Growth of Electoral Fraud in Non-Democracies: The Role of Uncertainty

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Abstract

Electoral fraud has become an integral part of electoral competition both in established democracies and less-than-democratic regimes. In this paper I study electoral fraud in the non-democratic setting. First, I present evidence of fraud sustainability and growth over the lifetime of non-democratic regimes in post-Soviet and Sub-Saharan countries. Second, I provide a formal model that rationalizes the observed tendency and explains how uncertainty can lead to growing fraud. Specifically, in a probabilistic voting model of electoral competition with falsifications, a corrupt incumbent faces two types of uncertainty: uncertainty about voters' attitude towards fraud and uncertainty about his true support, captured by a purely random component in the voters' utility over candidates. The model predicts that when uncertainty is sufficiently large, higher uncertainty about voters' fraud intolerance provides weaker incentives to commit fraud. Over time the incumbent becomes more certain about voters' reaction to fraud because of learning through Bayesian updating and, thus, as the deterrent role of fraud intolerance uncertainty declines, the incentives to commit fraud become stronger, providing a growing fraud profile.

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1 Introduction

Fair elections are fundamental to democracy. Over the last decades researchers mainly assumed that elections are well-functioning tools for converting public preferences into social choice. However, in reality, cases of manipulating electoral outcomes are quite widespread[1] even in established democracies. In less-than-democratic regimes, strategies to shape electoral results through political pressure and especially electoral fraud[2] are an integral part of political competition. International organizations exert tremendous effort to ensure transparency in elections. However, electoral fraud in non-democracies[3] seems to be not only persistent, but expanding.

All the main findings on electoral fraud are derived from the analysis of particular elections in a given country at a given moment (Lehoucq, 2003), while the issues of fraud dynamics get limited attention in academic literature. This is surprising, because studying the evolution of fraud seems to be extremely important from many different perspectives. Comprehensively studying political regimes, designing effective electoral legislation and, especially, assessing the effectiveness of electoral monitoring are much harder to do without an understanding of fraud dynamics. To assess the effect of an electoral reform on the integrity of elections or to study the role of international monitoring in improving electoral transparency, one has to understand how the electoral environment changes over time and what are the sources of those changes. For such purposes, studying fraud dynamics is crucial.

One reason why electoral fraud suffers from a relative lack of attention in the academic literature is the absence of a reliable measure of fraud. The inability to measure fraud in a consistent way precludes implementing reliable empirical research on fraud dynamics, which in turn discourages efforts towards a theoretical study of electoral fraud, as it is hard to test any theory in this field. As a result, a set of important questions including the question of increasing fraud as well as reasons and conditions for fraud occurrence still call for an explanation. This paper is intended to partially fill this gap by studying the role of uncertainty in fraud dynamics.

The contribution of the paper is twofold. Firstly, I discuss evidence suggesting a tendency of increasing electoral fraud in Post-Soviet and Sub-Saharan non-democratic countries. Secondly, I present a game-theoretical probabilistic voting model with fraud which rationalizes the observed tendency, suggesting evolution of uncertainty about voters' attitude towards fraud as a potential explanation for growing fraud. In particular, a model of electoral competition with falsifications explicitly distinguishes between two types of uncertainty that affect electoral outcomes: aggregate

[1]See the Literature Review section for references.

[2]Following Lehoucq (2003), I define fraud as any illegal act committed with the intent to shape an electoral result.

[3]Hereafter, by non-democracy I mean a country, which has been widely criticized for deviating from the principles of democracy although it has formal democratic institutions such as elections. Specifically, I use this term for the countries marked as Not Free or Partially Free in the Freedom House Index of Democracy.

uncertainty about true support, captured by a purely random component in voters' utility over candidates, and the incumbent's uncertainty about voters' fraud intolerance, represented by his subjective beliefs about the attitude of the voters towards fraud. These two uncertainties prevent the incumbent's learning about voters' fraud tolerance immediately after the first elections, providing him with a noisy signal about voters' true attitude towards fraud, which is used for Bayesian updating of the incumbent's beliefs. The model predicts that when there is large initial uncertainty on the incumbent's side, uncertainty about voters' attitude towards fraud negatively affects incentives to commit fraud. Over time this uncertainty decreases because of learning and, thus, provides the incumbent with greater incentives to commit fraud. One can doubt the significance of uncertainty in explaining increasing electoral fraud, suggesting a number of obvious reasons for the observed tendency like growing stakes of re-election and decreasing costs of fraud because of learning by doing (see for instance Simpser, 2008). However, costs and stakes determine the level of committed fraud only if there is uncertainty about the outcome of the elections. In this paper I focus on the pure effect of uncertainty and show that it can also provide incentives for increasing fraud. Furthermore, in contrast to conventional wisdom (e.g., Simpser, 2008) I demonstrate that uncertainty does not always increase the incentives to commit fraud, and the direction of the effect depends on the nature of the uncertainty.

The paper is organized as follows. In the next section I review the existing literature on the issues of electoral manipulations and, particularly, electoral fraud. I then discuss problems of measuring electoral fraud and provide some evidence from Post-Soviet and Sub-Saharan countries, suggesting that electoral fraud has been growing over time. Further, I develop a formal game-theoretic dynamic model of elections with falsifications and show how uncertainty could lead to increasing fraud.

2 Literature Review

Classic theories of electoral competition suggest that candidates can influence their chances of being elected only by choosing their policies. Nevertheless, in reality elections are often associated with a variety of not always legal activities that result in redistribution of votes in favor of one or another candidate. A wide stream of both theoretical and empirical literature focuses on different strategies that incumbents can use for influencing electoral outcomes. For instance, Glaeser and Schleifer (2005) show how an incumbent can engage in redistributive politics in order to shape the electorate in his favor - the so-called Curley effect. Further, a number of studies analyze political budget cycles when incumbents increase public expenditures or change their composition towards more visible goods in pre-election periods in order to attract votes, which is empirically documented by Akhmedov and Zhuravskaya (2004), Shi and Svensson (2006) and Guo (2009), for developing countries and, for instance, by Veiga and Veiga (2007) as well as Schneider (2010) for the developed ones. Political budget cycles are also widely studied theoretically starting from Rogoff (1990) and continued by, for example, Martinez (2009) who explicitly models how politicians can change their policies to improve reputation and, thus, increase chances for re-election.

Electoral fraud could be considered as another type of outcome-influencing strategies, widespread under autocracies and dictatorships where fraud tools are easily available in comparison to pure democracies. Chaturverdi (2005) and more recently Collier and Vicente (2010) study pre-election violence as an instrument for shaping electoral results through deterring opposition supporters from voting. To determine when fraud occurs and what should be done to prevent it, Sutter (2003) presents a simple model of rigged elections where the society decides how closely to monitor the elections, suggesting the provision of neutral observers, strengthening the protest option when fraud is detected, and reducing costs of monitoring by, for example, subsidization, as effective fraud prevention tools. However, though all the papers have focus on shaping electoral results through electoral fraud, their purposes, ways of modeling as well as the underlying assumptions do not allow for using them as a framework for studying fraud dynamics.

Simpser (2005) makes an exceptional attempt to study electoral fraud in a dynamic setting. He points out that rigged elections are often associated with very high victory margins implying that incumbents often irrationally commit excessive fraud. The author attempts to rationalize such behavior by formalizing the idea that excessive fraud can, first, deter future opposition coordination and turnout and, second, directly influence the beliefs of opposition supporters that elections will be corrupt and thus prevent their turnout. In a later paper, Simpser (2008) elaborates on this idea and comes up with a model that generates equilibrium with persistent but not growing excessive fraud. Also, the author briefly discusses the potential role of exogenous uncertainty, costs and stakes in his explanation of excessive fraud but concludes that these features cannot sufficiently explain the high victory margins observed in the data. However, the equilibrium outcome and the latter conclusion come from strict underlying assumptions, particularly, the assumption that opposition supporters, in the case of the incumbent's victory, get more utility when they abstain from voting than when they do vote (i.e. opposition supporters but not incumbent's supporters are discouraged from participating in the elections if the incumbent is very likely to win, which is a disputable assumption).

To summarize, the existing literature on electoral fraud cannot satisfactorily explain the observed patterns in the behavior of corrupt incumbents in non-democracies that I have documented in the next section. In particular, questions of sustainability and growth of electoral fraud over non-democratic regimes' lifetimes as well as reasons and conditions for fraud occurrence still call for an explanation. This paper presents a model that tries to partially fill the gap, theoretically studying how uncertainty can affect incentives to commit fraud in non-democratic setting and how it can contribute to explaining the increasing fraud profile.

3 Dynamics of Electoral Fraud in Non-Democracies

For the analysis of electoral fraud dynamics, two sets of countries with non-democratic regimes are used. When refer to a regime I mean a period in a country's history when there was either a single leader or several leaders from the same party or family whose ruling methods are considered to be less than democratic. Specifically, I focus on regimes that existed during the last 25 years, between 1988 and 2013, that have formal democratic institutions like direct presidential elections, that were rated as Not Free or Partially Free in the Freedom House Democracy Index (FHDI) for at least 2 years within their lifetimes, and that have survived for at least two terms.

The first country set consists of the Post-Soviet countries excluding Lithuania, Latvia and Estonia - democracies that entered the EU in 2003 - and Moldova, where there has been no clear regime since independence and, moreover, there have been no direct presidential elections between 1996 and 2012. Turkmenistan was excluded from the set as a country without direct presidential elections throughout its whole history. In the end, the first set of countries includes Armenia, Azerbaijan, Belarus, Georgia, Kazakhstan, Kyrgyzstan, Russia, Tajikistan, Ukraine and Uzbekistan. Two out of these countries, Georgia and Russia, are characterized by two regimes each. However, the first Russian regime (Yeltsin 1992-2000) is not considered in the further analysis since it is characterized by only one presidential elections (1996). Finally, the data set includes 11 non-democratic post-Soviet regimes in 10 countries (See Table 3 of the Appendix for the detailed list).

The second set consists of Sub-Saharan African countries that adopted elections in the early 1990s. From the total of forty nine countries of the region, eight were excluded as fully free and democratic countries (i.e., rated as Free in FHDI[4] in 1991-2013 with the exception of, at most, 1 year: Benin, Botswana, Cape Verde, Mali, Mauritius, Namibia, Sao Tome and Principe and South Africa). An additional two were excluded due to too few multiparty elections (less than two or with a gap of more than 10 years) since 1990 (Burundi, Eritrea). Five more were excluded that did not elect presidents by direct population vote (Angola, Ethiopia, Lesotho, Somalia and Swaziland), and three more countries were excluded that have not had a clear regime that has survived for at least two terms between 1990 and 2013 (Comoros, Guinea-Bissau and South Sudan). Finally, the set contains 31 Sub-Saharan countries, among which three (Central African Republic, Kenya and Senegal) are represented by two distinct regimes. This gives us 34

[4] Freedom House Democracy Index, www.freedomhouse.org/report-types/freedom-world (retrieved 20.03.2013)

non-democratic African regimes (See Table 4 of the Appendix for the entire list).

To credibly analyze the evolution of electoral fraud, one needs to have some objective measure of fraud. Literature (see, for example. Lehoucq, 2003) suggests several types of sources that can provide valuable information for building such a measure: press, opposition parties' archives with official acquisitions on fraud, complaints submitted to courts, scientific surveys and interviews with voters, and results of international electoral monitoring. However, all of these sources, except probably the last one, could be biased towards one or another candidate, and, thus, cannot provide fully objective information about fraud. In addition to the fact that comprehensive analysis of such data and collecting relevant information require tremendous effort, the partisan nature of the sources limits their usefulness in measuring fraud. Results of monitoring by electoral observation missions are also limited in their ability to provide useful information on dynamics of electoral misconduct: public reports mainly contain qualitative rather than quantitative information; monitoring techniques change over time; and the objectivity of the conclusions are often questionable.

Given that electoral fraud is a phenomenon which is hard to measure directly, the only way to assess fraud dynamics is to explore some indirect evidence. I analyze several data sources to obtain some evidence on the dynamics of electoral fraud in the countries of interest: empirical studies on electoral fraud, variables that might be strongly correlated with fraud such as political and press freedom, and election databases that contain some fraud proxies comparable across different elections. The analysis of all these sources provides a consistent observation: electoral fraud tends to grow over the lifetimes of non-democratic regimes.

I first explore academic empirical literature on fraudulent elections which would be an ideal source of evidence on fraud dynamics. Though empirical literature on electoral fraud dynamics is limited, there are a few papers that try to compare irregularities in consecutive elections in selected countries. Myagkov and Ordeshook (2008) suggest a statistical methodology, based on the analysis of the distribution of turnouts over different regions, and apply it to the official data for Russian federal elections between 1993 and 2007 to uncover electoral fraud. They find that ballot stuffing and some other forms of fraud in the mid-1990s were mentioned only in a few ethnic Russian regions but then spread to the other regions, both urban and rural, with noticeable acceleration during Putin's administration (2000-2008). The authors stress that once fraud occurs it becomes sustainable: they find that if fraud infected a precinct in Moscow for the first time in 2004 presidential elections, it is very likely to reoccur there in the 2007 parliamentary elections. An important finding of the paper is that the level of electoral fraud in Russian federal elections has been both sustainable and growing with the maturing of Putin's regime.

Increasing fraud dynamics in Russia after 2000 is also discussed by Treisman (2009) who comprehensively reviews the trends in voting in Russia since 1991. In a chapter devoted to electoral manipulations and fraud, by studying a variety of Russian electoral statistics the author finds that in the early 1990s the elections in Russia were almost clean, while since 2000 the electoral irregularities have become an integral part of electoral competition. Though manipulations were not serious enough to alter the outcomes, the fraud growth was noticeable. These finding are also supported by Mebane and Kalinin (2009), who explore data on Russian federal elections for 2003-2008, looking for deviations from Benford's Law[5] as well as other statistical anomalies that are likely to arise due to fraud. The results show unambiguously growth in electoral fraud in the 2000s: anomalies the methods detect are worse by the end of the period under study than at the beginning[6].

Using a methodology similar to Myagkov and Ordeshook (2008) and Mebane and Kalinin (2009), Levin et al (2009) analyze electoral data in Venezuela and find some evidence of increasing fraud over time. They analyze the data on two consecutive state-level referenda in 2007 and 2009 and, assuming constant voters' preferences, discover unusual patterns in voting behavior of selected regions that mainly benefit the incumbent. Specifically, the main finding of the paper is that most of the new votes in favor of Chavez in the 2009 referendum came from the regions with large abstention in 2007. Though this result cannot serve as strong evidence of fraud, it is more likely to be observed if fraud had really expanded between 2007 and 2009.

The main advantage of the discussed methodology is that it allows for the detection of electoral fraud based just on official election data. However, two main problems have to be mentioned. First, it is difficult (even impossible for the majority of African elections in the 1990s) to obtain such detailed data for all elections of interest. Second, electoral fraud is a comprehensive process, while the method detects mainly ballot stuffing and, thus, may underestimate the magnitude of fraud. Thus, one needs to adjust the evaluation of technical fraud by some measure of pre-election activity that directly affects election results. The main part of such activity consists of, for instance, controlling the media and pressuring the opposition (Schedler, 2002, Enikopolov et al, 2009). Hence, indexes of media and political freedom could be used as a proxy for pre-election manipulations.

First, I look at the dynamics of the Freedom House's Media Freedom Index[7], probably the most comprehensive cross country data set on press freedom. The index is an annual survey of media independence, which contains the assessment of the degree of print, broadcast, and internet freedom in practically every country in the world. It provides numerical rankings which reflect the legal environment for the media, political pressures that influence reporting, and economic factors that affect access to information in a particular country. The index is constructed annually

[5]The Benford's Law states that in large lists of real-life data, digits are distributed in a specific, non-uniform way.

[6]Mebane and Kalinin (2009), page 11.

[7]www.freedomhouse.org/report-types/freedom-press (retrieved 20.03.2013)

from 1980 to the present, using approximately the same methodology during the entire span, which allows for cross country and time comparisons. I use the index for the countries of interest starting from 1994 (the first year for which the index is available for all the countries of interest) or the year of the first multiparty elections in a country, whichever number is higher, till 2013. The results are summarized in Table 5 and Table 6 of Appendix A. Columns "Press Freedom" contain coefficients and their standard errors in a regression of the index value on time for each regime. Since higher index values correspond to lower press freedom, a positive coefficient implies decreasing of press freedom over time. Generally, the results show that over time politicians in the countries of interest put more pressure on media: 8 post-Soviet regimes out of 11 demonstrate a statistically significant positive time trend, while only 2 (Georgia and Tajikistan) shows clear improvement in press freedom. In 15 out of 34 African regimes, press freedom has significantly declined over time, 11 countries demonstrate no significant changes, and only 8 show some improvement.

Further, political freedom could also signal fraud-related pre-election activity, reflecting the transparency of the political environment in a given country. Another popular index, Freedom House Political Rights Index [8], captures the assessment of global political rights at country level. I use the index for 1994-2013 to analyze time trends in political freedom in the countries of interests. The results are summarized in Table 5 and Table 6 of Appendix A. The evolution of political freedom in the countries of interest, shows a picture similar to the case of press freedom. Of the Post-Soviet regimes, 7 out of 11 demonstrate significant growth of the index over time, which corresponds to a decrease in political freedom, 3 (Azerbaijan and 2 Georgian regimes) have no significant changes and in just 1 (Tajikistan) there is an improvement in political freedom overtime. In Africa, for 12 regimes the index grows significantly, for another 15 the trend is not clear, and only 7 show improving political freedom.

The World Bank Database of Political Institutions (Beck et al, 2001) is a popular regularly updated[9] source of data (1975-2009) on political systems and elections around the World. The database contains a dummy variable fraud that captures extra-constitutional electoral irregularities and equals to 0 only if elections are considered to be fair and 1 otherwise. Though the variable is just a dummy that does not allow for measuring the magnitude of fraud, and there are some questions how the variable is constructed (for instance, all the elections in Russia and Zimbabwe as well as in Uzbekistan since 2005 are marked as not fraudulent), one can get some inference about the time trends in electoral fraud by looking for a switch from not fraudulent elections to rigged ones within a regime's lifetime. The results of the analysis of the DPI dataset

[8] www.freedomhouse.org/report-types/freedom-world, Report "Country Ratings and Status by Region, FIW 1973-2013" (retrieved 20.03,2013)

[9] Last update of the DPI: April 2010.

are summarized in Table 5 and Table 6 of Appendix A: "+" sign means that a switch from not-fraudulent elections to fraudulent during a regime lifetime was observed, "–" sign stands for an opposite switch. Out of 45 regimes from both sets, a switch is observed for 15: in 11 countries there is a switch from not fraudulent to fraudulent, and only in 4 - vice versa.

Next, I examine another popular election data set, National Elections Across Democracy and Autocracy-NELDA (Hyde and Marinov, 2012), which provides detailed information on election events between 1960 and 2006. I analyze the dynamics of several fraud related variables contained in the data set. Specifically, for each elections there are three binary answers for the following three questions: "Before elections, are there significant concerns that elections will not be free and fair?", "Is there evidence that the government harassed the opposition?", and "Were there allegations by Western monitors of significant vote-fraud?". As in the case of the analysis of DPI data set, I am looking for switches between answers "no" and "yes" to these questions for elections within each regime of interest. Switch from "no" to "yes" to any of these questions implies clear decreasing in electoral integrity, from "yes" to "no" - an improvement, while the absence of the switches is inconclusive. Since the data spans till 2006 only, there are just 36 regimes for which at least two observations are available. The results of the analysis are presented at Table 7 of the Appendix A. The results show weak evidence of growing fraud. For the first question there are 8 switches from "no" to "yes" (marked with "+" in Table 7), 5 switches form "yes" to "no" (marked with "–") and 3 cases where both switches were observed ("both"). The other 20 regimes did not have switches. For the second question there are 9 "+" marks, 4 "–" marks and 3 regimes with both types of switches. For the third question there are 7 "no"-to-"yes" switches, 4 "yes"-to-"no" switches, 1 regime demonstrated switches of both types, and for 6 regimes data are not available.

Further, evolution of the victory margin[10] could also provide some information about fraud. For example, Simpser (2005, 2008) argues that that fraudulent elections are strongly associated with high victory margins: corrupt politicians tend to win elections with large advantage. Given this correlation, positive time trend in victory margins may be a signal of growing fraud. The number of presidential elections within the lifetime of a regime varies across countries of interest between 2 and 5. As a result, standard errors for time coefficients are relatively large even if the trend is obvious. Table 5 and Table 6 contains the results of the analysis[11]: out of 44 regimes [12], 27 demonstrate clear growth in the victory margin over time.

To summarize, electoral fraud is a difficult-to-measure phenomenon, and there is no ob-

[10]The victory margin is the difference between the shares of votes cast for the winner and the first runner-up.

[11]absence of standard error means two observations

[12]Several elections in the dataset were boycotted by opposition, and thus victory margins for these elections are irrelevant. One of such elections were held in Cote d'Ivoire, a regime that had only two presidential elections over its lifetime. As a result, there is only 1 data point for this regime

jective measure that reliably reflects the magnitude of electoral misconduct. Yet, there is some indirect evidence and measures of fraud-related phenomena that can give an inference on the evolution of electoral fraud over time. The analysis of such indirect evidence in non-democratic regimes in Africa and in Post-Soviet countries suggests that electoral fraud tends to grow over time in the majority of the cases. The following section presents a formal model of fraudulent elections to show how uncertainty can contribute to the observed trends.

4 The Model

4.1 General Setup

This section develops a game-theoretical model of political competition between a corrupt incumbent and a challenger. The incumbent faces a continuum of voters of measure one. Before the elections, the incumbent chooses the level of fraud. The incumbent derives utility only from remaining in office and thus, from his point of view, fraud is just an instrument to manipulate the probability of being re-elected. This eliminates the potential commitment problem typical for non-democracies (Acemoglu and Robinson (2006), Chapter 5) because the incumbent has no incentive to change his policy before the next election campaign as fraud does not affect the incumbent's utility. The level of chosen fraud corresponds to a unique number $f \in [0, 1]$ - the share of votes that the incumbent can add to his true support. Hereafter, without loss of generality, I assume that the amount of fraud in the model equals the increment in percentage of votes.

All the voters dislike fraud in the same way. Voter i has utility from fraud f

$$V_i(f) = -\beta f, \tag{1}$$

where $\beta \in [0, 1]$ is an intolerance parameter that captures voters' attitude towards fraud. Parameter β has a true value, which is, however, unknown to the incumbent. Yet, the incumbent has prior beliefs about its value: $\beta \sim N(\beta_1, \epsilon_1^2)$. Thus, the intolerance parameter β is subject to uncertainty, which I refer to as fraud intolerance uncertainty. Challenger has no option to commit fraud.

The elections are modeled in a modified version of the standard probabilistic voting framework (presented for the first time in Lindbeck and Weibull (1987), later used in Persson and Tabellini (2000) and more recently in, for example, Gregory et al (2011)), where voter i votes for the incumbent, who commits fraud f against the challenger, if

$$V_i(f) + \sigma_i + \delta > 0, \tag{2}$$

where σ_i is an individual-specific time-constant preference over the incumbent. Across all the voters, σ_i, which captures a relative ideological bias towards the incumbent, is distributed uniformly over $[-\frac{1}{2\phi}, \frac{1}{2\phi}]$. The distribution is common knowledge.

Uncertainty about voters' preferences, that I hereafter call aggregate electoral uncertainty, is captured by δ, a common for all the voters component, which represents a random preference for the incumbent shared by all voters and which is unknown to the candidates prior to election day. This component is the utility an individual derives from all the incumbent's policies other than fraud relative to all other policies of the challenger. Prior to elections the value of this component is drawn from a zero-mean normal distribution: $\delta \sim N(0, \psi^2)$, which is known to the incumbent.

If elected, the incumbent gets benefits from remaining in office. The benefits are normalized to 1. The direct costs of fraud are $c(f)$ such that $c(0) = 0$, $c'(0) = 0$, $c'(f) > 0$, $c''(f) > 0$.

Also, $c(1)$ and $c'(1)$ are assumed to be relatively large numbers to guarantee that falsifying 100% of the votes is extremely costly. The incumbent chooses the level of fraud to maximize his expected benefits.

The timing of the game is as follows:

1. The incumbent chooses the level of fraud $f \in [0, 1]$.

2. The voters anticipate f, the elections take place, the results are adjusted by the level of fraud and are announced, and the winner takes office.

3. The payoffs are realized.

Note that correct anticipation of the fraud level by voters is possible only under the assumption of no private information on the side of the incumbent. This means that to anticipate the level of fraud correctly, the voters must know preferences, costs and benefits of fraud as well as the incumbent's beliefs. If one considers this assumption to be too strict, it can be assumed that fraud is fully observable by voters. The latter assumption is not as strict as it seems from first sight because fraud, as discussed above, is a comprehensive process including controlling media and threatening the opposition that mainly takes place before elections, which is well observed by voters. With any of these two assumptions the following analysis is valid.

4.2 One Period Analysis

I start with the analysis of a one-period model. For any given level of fraud f, voter i votes for the incumbent if

$$V(f) + \sigma_i + \delta > 0, \tag{3}$$

$$-\beta f + \sigma_i + \delta > 0, \tag{4}$$

$$\sigma_i > \beta f - \delta. \tag{5}$$

Then, the probability that a randomly picked voter votes for the incumbent is

$$P(\sigma_i > \beta f - \delta) = 1 - P(\sigma_i \leq \beta f - \delta) = 1 - \frac{\beta f - \delta + \frac{1}{2\phi}}{1/\phi} = \frac{1}{2} - \phi(\beta f - \delta). \tag{6}$$

This is exactly equal to the true share of votes cast for the incumbent for a given realization of δ as there is a continuum of voters of measure 1:

$$\Pi_I = \frac{1}{2} - \phi(\beta f - \delta). \tag{7}$$

Note that if elections are clean, i.e., fraud is zero, in expectation each candidate gets exactly half of the votes.

Given the fraud level, the probability that the incumbent wins the elections under the majority rule is then

$$P_w = P(\Pi_I + f \geq \frac{1}{2}) = P\left(\frac{1}{2} - \phi(\beta f - \delta) + f \geq \frac{1}{2}\right) = P\left(\beta - \delta/f - 1/\phi \leq 0\right). \tag{8}$$

Denote $X = \beta - \delta/f - 1/\phi$. From the incumben's point of view X is a random variable which, given his priors about β and distribution of δ, follows $N\left(\beta_1 - \frac{1}{\phi}, \epsilon_1^2 + \frac{\psi^2}{f^2}\right)$.

Given the expected share of votes, the incumbent chooses the level of fraud f such that it maximizes his expected benefit:

$$\max_f P_w(f) - c(f). \tag{9}$$

The problem can be rewritten as:

$$\max_f G_X(0) - c(f), \tag{10}$$

where $G_X(z) = \frac{1}{\sqrt{2\pi s^2}} \int_{-\infty}^{z} e^{-\frac{(x-\mu)^2}{2s^2}} \mathrm{d}x$, $\mu = \beta_1 - \frac{1}{\phi}$, and $s^2 = \epsilon_1^2 + \frac{\psi^2}{f^2}$.

Proposition 1. *Solution f to maximization problem (10) satisfies:*

$$-\frac{\psi^2}{f^3}\frac{\mu}{s^2} g_X(0) - c'(f) = 0, \tag{11}$$

where $g_X(z) = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(z-\mu)^2}{2s^2}}$.
Proof: *See the Appendix.*

Proposition 2. *There is unique $f^* \in (0,1)$ that satisfies the first-order condition (11) if and only if $\mu = \beta_1 - \frac{1}{\phi} < 0$.*

Proof: *See the Appendix.*

To understand the condition for uniqueness, recall that the incumbent's expected vote share is $E\Pi_I = E(\frac{1}{2} - \phi(\beta f - \delta) + f) = \frac{1}{2} + f(1 - \phi\beta_1)$, which is an increasing function of fraud if and only if $\phi\beta_1 < 1$ or $\beta_1 - 1/\phi < 0$, otherwise committing fraud does not make sense because it hurts the incumbent in terms of votes. This can happen when people strongly dislike fraud (e.g., β is relatively high), or when there is little heterogeneity among the electorate in terms of ideology (ϕ is high). In the latter case, by committing fraud, which is disliked by everyone, the incumbent loses a relatively large number of his ideological supporters (those with $\sigma_i > 0$) as he is ideologically too close to the challenger who does not commit any fraud. Thus, the condition $\beta_1 - 1/\phi < 0$ guarantees that in expectation committing fraud makes sense, i.e., it provides incumbent with a higher official vote share than he would get without committing fraud.

The second condition (sufficiently large uncertainty) guarantees that the left-hand side of the first-order condition (11) intersects the marginal cost function at a point between 0 and 1. It is not binding for any reasonable parameter values mainly because marginal cost at $f = 1$ is a relatively large number under the assumption that stealing 100% of the votes is extremely costly.

4.3 Multi-Period Setup

onsider a multi-period setup where there is a sequence of elections. In the end of period 1 the incumbent observes his vote share:

$$v_1 = \frac{1}{2} - \phi(\beta f_1 - \delta_1) + f_1, \tag{12}$$

where f_1 is the first-period fraud. The incumbent knows the exact values of ϕ and f_1, but does not know β and δ_1, and thus cannot decompose the observed value $\beta f_1 - \delta_1$. This value may be rewritten as $m = \beta - \frac{\delta_1}{f_1}$. Observing his announced vote share (12) by the incumbent is equivalent to observing m. Since δ is distributed with zero mean, m can be interpreted as an unbiased signal about true value of β, which is then used for Bayesian updating of the prior beliefs.

Because δ is drawn from normal distribution $N(0, \psi^2)$, signal m is also distributed normally: $m \sim N(\beta, \frac{\psi^2}{f_1^2})$. Given the distribution of the signal and priors $\beta \sim N(\beta_1, \epsilon_1^2)$, the posterior beliefs about β are:

$$\beta | m \sim N\left(\frac{\beta_1\psi^2 + \epsilon_1^2 f_1^2 m}{\psi^2 + \epsilon_1^2 f_1^2}, \frac{\epsilon_1^2\psi^2}{\psi^2 + \epsilon_1^2 f_1^2}\right). \tag{13}$$

It is important to note that fraud in the first period affects the beliefs about β in the second period. Specifically, the variance of the beliefs is lower if the first period fraud is higher. Also, note that if there is no fraud in the first period, the beliefs about β do not change: when there is no fraud, there is no way to learn anything about voters' response to it.

Given the updated beliefs, in period 2 the incumbent solves

$$\max_{f_2} G_Y(0) - c(f_2), \tag{14}$$

where $Y = \beta - \frac{\delta_2}{f_2} - \frac{1}{\phi}$, $G_Y(z) = \frac{1}{\sqrt{2\pi s_2^2}} \int_{-\infty}^{z} e^{-\frac{(x-\mu_2)^2}{2s_2^2}} \mathrm{d}x$, $\mu_2 = \frac{\beta_1 \psi^2 + \epsilon_1^2 f_1^2 m}{\psi^2 + \epsilon_1^2 f_1^2} - \frac{1}{\phi}$, and $s_2^2 = \frac{\epsilon^2 \psi^2}{\psi^2 + \epsilon^2 f_1^2} + \frac{\psi^2}{f_2^2}$.

Note that δ_2 is again drawn from the same commonly known distribution $N(0, \psi^2)$ independently from the first-period draw. Further, I will explore the case when δ follows a random walk and show that the results do not substantially differ from the case with independent draws. However, random walk generates an additional effect (to be discussed further) that can contribute to a growing fraud profile and cannot be distinguished from the uncertainty effect. The independent draw assumption eliminates this effect, allowing me to study purely the role of uncertainty in fraud dynamics.

Similar to the the first-period case (see Proposition 1), the second period first-order condition takes the following form:

$$-\frac{\psi^2}{f_2^3} \frac{\mu_2}{s_2^2} g_Y(0) - c'(f_2) = 0, \tag{15}$$

where $g_Y(z) = \frac{1}{\sqrt{\pi s_2^2}} e^{-\frac{(z-\mu_2)^2}{s_2^2}}$.

To understand the conditions under which growing fraud occurs note that the only components of the maximization problem that are different between two periods are mean and variance of the subjective distribution of β. Recall, that from the incumbent's point of view in period 1 $\beta \sim N(\beta_1, \epsilon_1^2)$, and in period 2, given a signal $\beta | m \sim N(\beta_2, \epsilon_2^2) = N\left(\frac{\beta_1 \psi^2 + \epsilon_1^2 f_1^2 m}{\psi^2 + \epsilon_1^2 f_1^2}, \frac{\epsilon_1^2 \psi^2}{\psi^2 + \epsilon_1^2 f_1^2}\right)$.

Thus, beliefs are the only thing that determines the difference in optimal choices of fraud in two different periods. For clarity, I separate the effect of changes in beliefs on the mean effect and the variance effect, i.e., changes in optimal fraud between two periods in response to changes in the mean and variance of beliefs, respectively. Further note that variance of the beliefs in the second period is always lower for any $f_1 > 0$: $\epsilon_2^2 = \frac{\epsilon_1^2 \psi^2}{\psi^2 + \epsilon_1^2 f_1^2} < \frac{\epsilon_1^2}{1 + \epsilon_1^2 f_1^2 / \psi^2} < \epsilon_1^2$. The following proposition states that this decrease leads to higher optimal level of fraud.

Proposition 3. *For some prior beliefs about true value of β distributed according to $N(\beta_0, \epsilon_0^2)$ optimal fraud is decreasing in ϵ_0^2 if $\epsilon_0^2 + \psi^2 > (1/\phi - \beta_0)^{2/3}$.*
Proof: *See the Appendix.*

Proposition 3 says that when there is sufficiently large uncertainty, no matter of what type, on the incumbent's side an increase in fraud intolerance uncertainty leads to lower equilibrium level of fraud. Note that there are two opposite effects of uncertainty on committing fraud. On the one hand, the incumbent is afraid of committing too much fraud when he has doubts on how voters react to falsification as the intolerance parameter could easily appear to be high enough to make fraud hurting instead of benefiting (see Proposition 1). On the other hand, higher uncertainty implies that investment in fraud becomes less efficient. According to Proposition 3, when uncertainty is relatively high the first effect dominates. Finally, note that the condition is sufficient, meaning that the optimal fraud is decreasing in subjective uncertainty under even less strict circumstances.

Thus, more precise beliefs under sufficiently large uncertainty provide incentives to commit higher fraud, implying that the variance effect pushes the optimal fraud up. The next question is the direction of the mean effect. To answer it one first needs to know how changes in the mean of beliefs affect the optimal fraud.

Proposition 4. *For some prior beliefs about true value of β distributed according to $N(\beta_0, \epsilon_0^2)$ optimal fraud is decreasing in β_0 if $\epsilon^2 + \psi^2 > (1/\phi - \beta_0)^2$.*
Proof: *See the Appendix.*

This result, when optimal fraud is not always decreasing in the expected voter's intolerance, could seem counterintuitive. To understand it note that higher fraud intolerance should decrease incentives to commit fraud because with higher value of β_0, keeping the variance ϵ^2 fixed, the probability that true β will appear to be high enough to make fraud electorally detrimental to the incumbent instead of beneficial (see Proposition 1) is now higher. But according to Proposition 3, increased uncertainty about fraud intolerance decreases incentives to commit fraud only when the uncertainty is sufficiently high. Thus, an increase in β_0 induces lower fraud only when uncertainty is relatively high.

The second period mean $\frac{\beta_0\psi^2+\epsilon^2 f_1^2 m_1}{\psi^2+\epsilon^2 f_1^2} - < \beta_0 > \beta_0$ if $m_1 > \beta_0$, and $\frac{\beta_0\psi^2+\epsilon^2 f_1^2 m_1}{\psi^2+\epsilon^2 f_1^2} - < \beta_0$ if $m_1 < \beta_0$. Thus, according to Proposition 4, if $m_1 < \beta_0$ and ϵ^2 is high enough, the mean effect pushes the optimal level of fraud up as well as the variance effect, resulting in unambiguously increasing fraud over two periods. If $m_1 > \beta_0$, the updated mean is higher than the prior mean, and in this case the mean and the variance effects affect optimal fraud in opposite ways, and the resulting direction depends on the values of the model parameters and the realized value of the signal. Specifically, the higher the signal, the more likely the mean effect dominates the variance effect, implying a decrease in fraud. Thus, there is a threshold value for signal m_1^* such that if $m_1 > m_1^*$ then the second period optimal fraud is lower than the first one; if $m_1 \leq m_1^*$ then the optimal fraud increases between the two periods and $m_1^* > \beta_0$.

Because m_1 is distributed symmetrically around the true β, realization of the signal is more likely to be below the threshold value m_1^* implying that it is more likely to observe increasing fraud rather than decreasing, if prior beliefs are unbiased ($\beta_0 = \beta$). The likelihood increases if the incumbent overestimates ($\beta_0 > \beta$).

The analysis could be easily extended to the case of a multiple period game under the assumption of a myopic incumbent. Here, myopia means that the incumbent does not invest in learning by strategic committing excessive fraud. Fully rational incumbent could have incentives to choose relatively more fraud in the first period bearing some extra risk and extra costs in exchange for faster learning the true value of β. However, the assumption of a fully rational incumbent seems exaggerated, taking into account some features of the real-life electoral environment, where, for example, the length of electoral cycles is rarely less than 4-6 years, which is probably too long to assume strategic fraud commitment.

The crucial thing for the results of the multi period analysis is the conditions stated in Propositions 3 and 4. The analysis is indeed valid only if the conditions hold over time. Note that both conditions require ϵ^2 and ψ^2 to be sufficiently large, ϵ^2 is decreasing over time due to learning and ψ^2 is constant over time. Thus, eventually the conditions break down. However, the higher ψ^2 and the initial value of ϵ^2, the later the break occurs, allowing the analysis to be valid for sufficiently large number of periods. The following section presents the results of the simulation of the multi-period model.

4.4 Simulation of the Multi-Period Model

The multi-period setup assumes T periods. At the beginning of a period t the incumbent solves

$$\max_{f_t} G_H(0) - c(f_t), \tag{16}$$

where $H = \beta - \frac{\delta_t}{f_t} - \frac{1}{\phi}$, $G_H(z) = \frac{1}{\sqrt{2\pi s^2}} \int_{-\infty}^{z} e^{-\frac{(x-\mu)^2}{2s^2}} \mathrm{d}x$, $\mu = \mu_t(m_{t-1}, f_{t-1} - 1/\phi)$, and $s^2 = s_t^2(m_{t-1}, f_{t-1} + \frac{\psi^2}{f_t^2})$, $_t$ and s_t^2 are mean and variance of the incumbent's beliefs about β at period t, which are both functions of the previous period fraud and signal.

To analyze the dynamics of fraud I simulate the multi-period model with a sequence of 6 periods. I repeat the sequence of elections 30 times, starting from the same parameter values. In each period of a sequence I solve the incumbent's problem given a quadratic cost function, resolve uncertainty by randomly drawing a value of δ_t from the specified distribution, and update the incumbent's beliefs. If the incumbent has lost elections, the game is over, otherwise the next period starts.

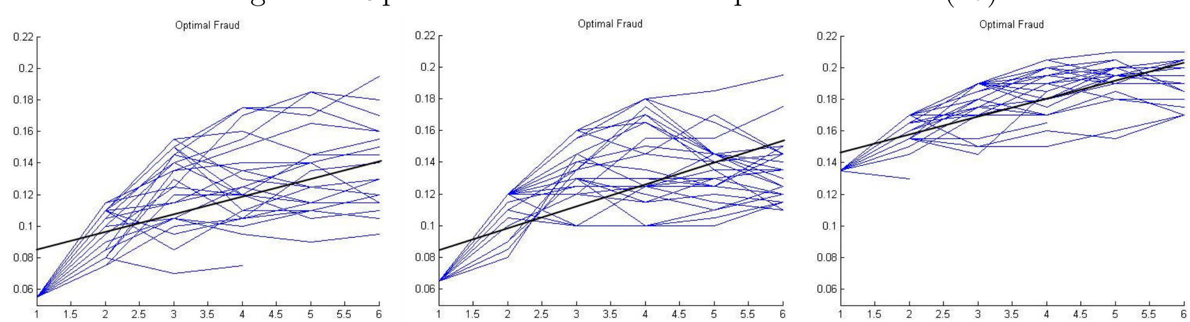
The benchmark model parameters are as follows: $\beta = 0.15$, $\epsilon = 0.05$, $\psi = 0.001$, $\phi = 6$. Prior beliefs are unbiased. The choice of the parameter values is not a result of calibration due to the obvious reasons discussed above. Instead, the parameters are chosen in a way that guarantees reasonable relationships between them. First, β and ϕ are set such that committing fraud makes sense (see Proposition 2). Second, ϵ is chosen such that it guarantees reasonable uncertainty of the value of β, allowing the incumbent to assign a relatively high probability to an outcome where the voters are fraud intolerant and high fraud becomes electorally detrimental to the incumbent (see Proposition 2). Finally, variance ψ of the purely random component δ is chosen such that it is relatively lower than ϵ, guarantying that a toss of a coin does not decide too much in the model and the incumbent does not lose very often because of bad luck.

To show that the exact values of the parameters do not exclusively determine the model predictions, I simulate the model for another two parameter sets, chosen in the same way as described above, in addition to the benchmark one. As a result, the model is simulated for three distinct sets:

$$(\beta, \epsilon, \psi, \phi) = (0.15, 0.05, 0.001, 6), (0.5, 0.4, 0.01, 1.5), (0.8, 0.25, 0.025, 1). \quad (17)$$

In the following figures each kinked line represents optimal fraud as a function of time.

Figure 1: Optimal fraud for different parameter sets (17).



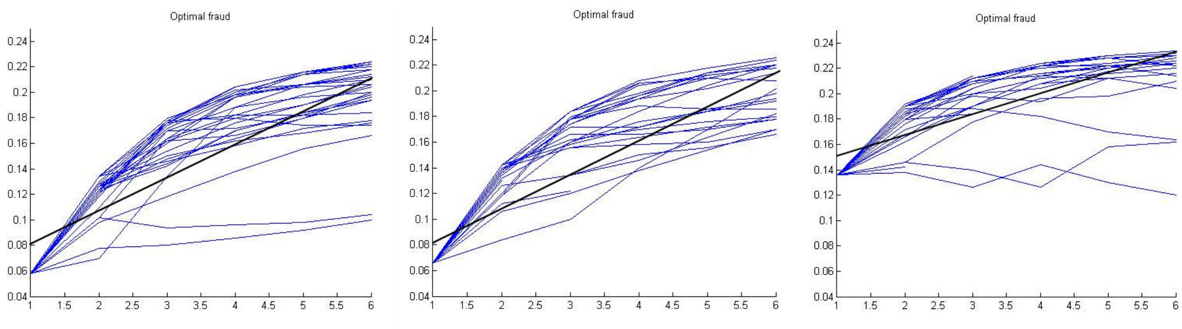
To document the significance of the growing trend I simulate the model with a sequence of 6 periods 1000 times for the parameter sets used above and run a simple regression of optimal fraud on time. The results show that in the model, fraud on average grows by about 1.1-1.5 percentage points every period.

One of the essential model components is the aggregate electoral uncertainty captured by parameter δ. The benchmark dynamic model assumes that every period δ is independently drawn from the same normal distribution. One can argue that non-partisan preferences over candidates could be time dependent. Finally, I allow for this by making δ follow a random walk instead of being independently drawn: $\delta_t \sim N(\delta_{t-1}, \psi^2)$. Again, the results of the time regressions demonstrate the significance of fraud growth.

Variable	Set 1	Set 2	Set 3
Time	0.01531***	0.01330***	0.01165***
	(0.00019)	(0.00018)	(0.00013)
Const	0.06175***	0.07278***	0.13356***
	(0.00072)	(0.00070)	(0.00049)
N	1000	1000	1000

Table 1: Time trend of optimal fraud for the benchmark model

Figure 2: Optimal fraud in the model with random walk for different parameter sets (17).



Variable	Set 1	Set 2	Set 3
Time	0.02632***	0.02521***	0.01611***
	(0.00023)	(0.00022)	(0.00016)
Const	0.05486***	0.06517***	0.13651***
	(0.00085)	(0.00086)	(0.00061)
N	1000	1000	1000

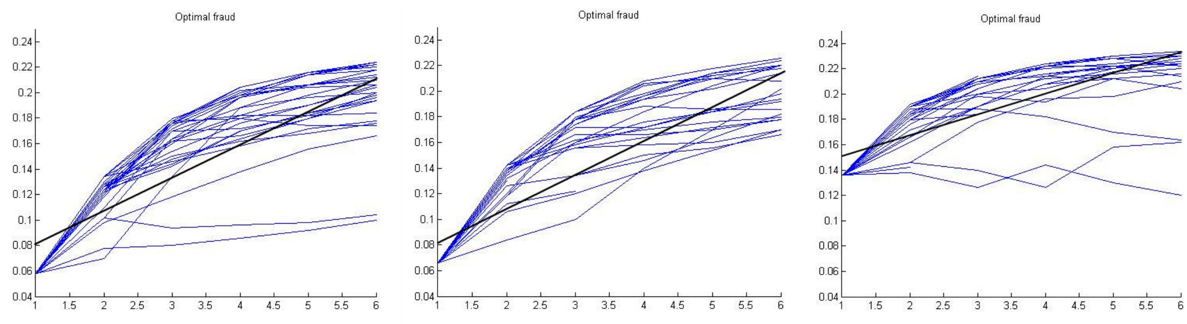
Table 2: Time trend of optimal fraud for the model with random walk

Indeed, random walk for aggregate uncertainty seems to be more realistic than independent draws. However, the growing fraud profile in this case could be a result of two effects: in addition to the effect of learning about fraud tolerance, there is also an effect of aggregate uncertainty. In contrast to the benchmark case where the aggregate uncertainty was constant over time, it is growing due to the random walk process. As a result, incentives to commit higher fraud over time are increased not only by more precise beliefs about β, but also by higher aggregate uncertainty about electoral support. Thus, for the benchmark case the assumption on independent draws for the aggregate uncertainty component allows for distinguishing two uncertainty time effects and, thus, seems more plausible for the purposes of the paper.

To summarize, the model simulation results in a consistent growing fraud profile which is robust to parameter choice and underlying law of motion for aggregate electoral uncertainty. When uncertainty is sufficiently large, incentives to commit fraud increase when an incumbent's uncertainty about β decreases. Over time an incumbent's beliefs about β becomes more precise because of learning and, thus, the deterrent role of uncertainty about fraud intolerance declines, implying that the incentives to commit fraud become stronger, leading to a growing profile.

An important observation is that fraud generally grows at a decreasing rate, which is a result of fast learning that mainly takes place in early periods. To clearly illustrate the speed of information gathering, in Figure 3, I represent the evolution of uncertainty for the three parameters sets already used above. One can notice that the standard deviation of subjective beliefs rapidly decreases in the few first periods.

Figure 3: Optimal fraud in the model with random walk for different parameter sets (17).



Finally, it is worth stressing that the model puts aside the effects of the growing stakes of re-election and decreasing costs of fraud because of learning-by-doing, focusing purely on the role of uncertainty in fraud dynamics. Obviously, being introduced into the model, these components would just magnify the results, making the model predictions even stronger.

5 Conclusion

This paper consists of two parts. The first part explores different available information on electoral fraud in post-Soviet and Sub-Saharan countries. All the explored sources, including the academic literature, electoral data and data on political freedom, provide consistent, though indirect, evidence of growing fraud: electoral manipulations tend to grow over the lifetime of a non-democratic regime.

The second part provides a theoretical model of electoral competition with falsifications designed in the traditional probabilistic voting framework which specifically studies the effect of uncertainty on the incumbent's incentives to commit fraud. The model explicitly distinguishes between two types of uncertainty that affect electoral outcomes: aggregate electoral uncertainty, captured by a purely random component in voters' utility over candidates, and subjective uncertainty about voters' fraud intolerance, represented by the variance of an incumbent's beliefs about the fraud intolerance component in voters' utility. The most important findings of the model are as follows.

First, when uncertainty (no matter of what type) is sufficiently large, incentives to commit fraud increase when an incumbent's uncertainty about fraud intolerance decreases. Second, because in the multi-period setup an incumbent's uncertainty decreases as a result of learning through Bayesian updating, increasing fraud is more likely to be observed. Third, optimal fraud demonstrates an increasing profile at a decreasing rate. This is explained by fast learning that mainly takes place in early periods: the incumbent quickly absorbs information about the true value of the intolerance component in voters' utility function. The predictions are robust to functional forms of the model components as well as choice of parameter values.

Appendix A

Country	Regime Years	Elections	Leaders
Armenia	1998-	1998, 2003, 2008	Kocharyan, Sargsyan
Azerbaijan	1993-	1993, 1998, 2003, 2008	H. Aliyev, I. Aliyev
Belarus	1994-	1994, 2001, 2006	Lukashenko
Georgia 1	1995-2003	1995, 2000	Shevardnadze
Georgia 2	2004-	2004, 2007	Saakashvili
Kazakhstan	1991-	1991, 1999, 2005	Nazarbaev
Kyrgyzstan	1990-2005	1991, 1995, 2000	Akayev
Russia	2000-	2000, 2004, 2008, 2012	Putin, Medvedev
Tajikistan	1992-	1995, 1999, 2006,	Rahmon
Ukraine	1994-2004	1994, 1999	Kuchma
Uzbekistan	1990-	1991, 2000, 2007	Kerimov

Table 3: Regimes. Post-Soviet Countries.

Country	Regime Years	Elections	Leaders
Burkina Faso	1987-	1991, 1998, 2005, 2010	Compaore
Cameroon	1982-	1992, 1997*, 2004, 2011	Biya
CAR 1	1993-2003	1993, 1999	Patasse
CAR 2	2003-	2005, 2011	Bozize
Chad	1990-	1996, 2001, 2006, 2011	Deby
Congo (Brazzaville)	1997-	2002, 2009	Sassou-Nguesso
Cote d'Ivoire	1960-1999	1990, 1995*	Houphouet-Boigny, Bedie
Djibouti	1977-	1993, 1999, 2005*, 2011	Aptidon, Guelleh
DR Congo (Kinshasa)	1997-	2006, 2010	Kabila, Kabange
Equatorial Guinea	1987-	1992, 2006, 2009	Mbasogo
Gabon	1987-	1991, 1998, 2005, 2010	O. Bongo, A. Bongo
the Gambia	1994-	1996, 2001, 2006, 2011	Jammeh
Ghana	1982-2000	1992, 1996, 2000	Rawlings
Guinea	1984-2008	1993, 1998, 2003	Conte
Kenya 1	1978-2002	1992, 1997, 2002	Moi, Kenyatta
Kenya 2	2002-	2002, 2007	Kibaki
Liberia	2005-	2005, 2011	Johnson-Sirleaf
Madagascar	2001-2009	2001, 2006	Ravalomanana
Malawi	1994-	1994, 1999, 2004, 2009	Muluzi, Mutharika
Mauritania	1984-2005	1992, 1997*, 2003	Taya
Mozambique	1986-	1994, 1999, 2004, 2009	Chissano, Guebuza
Niger	1999-2010	1999, 2004	Tandja
Nigeria	1987-	1999, 2003, 2007, 2011	Obasanjo, Yar'Adua
Rwanda	1994-	2003, 2010	Bizimungu, Kagame
Senegal 1	1960-2000	1983, 1988, 1993, 2000	Diouf
Senegal 2	2000-2012	2000, 2007, 2012	Wade
Seychelles	1977-	1993, 1998, 2001, 2006, 2011	Rene, Michel
Sierra-Leone	2007-	2007, 2012	Koroma
Sudan	1989-	1996, 2000, 2010	al-Bashir
Tanzania	1960-	1995, 2000, 2005, 2010	Nyerere, Mwinyi, Mkapa, Kikwete
Togo	1993-	1993*, 1998, 2003, 2005, 2010	Eyadema, Gnassingbe
Uganda	1986-	1996, 2001, 2006, 2011	Museveni
Zambia	1991-2011	1991, 1996, 2001, 2006 2008, 2011	Chiluba, Mwananwasa, Banda
Zimbabwe	1987-	1990, 1996*, 2002, 2008	Mugabe

*Elections were boycotted by opposition parties.

Table 4: Regimes. African Countries

Country	Political Rights	Press Freedom	Victory Margin	DPI
Armenia	0.161 (0.026)***	0.767 (0.092)***	7.6 (2.34)	+
Azerbaijan	0	0.495 (0.092)***	10.7 (7.16)	+
Belarus	0.124 (0.018)***	1.263 (0.192)***	24.7 (4.50)	no
Georgia 1	0.050 (0.066)	-2.351 (0.564)***	8.2	no
Georgia 2	0.083 (0.065)	-0.067 (0.560)	-66.3	no
Kazakhstan	0.023 (0.009)**	1.167 (0.044)***	12.3 (1.81)	+
Kyrgyzstan	0.158 (0.040)***	1.790 (0.260)***	5.6	no
Russia	0.099 (0.022)***	2.088 (0.151)***	6.3 (6.88)	no
Tajikistan	-0.033 (0.013)**	-1.232 (0.225)***	-20.9	no
Ukraine	0.145 (0.031)***	3.200 (0.306)***	11.4	+
Uzbekistan	0.022 (0.009)**	1.021 (0.124)***	-0.3	no

Table 5: Fraud Proxies Dynamics. Post-Soviet Countries

Country	Political Rights	Press Freedom	Victory Margin	DPI
Burkina Faso	0.006 (0.016)	0.189 (0.043)***	-4.8 (0.28)	+
Cameroon	-0.035 (0.015)**	-0.726 (0.126)***	31.7 (10.31)	+
CAR 1	0.309 (0.088)***	-0.752 (0.661)	16.6	no
CAR 2	0	0.321 (0.216)	13.7	no
Chad	0.081 (0.014)***	0.293 (0.110)**	13.7 (3.12)	–
Congo (Brazzaville)	0.082 (0.038)*	0.091 (0.162)	-15.5	no
Cote d'Ivoire	0	2.800 (0.901)**		+
Djibouti	0.023 (0.027)	0.917 (0.181)***	11.3 (1.15)	no
DR Congo (Kinshasa)	0.172 (0.062)**	0.429 (0.152)**	0.5	n/a
Equatorial Guinea	-0.053 (0.025)*	1.009 (0.085)***	-2.8 (0.06)	no
Gabon	0.083 (0.012)***	1.304 (0.156)***	-1.2 (12.54)	no
the Gambia	-0.103 (0.051)*	1.185 (0.193)***	12.3 (2.85)	–
Ghana	-0.383 (0.053)***	-0.075 (0.038)	-20.9 (6.18)	–
Guinea	0.022 (0.012)*	0.209 (0.231)	29.2 (17.20)	no
Kenya 1	0.127 (0.092)	2.103 (0.444)***	-20.6 (11.14)	+
Kenya 2	0.145 (0.031)***	-0.900 (0.250)***	-28.6	no
Liberia	0	-1.000 (0.244)***	62.6	no
Madagascar	0.366 (0.072)***	2.883 (0.506)***	27.5	no
Malawi	0.082 (0.029)***	1.153 (0.180)***	6.6 (5.36)	no
Mauritania	-0.099 (0.031)***	-1.000 (0.263)***	18.6	no
Mozambique	0.053 (0.013)***	-0.425 (0.173)**	4.7 (5.07)	no
Niger	-0.021 (0.073)	0.647 (0.438)	11.2	no
Nigeria	-0.114 (0.126)	-0.800 (0.400)	2.5 (6.31)	no
Rwanda	0 (0)	-0.050 (0.153)	-3.4	no
Senegal 1	0.039 (0.022)*	-2.702 (1.012)**	-43.4	–
Senegal 2	0.042 (0.044)	2.157 (0.229)***	-24.3 (27.88)	no
Seychelles	0	0.596 (0.138)***	-8.24 (8.10)	n/a
Sierra-Leone	-0.143 (0.082)	-2.300 (0.277)***	12.1	no
Sudan	0	-0.704 (0.127)***	-30.4	n/a
Tanzania	-0.104 (0.015)***	0.037 (0.056)	1.9 (8.98)	+
Togo	-0.066 (0.018)***	-0.040 (0.174)	2.5 (1.12)	no
Uganda	0.029 (0.030)	1.338 (0.111)***	-5.2 (6.12)	+
Zambia	-0.004 (0.036)	0.254 (0.169)	-14.6 (5.75)	+
Zimbabwe	0.089 (0.018)***	2.107 (0.338)***	5.1 (32.88)	no

Table 6: Fraud Proxies Dynamics. African Countries

Country	Question 1	Question 2	Question 3
Armenia	+	no	no
Azerbaijan	no	no	no
Belarus	+	+	+
Burkina Faso	–	–	no
Cameroon	no	–	–
CAR 1	–	+	no
Chad	+	no	no
Cote dIvoire	no	no	n/a
Djibouti	no	both	n/a
Equatorial Guinea	no	no	–
Gabon	–	no	n/a
Gambia	–	both	n/a
Georgia 1	+	no	+
Ghana	both	no	no
Guinea	no	no	n/a
Kazakhstan	no	no	no
Kenya 1	no	no	–
Kyrgyz Republic	no	+	+
Madagascar	–	+	–
Malawi	+	–	no
Mauritania	no	+	no
Mozambique	no	no	no
Niger	+	no	no
Nigeria	no	no	no
Russia	+	+	+
Senegal 1	no	no	no
Sudan	no	+	no
Tajikistan	no	+	no
Tanzania	both	no	no
Togo	no	–	both
Uganda	no	no	+
Ukraine	no	+	+
Uzbekistan	+	no	n/a
Zambia	no	no	no
Zambia	both	no	no
Zimbabwe	no	both	+

Table 7: NELDA Database

Appendix B

Proof of Proposition 1. The first-order condition to the maximization problem (10):

$$\frac{dG_X(0)}{df} = c'(f),$$

$$\frac{dG_X(0)}{df} = \frac{dG_X(0)}{ds^2}\frac{ds^2}{df} = -\frac{2\psi^2}{f^3}\frac{dG_X(0)}{ds^2},$$

$$\frac{dG_X(0)}{ds^2} = \frac{1}{2\sqrt{2\pi}s^3}\int_{-\infty}^{0} e^{-\frac{(x-\mu)^2}{2s^2}}\,\mathrm{d}x + \frac{1}{2\sqrt{2\pi s^2}}\int_{-\infty}^{0}\frac{d}{ds}\left(e^{-\frac{(x-\mu)^2}{2s^2}}\right)\mathrm{d}x = -\frac{1}{2s^2}G_X(0) + I, \quad (18)$$

$$I = \frac{1}{2\sqrt{2\pi s^2}}\int_{-\infty}^{0}\frac{d}{ds}\left(e^{-\frac{(x-\mu)^2}{2s^2}}\right)\mathrm{d}x = \frac{1}{2\sqrt{2\pi s^2}s^2}\int_{-\infty}^{0}\frac{(x-\mu)^2}{2s^2}e^{-\frac{(x-\mu)^2}{2s^2}}\,\mathrm{d}x.$$

Using a substitution $y = \frac{x-\mu}{\sqrt{2s^2}}$:

$$I = \frac{1}{\sqrt{\pi}s^2}\int_{-\infty}^{-\mu/\sqrt{2s^2}} y^2 e^{-y^2}\,\mathrm{d}y. \quad (19)$$

Integrating by parts with substitution $U = z$ and $dV = y^2e^{-y^2}dz$:

$$\int_{-\infty}^{a} y^2 e^{-y^2}\,\mathrm{d}y = -\frac{1}{2}ae^{-a^2} + \frac{1}{2}\int_{-\infty}^{a} e^{-y^2}\,\mathrm{d}y.$$

Applying this result to the integral (19):

$$I = \frac{1}{\sqrt{\pi}s^2}\int_{-\infty}^{-\mu/\sqrt{2s^2}} y^2 e^{-y^2}\,\mathrm{d}y = \frac{1}{\sqrt{\pi}s^2}\left(\frac{1}{2}\frac{\mu}{\sqrt{2s^2}}e^{-\frac{\mu^2}{2s^2}} + \frac{1}{2}\int_{-\infty}^{\mu/\sqrt{2s^2}} e^{-y^2}\,\mathrm{d}y\right).$$

Going back to original notation:

$$I = \frac{1}{2s^2}\left(\frac{\mu}{\sqrt{2\pi s^2}}e^{-\frac{\mu^2}{2s^2}} + G_X(0)\right) = \frac{1}{2s^2}\left(\mu g_X(0) + G_X(0)\right).$$

Plugging the last expression to (18):

$$\frac{dG_X(0)}{ds^2} = -\frac{1}{2s^2}G_X(0) + I = -\frac{1}{2s^2}G_X(0) + \frac{1}{2s^2}\left(\mu g_X(0) + G_X(0)\right) = \frac{\mu}{2s^2}g_X(0).$$

Combining all the results:

$$\frac{dG_X(0)}{df} = -\frac{2\psi^2}{f^3}\frac{dG_X(0)}{ds^2} = -\frac{2\psi^2}{f^3}\frac{\mu}{2s^2}g_X(0) = -\frac{\psi^2}{f^3}\frac{\mu}{s^2}g_X(0).\square$$

Proof of Proposition 2. The incumbent's expected share of votes $\Pi_I = \frac{1}{2} + (1 - \phi\beta_1)f$ is increasing in f if and only if $\beta_1 - 1/\phi < 0$. Otherwise, it is always optimal to choose zero fraud. The left hand side of the first-order condition (11) $L(f) = -\frac{2\psi^2}{f^3}\frac{2s^2}{g_X(0)}$ is a strictly decreasing function of fraud whenever the first part of the proposition is satisfied. Because marginal cost $c'(f)$ is an increasing function of fraud, and $L(0) = \frac{1/\phi - \beta_1}{\psi\sqrt{2\pi}} > 0 = c'(0)$, there is a unique intersection between $L(f)$ and $c'(f)$. To show that the intersection point is between 0 and 1, it is sufficient to show that $L(1) < c'(1)$:

$$L(1) = \frac{\psi^2(1/\phi - \beta_1)}{\epsilon_1^2 + \psi^2} \frac{1}{sqrt 2\pi(\epsilon_1^2 + \psi^2)} e^{-\frac{1/(\phi-\beta_1)^2}{\epsilon_1^2+\psi^2}} < \frac{1/\phi - \beta_1}{\sqrt{2\pi(\epsilon_1^2 + \psi^2)}}.$$

Thus, to have optimal fraud less than 1, it is sufficient to have $\frac{1/\phi-\beta_1}{\sqrt{2\pi(\epsilon_1^2+\psi^2)}} < c'(1)$. □

Proof of Proposition 3. Consider the first-order condition (11) and denote $L = -\frac{2\psi^2}{f^3}\frac{2s^2}{g_X(0)} - c'(f)$. Denote the solution for the first-order condition (11) as f^* and use the implicit function theorem:

$$\frac{\partial f^*}{\partial \epsilon_0^2} = -\frac{\partial L/\partial \epsilon_0^2}{\partial L/\partial f^*}.$$

Since $\mu < 0$

$$\frac{\partial L}{\partial f^*} = \frac{\psi^2 \mu g_X(0)}{({f^*}^2\epsilon_0^2 + \psi^2)^2}\left(\frac{\psi^2\mu^2}{f^2\epsilon_0^2 + \psi^2} + 3\epsilon_0^2\right) - c''(f) < 0.$$

Further

$$\frac{\partial L}{\partial \epsilon_0^2} = -\frac{\psi^2 \mu g_X(0) f^*}{2({f^*}^2\epsilon_0^2 + \psi^2)^2}\left(\frac{\mu^2}{s^2} - 3\right).$$

The latter expression is negative if and only if $\frac{\mu}{s^2} < 3$. Because $\frac{\mu^2}{s^2} = \frac{\mu^2}{\epsilon_0^2+\psi^2/{f^*}^2} < \frac{\mu^2}{\epsilon_0^2+\psi^2}$ to guarantee $\frac{\partial L}{\partial \epsilon_0^2} < 0$ it is sufficient to have $\frac{\mu^2}{\epsilon_0^2+\psi^2} < 3$ or $\epsilon_0^2 + \psi^2 > \mu^2/3$, which in turns guarantee $\frac{\partial f^*}{\partial \epsilon_0^2} < 0$. □

Proof of Proposition 4. Consider the first-order condition (11) and denote $L = -\frac{2\psi^2}{f^3}\frac{2s^2}{g_X(0)} - c'(f)$. Denote the solution for the first-order condition (11) as f^* and use the implicit function theorem:

$$\frac{\partial f^*}{\partial \epsilon_0^2} = -\frac{\partial L/\partial \epsilon_0^2}{\partial L/\partial f^*}.$$

Since $\mu < 0$

$$\frac{\partial L}{\partial f^*} = \frac{\psi^2 \mu g_X(0)}{({f^*}^2 \epsilon_0^2 + \psi^2)^2} \left(\frac{\psi^2 \mu^2}{f^2 \epsilon_0^2 + \psi^2} + 3\epsilon_0^2 \right) - c''(f) < 0.$$

Thus, to prove the proposition it is enough to show that $\frac{\partial L}{\partial \mu} < 0$.

$$\frac{\partial L}{\partial \mu} = -\frac{\psi g_X(0)}{f^*({f^*}^2 \epsilon_0^2 + \psi^2)} \left(1 - \frac{\mu^2}{s^2} \right).$$

The latter expression is negative whenever $1 - \frac{\mu^2}{s^2} < 0$ or $\epsilon_0^2 + \psi^2 > \mu^2$.$\square$

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