWG offers variants of games (Entry, Niche) that we encountered before. The indicated paths denote sequential equilibria (and hence weak perfect Bayesian equilibria, or WPBE.)

Why?

They constitute WPBE because beliefs and strategies are consistent. The beliefs, of course, are consistent only by default: they can not be derived from Bayes' Rule. So everything goes, even beliefs of [1] at the left-most decision node in the information sets of Firm I in Figure 9.D.1(a) and Figure 9.D.1(b), as indicated. These beliefs are unreasonable because
[Entry] for E, In_1 is strictly dominated by In_2.
[Niche] for E, Small is strictly dominated by Out.
Given these (unreasonable) beliefs about E's actions, I's chosen actions are sequentially rational.

How can be that they are sequential equilibria?
Take Figure 9.D.1(a):
Let $(p_{Out}, p_{In1}, p_{In2})$ denote a completely mixed strategy profile of player E,
let (q_F, q_A) denote a mixed strategy profile for of player I, and
let $(\mu, 1-\mu)$ denote the system of beliefs associated with player I's information set, with μ corresponding as usual the leftmost decision node (here the one reached by In_1)
Note that the WPBE that is specified has
- the strategy profile S = ((1,0,0), (1,0)) and
- the system of beliefs b = (1,0).

Now consider a sequence of completely mixed strategy profiles $\{S^k\} \to S$.
$\{S^k\} \to S$ iff $((1 - \varepsilon^k_1 - \varepsilon^k_2, \varepsilon^k_1, \varepsilon^k_2), (1 - \varepsilon^k_3, \varepsilon^k_3)$ and $\varepsilon^k_{1,2,3} \to 0$ as $k \to \infty$;
note that $\{S^k\}$ is a completely mixed strategy profile iff $\varepsilon^k_{1,2,3}$ are positive and sufficiently small. Let b^k a system of beliefs $(\mu^k, 1-\mu^k)$ that is consistent with S^k. Since every S^k consists of completely mixed strategies, b^k is uniquely determined by Bayes' Rule as $\mu^k = \varepsilon^k_1/(\varepsilon^k_1 + \varepsilon^k_2)$.
Can we choose the sequences $\{\varepsilon^k_1\}, \{\varepsilon^k_2\}, \{\varepsilon^k_3\}$ so that $b^k \to b$?
Try $\varepsilon^k_1 = \varepsilon^k_3 = 1/(k+1)$ and $\varepsilon^k_2 = 1/(k+1)^2$.

Figure 9.D.1(b), analogously.

Remark 2 (forward induction and unreasonable beliefs): Recall that for E, In_1 in Fig 9.D.1(a) is strictly dominated by In_2. So firm I's belief about E's action is not reasonable. Imagined speech: "I, E, have entered. Note that I have a strictly dominant strategy which I shall pursue. You better take note (and accommodate me)."

Remark 3 (forward induction and unreasonable beliefs): Recall that for E, Out in Fig 9.D.1(b) strictly dominates Small Niche. So firm I's belief about E's action is not reasonable. Imagined speech: "I, E, have entered. Note that I could have entered Small Niche or Large Niche. Note also that my out option strictly dominates Small Niche. Since Out dominates Small Niche, my having entered ought to suggest to you that I have entered Large Niche. You better take note (and play Small Niche because otherwise you get burned badly)."

[Also discussion of Figure 9.D.3]

Remark 4: Nice try.

But no without problems. What if players make mistakes?

Then in Fig 9.D.1(a), the Incumbent's response to E might be: "Of course, E is telling me this. It has made a mistake and now is trying to make the best of it by convincing me to accommodate."

Then in Fig 9.D.1(b) , the Incumbent's response to E might be: "Forget it! I think you just made a mistake - and even if you did not, I'm going to target the large niche!"

Forward induction in normal form games

[based on Cachon and Camerer, “Loss Avoidance and Forward Induction in Experimental Coordination Games”, Quarterly Journal of Economics 1996, 165 - 194.]

MEDIAN/MINIMUM EFFORT GAME PAYOFF TABLE

	Median/minimum action						
Your action	1	2	3	4	5	6	7
1	140	150	140	110	60	-10	-100
2	130	160	170	160	130	80	10
3	100	150	180	190	180	150	100
4	50	120	170	200	210	200	170
5	-20	70	140	190	220	230	220
6	-110	0	90	160	210	240	250
7	-220	-90	20	110	180	230	260

The game is a more complex example of the kind of coordination games that have been used to study macroeconomic cycles, technology adoption, sticky prices, organizational design, bank runs, stag hunts, etc.

The game has two important features:

- Players’ payoffs are decreasing with the (absolute) difference between their own action and the median/minimum (of the actions of all participants, here 9)
- The seven pure-strategy equilibria are Pareto-ranked: the higher an equilibrium is ranked (in terms of the number of the supporting action) the better. If payoff-dominance were a universally accepted selection principle, then 7 should be chosen.

Empirically,

- median games resulted typically in actions that create a median of 4-5 in the first round. That initial value then acted as precedent that subjects had trouble to extract themselves from.
  Famous paper: Van Huyck, Battalio, & Beil (1991), see also van Huyck, Battalio, & Rankin (1996)
- minimum games typically resulted in action choices that were lower in the first round. Unravelling to the lowest-ranked pure-strategy equilibrium (!) was quick.
  Famous paper: Van Huyck, Battalio, & Beil (1990)
- 1. and 2. true for games with “small” number of rounds (but see for example Berninghaus & Ehrhart (1998) or no pre-play communication (but see for example Blume & Ortmann (2000/3), or earlier, Van Huyck, Battalio, & Beil (1993) who show that adding a pre-play auction each period enables experimental participants to coordinate on the Pareto-efficient equilibrium. VHBB’s interpretation is that the price of the right to play serves as a means of ‘tacit communication’, and allows participants to eliminate equilibria with payoffs lower than that price, and thus to reduce the strategic uncertainty resulting from the multiplicity of equilibria. Cachon & Camerer (1996) show that asking participants to pay a fixed price for participation also enables participants to coordinate on the Pareto-efficient equilibrium.)

Cachon & Camerer use forward induction as follows:

In their Opt Out sessions they had subjects pay the coordination game for rounds 1 - 3; in these three rounds subjects ‘ entry costs were zero and subjects could not opt out. In rounds 4 - 6 the entry fee was raised from $0 to $1.85. (This entry fee was chosen because of the empirical fact summarized under 1. above.) In rounds 7 - 9 the entry costs was raised from $1.85 to $2.25.
Imagined speech: “Look, I could have opted out if I had wanted to, thereby earning $0. Since I’m willing to pay $2.25 you can safely assume that I will play at least “6". And so should you. Because if you don’t, then you will loose (although I, admittedly, will loose more).”

In their Must Play sessions they had subjects pay the coordination game for rounds 1 - 3; in these three rounds subjects ‘ entry costs were zero and subjects could not opt out. In rounds 4 - 6 the entry fee was raised from $0 to $1.85. In rounds 7 - 9 the entry costs was raised from $1.85 to $2.25. In contrast to the Opt Out sessions, in the Must Play sessions subjects could not opt out. Hence they also could not make imagined speeches as before.

Purpose of this treatment: To assess whether forward induction or loss avoidance made the difference.

What is loss avoidance?
It is here used a selection principle (such as forward induction, or precedence, or payoff dominance): “It says that players do not pick strategies that result in certain losses for themselves, if other (equilibrium) strategies are available. People only pick (and expect others to pick) strategies that *might* result in a gain. ... The loss-avoidance principle is a game-theoretic cousin of findings from research on individual choice that highlight the psychological differences between gains and losses. ... [it] guides players’ beliefs about the behavior of others.” (CC p. 167)

Results?
[See Figure 1, Table V, Figure 5]

Indefinitely (infinitely) repeated games

[Drawing on Kreps (both), Binmore, Osborne/Rubinstein, and also MWG 12D and 12App]

The main idea:

If a game is played repeatedly then the mutually desirable outcome (Nash equilibrium) *may* differ from that of the "stage game".

The classic examples:

	1,1	-1,2		1,1	0,0		played once
PDGs such as			or				played finitely often
	2,-1	0,0		2,-1	0,0		played indef. often

If played once, the outcome in the lower right corner is the likely outcome under the standard assumptions of deductive game theory.
If played finitely often, then by Proposition 9.B.4, the outcome in the lower right corner is again the likely outcome under standard assumptions of deductive game theory.

Let's take the 1sPDG and bestow some context on it (Simon 1951)

> P must decide whether to accept employment with A.
> A then decides whether to exploit P or not.
> P does not like to be exploited and will work for A only if A does not exploit her.
> However, when P accepts employment, then it's in A's interest to exploit P. P – by backward induction correctly anticipating this – will decline the contract, making both worse off than the could have been otherwise. At least that's the game-theoretic story if the game is played once, or finitely often.

Solutions:

Commitment devices
Third-party enforcement

Reputational enforcement (indefinitely)

Imagine that A now plays this game not just against one P but many P's indexed P_1, P_2, ...

> Each P_n is only interest in the payoff from his interaction for A. For A, however, an outcome is now a sequence of results -- what happens with P_1, what happens with P_2, etc. ... – and hence the sum of payoffs u_1, u_2, etc., duly discounted: $u_1 + *u_2, + *^2 u_3 + ...$ where * , (0,1). Crucially, prospective employee P_n, when deciding whether to take employment is aware of A's history of treatment of workers.

This set-up changes the decision problem for A who now has to choose the payoffs of exploiting and not exploiting. Whether the former payoff-dominates the latter is a function of the strategies being used.

For example, let's assume that all workers have the following decision rule, "Never seek employment with an employer who in the past exploited a worker", then the employer would face a "grim" or "trigger" strategy. (This is nothing but what MWG, p. 401, call the "Nash reversion strategy: Firms cooperate until someone deviates, and any deviation triggers a permanent retaliation in which both firms thereafter set their prices equal to cost, the one-period Nash strategy.")

In terms of our example, the decision problem of the employer becomes $1 + *1 + *^2 + ... > 2 + 0 + 0 + ...$ which holds if * > 1/2. So, for all * > ½, the "grim" or "trigger" strategy constitutes a subgame perfect equilibrium in the PA game as parameterized above.

(Compare to equations 12.D.1 and 12.D.2: note that the above result is nothing but Proposition 12.D.1 for the 1sPDG. Proposition 12.D.1 deals with the 2sPDG case of the indefinitely repeated Bertrand duopoly game. Proposition tells us what happens if the condition that * > ½ does not hold.)

What's basic economic story drives these results (and the resultant subgame perfect equilibria?)

Obviously, this result also applies for A repeatedly interacting with the same P ... .

A cool result here (which is highly applicable to experimental work): Whenever two people interact in that kind of scenario and you watch them for just one instance, you might think they are really nice, altruistic, what not, when in fact they are just self-interested utility maximizers (!)

In Binmore's words, "We like to tell ourselves that it is from the goodness of our hearts that we help our neighbors. It is no accident, however, that what our hearts commend often turns out to coincide with what our heads would command if given the chance." (Fun & Games, p. 348)

To repeat, everything said above is true not just for the 1sPDG (and hence all kinds of reputation games including self-command games, e.g., Meardon & Ortmann1996a,b), but also for 2sPDGs, Bertrand or Cournot games, etc. (Ortmann & Meardon 1995).

Please note that there is a very useful paragraph on the top of page 403 MWG that points out that the discount factor can be interpreted in several ways (e.g., it could denote the probability of some interaction coming to an end, etc.)

Problems with the argument we just went through!

(1) A has to always have a reputation to protect, hence this game should have an infinite tree. Plus the value of the short-term deviation should never be higher than the PV of the future payoffs. If the game tree is finite, our reputation construction might collapse. Dito, as we noted, if future payoffs are heavily discounted.

(2) The equilibrium we have identified is but one of many, many, many. Another one is ... ? [The Nash equilibrium of the stage game - see Proposition 9.B.4. for the basic argument in finite move games.]]

(3) Our reputation construction depends crucially on each P_n seeing precisely how A has behaved previously.

(4) Life is more complicated. It's not at all clear that a manager has the same interest that the firm has in protecting the firm's reputation, although it's all a question of optimal mechanism design (see Ortmann & Squire, JEBO 2000)

(5) The world is not as stationary as we pretend here.

Of all these problems (2) is arguably the most important one. Or, dependent on your view, disturbing one (See section 8.2. in Osborne & Rubinstein).

The possibility of not just many, but infinitely many equilibria (that fulfill individual rationality) in the PDG, or for that matter all finite games, is known as folk theorem. See MWG p. 404:

"Any feasible discounted payoffs that give each player, on a per-period basis, more than the lowest payoff that he could guarantee himself in a single play of the simultaneous-move component game can be sustained as the payoffs of a SPNE if players discount the future to a sufficiently small degree."

More on the folk theorem:

The study of indefinitely repeated games is very demanding technically, hence let's simplify discussions:
first, let's restrict attention to strategies that can be represented by finite automata (for otherwise strategy sets are way too large & complicated)
second, let's assume no discounting (for otherwise might have situations of strategic interaction with finite horizon)
third, let's restrict players to use of pure strategies (for otherwise strategy sets even more complicated, also would have monitoring problems).

Specifically, let's restrict attention to games that can be represented by finite automata = computing machines with finite numbers of possible states. Think of them as algorithms that people have to design and have to deliver before they play.

Technically, we are dealing with Moore machines which when stimulated by an appropriate input respond with some output. The machine chosen by P_1 will have P_2's stage game outputs as its possible inputs; it responds to what P_2 does at the nth stage by chosing an action at the (n+1)th stage. And vice versa.

[Illustration: Hawk, Dove, Grim, Tit-for-Tat, ... ;
circles represent possible states of the world, letter written inside of circle is output that machine offers; arrows indicate transitions; the arrow coming from nowhere indicates the machine's initial state.]

One can now study the set of possible pairings (e.g., {Hawk, Hawk}, {Grim, Grim}, {Tit-for-Tat, Tit-for-Tat}) of such algorithms and construct a restricted strategic form for <u>average</u> payoffs of such interactions. Alternatively, we can illustrate the result pictorially ....

[Illustrate: Cooperative payoff region, ... and {Hawk, Hawk}, {Dove, Dove}, {Grim, Grim}, {Tit-for-Tat, Tit-for-Tat} , ...]

[Binmore pp. 370 - 376 gives a nice discussion of a somewhat more technical nature for finite general games that introduces strategy combinations that lead to cycles.]

Remark 1:

$m^+ = \min_{t,T}\{\max_{s,S} B(s,t)\}$ = min value {max value for all rows, for each column}
$m^- = \max_{s,S}\{\min_{t,T} B(s,t)\}$ = max value {min value for all columns, for each row}

Remark 2:
The minimax point m^+ of the 2sPDG is {2,2} with payoffs (0,0) - see notes above.

Note:
G* is an indefinitely repeated game whose stage game is G.

Lemma (8.4.1, Binmore 371):
Any outcome of G* is necessarily a point of the cooperative payoff region of the one-shot game G.

Lemma (8.4.2, Binmore 373):
Any Nash equilibrium of G* assigns each player at least his or her minimax value in the one-shot game G. (This is an individual rationality requirement. Note that it can reduce the cooperative payoff region.)

Folk Theorem (slightly more formal):
Let X be the cooperative payoff region of a finite one-shot game G, and let m^+ be its minimax point. Then the outcomes corresponding to Nash equilibria in pure strategies of the game G* (the indefinitely repeated stage game) are dense in the set

$$Y = \{x: x \in X \text{ and } x \geq m^+\}.$$

Remark 1: The rational numbers are dense in the set R of all real numbers in the sense that each real number can be approximated arbitrarily close by a rational number. In other words, there are many, many, many Nash equilibria out there.

Remark 2:

For the 2sPDG the minimax is equal to the maximin, and it is the payoff that someone can minimally guarantee one-self (and that one can restrict someone else to) through the appropriate choice of strategy, for our parametrization it is {2,2} = {Hawk, Hawk}, with payoff (0,0). (Von Neumann's minimax theorem states that $m^+ = m^-$
but this assumes that mixed strategies are allowed.)

Remark 3:
Intuitively, what's going on is this.

There are some clear-cut pure-strategy pairs such as {Hawk, Hawk}, {Grim, Grim}, {Dumpty, Humpty}, {Humpty, Dumpty} that span that part of the cooperative payoff region of the stage game that is individually rational. Then all convex combinations of such pairs clearly also lie in that part of the cooperative payoff region whose "threat point" is the Nash equilibrium of the stage game (which is always a Nash equilibrium of the repeated game, whether repetition is finite or infinite.)

Remark 4:
The Nash equilibria identified do not have to be subgame perfect (but a version of the folk theorem holds with Nash equilibria replaced by subgame-perfect equilibria.)